

# Inverse Dominating Set in Neutrosophic Graphs

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## Abstract

In this paper, the concept of inverse domination in neutrosophic graph is established. The definition of inverse domination number, inverse dominating set, inverse split and non split dominating sets in neutrosophic graph are developed with suitable examples here. Also, the theorems in inverse domination in neutrosophic graph and the bound on inverse domination number in neutrosophic graph are derived.

**Keywords:** Neutrosophic graph, Inverse dominating set, Inverse dominating number, Neutrosophic path, Minimum dominating set.

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## 1 Introduction

The first definition of fuzzy graph was proposed by Kaufmann [11], from the fuzzy relations introduced by Zadeh [29]. Rosenfeld [20] introduced another elaborated definition including fuzzy vertex and fuzzy edge and several fuzzy analogs of graph theoretic concepts such as paths, cycles, fuzzy connectedness etc.. Some other concepts such as fuzzy automorphic graphs, neighborhood set in fuzzy graphs, fuzzy intersection graphs, fuzzy line graphs and algorithmic aspects of dominations in graphs were introduced in [4,5,13,14,15] respectively. The concept of domination in fuzzy graphs was investigated by A. Somasundaram and S. Somasundaram in [26,27] and A. Somasundaram presented the concepts of independent domination, total domination, connect domination of fuzzy graphs. Also, inverse domination, inverse split and non split domination in graph and fuzzy graphs were studied in [6,7,8,10,12]. The first definition of intuitionistic fuzzy graph was proposed by Atanssov[2,3] and Nagoorani et al.[18]. Domination in intuitionistic fuzzy graphs was investigated by R. Parvathi and G. Thamizhendhi[19]. Neutrosophic set proposed by Smarandache [22] is a powerful tool for dealing incomplete and indeterminate problems in the real world which is the generalization of fuzzy sets and intuitionistic fuzzy sets. Fuzzy graph and intuitionistic approaches are failed in some applications when indeterminacy occurs. So Smarandache defined four main categories of neutrosophic graphs in[23,24,25,28]. M.Mullai introduced the concept of domination in neutrosophic graphs[17]. In this paper, inverse domination in neutrosophic graph is developed with suitable examples and some theorems are explored.

## 2 Preliminaries

In this section, the basic definitions involving domination in fuzzy, intuitionistic fuzzy and neutrosophic graphs are outlined.

**DEFINITION 2.1 [16].** Let  $V$  be a finite nonempty set. Let  $E$  be the collection of all two- element subsets of  $V$ . A fuzzy graph  $G = (\sigma, \mu)$  is a set with two functions  $\sigma : V \rightarrow [0, 1]$  and  $\mu : E \rightarrow [0, 1]$  such that,  $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$  for all  $x, y \in V$ . We write  $\mu(xy)$

**DEFINITION 2.2 [9]** Let  $G = (\sigma, \mu)$  be a fuzzy graph on  $V$  and  $S$  be the subset of  $V$ . The fuzzy cardinality of  $S$  is defined to be  $\sum_{v \in S} \sigma(v)$

**DEFINITION 2.3 [10].** A fuzzy graph  $G$  is said to be complete if  $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ , for all  $u, v \in V$ .

**DEFINITION 2.4 [9].** The domination number of a fuzzy graph  $G$  is the minimum cardinality taken over all dominating sets in  $G$  and is denoted by  $\gamma(G)$  or simply  $\gamma$ .

**DEFINITION 2.5 [9].** Let  $G = (\sigma, \mu)$  be a fuzzy graph on  $V$ . A subset  $S$  of  $V$  is said to be an independent set if  $\mu(uv) < \sigma(u) \wedge \sigma(v)$  for all  $u, v \in S$ . The maximum fuzzy cardinality taken over all independent sets in  $G$  is called the independence number of  $G$  and is denoted by  $\beta_0(G)$ .

**DEFINITION 2.6 [10].** The complement of a fuzzy graph  $G$  denoted by  $\bar{G}$  is defined to be  $\bar{G} = (\sigma, \bar{\mu})$ , where  $\bar{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$ .

**DEFINITION 2.7 [18].** An intuitionistic fuzzy graph (IFG) is of the form  $G = (V, E)$ , where  $V = \{v_1, v_2, v_3, \dots, v_n\}$  such that  
i)  $\mu_1 : V \rightarrow [0, 1]$ ,  $\gamma_1 : V \rightarrow [0, 1]$  denote the degree of membership and non membership of the element  $v_i \in V$  and  $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ , for every  $v_i \in V$ , ( $i = 1, 2, \dots, n$ ).  
ii)  $E \subseteq V \times V$ , where  $\mu_2 : V \times V \rightarrow [0, 1]$  and  $\gamma_2 : V \times V \rightarrow [0, 1]$  are such that  $\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$ ,  $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \wedge \gamma_1(v_j)$  and  $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ .

**DEFINITION 2.8 [10].** An arc  $(v_i, v_j)$  of an intuitionistic fuzzy graph  $G$  is called an strong arc if,  $\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$  and  $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \wedge \gamma_1(v_j)$ .

**DEFINITION 2.9 [9].** Let  $G = (V, E)$  be an intuitionistic fuzzy graph. Then, the cardinality of  $G$  is defined to be  
 $|G| = \left\{ \sum_{v_i \in V} \left[ \frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] + \sum_{v_i \in V} \left[ \frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right] \right\}$

**DEFINITION 2.10 [9].** Let  $G = (V, E)$  be an IFG. The vertex cardinality of  $G$  is defined by  $|V| = \left\{ \sum_{v_i \in V} \left[ \frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] \right\}$  for all  $v_i \in V$ .  
The edge cardinality of  $G$  is defined by  $|E| = \left\{ \sum_{v_i \in V} \left[ \frac{1 + \mu_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2} \right] \right\}$  for all  $(v_i, v_j) \in E$ .

The vertex cardinality of an intuitionistic fuzzy graph is called the order of  $G$  and it is denoted by  $O(G)$ . The cardinality of the edges in  $G$  is called the size of  $G$  and is denoted by  $S(G)$ .

**DEFINITION 2.11 [21].** Let  $X$  be a space of points (object) with generic elements in  $X$  denoted by  $x$ . Then the neutrosophic set  $A(NSA)$  is an object having the form

$$A = \{ \langle x : \mu_A(x), \gamma_A(x), \delta_A(x) \rangle, x \in X \},$$

where the functions  $\mu, \gamma, \delta : X \rightarrow ]0^-, 1^+[$  define respectively the truth membership function, an indeterminacy-membership function, and a falsity-membership function of the element  $x \in X$  to the set  $A$  with the condition

$$0^- \leq \mu_A(x) + \gamma_A(x) + \delta_A(x) \leq 3^+$$

The functions  $\mu_A(x)$ ,  $\gamma_A(x)$ , and  $\delta_A(x)$  are real standard or nonstandard subsets of  $]0^-, 1^+[$

**DEFINITION 2.12 [21].** Let  $X$  be a space of points (object) with generic elements in  $X$  denoted by  $x$ . A single valued neutrosophic set  $A(SVNS A)$  is characterized by truth-membership function  $\mu_A(X)$ , an indeterminacy-membership function  $\gamma_A(X)$ , and a falsity-membership function  $\delta_A(X)$ . For each point  $x$  in  $X$ ,  $\mu_A(x)$ ,  $\gamma_A(x)$  and  $\delta_A(x) \in [0, 1]$ . A SVNS  $A$  can be written as

$$A = \{ \langle x : \mu_A(x), \gamma_A(x), \delta_A(x) \rangle, x \in X \}.$$

**DEFINITION 2.13 [21].** A single valued neutrosophic graph (SVN-graph) with underlying set  $V$  is defined to be a pair  $G = (A, B)$  where,

(i) The functions  $\mu_A : V \rightarrow [0, 1]$ ,  $\gamma_A : V \rightarrow [0, 1]$  and  $\delta_A : V \rightarrow [0, 1]$  denote the degree of truth-membership, degree of indeterminacy-membership, and degree of falsity-membership of the element  $v_i \in V$ , respectively and

$$0 \leq \mu_A(v_i) + \gamma_A(v_i) + \delta_A(v_i) \leq 3, \forall v_i \in V (i = 1, 2, 3, \dots, n)$$

(ii) The functions  $\mu_B : E \subseteq V \times V \rightarrow [0, 1]$ ,  $\gamma_B : E \subseteq V \times V \rightarrow [0, 1]$  and  $\delta_B : E \subseteq V \times V \rightarrow [0, 1]$  are defined by  $\mu_B(\{v_i, v_j\}) \leq \min(\mu_A(v_i), \mu_A(v_j))$ ,  $\gamma_B(\{v_i, v_j\}) \geq \max(\gamma_A(v_i), \gamma_A(v_j))$  and  $\delta_B(\{v_i, v_j\}) \geq \max(\delta_A(v_i), \delta_A(v_j))$ , denote the degree of truth-membership, degree of indeterminacy-membership, and degree of falsity-membership of the edge  $(v_i, v_j) \in E$  respectively, where  $0 \leq \mu_B(\{v_i, v_j\}) + \gamma_B(\{v_i, v_j\}) + \delta_B(\{v_i, v_j\}) \leq 3$ , for all  $\{v_i, v_j\} \in E$ ,  $(i, j = 1, 2, 3, \dots, n)$

**DEFINITION 2.14 [10].** An arc  $(v_i, v_j)$  of a neutrosophic graph  $G$  is called an strong arc if  $\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\}$ ,  $\gamma_2(v_i, v_j) \leq \max\{\gamma_1(v_i), \gamma_1(v_j)\}$ ,  $\delta_2(v_i, v_j) \leq \max\{\delta_1(v_i), \delta_1(v_j)\}$ .

### 3 Inverse Dominating Set in Neutrosophic Graphs

This section includes the definition of vertex and edge cardinalities, inverse dominating set, inverse dominating number, inverse split and non split dominating sets, thoerems and results in a neutrosophic graph  $G$ .

**DEFINITION 3.1** Let  $G = (V, E)$  be an neutrosophic graph. Then, the cardinality of  $G$  is defined to be  $|G| = \{ \sum_{v_i \in V} [\frac{1+\mu_1(v_i)-\gamma_1(v_i)-\delta_1(v_i)}{3}] + \sum_{v_i \in V} [\frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)-\delta_2(v_i, v_j)}{3}] \}$

**DEFINITION 3.2** Let  $G = (V, E)$  be an neutrosophic graph. The vertex cardinality of  $G$  is defined by  $|V| = \{ \sum_{v_i \in V} [\frac{1+\mu_1(v_i)-\gamma_1(v_i)-\delta_1(v_i)}{3}] \}$  for all  $v_i \in V$ .

The edge cardinality of  $G$  is defined by  $|E| = \{ \sum_{v_i \in V} [\frac{1+\mu_2(v_i, v_j)-\gamma_2(v_i, v_j)-\delta_2(v_i, v_j)}{3}] \}$  for all  $(v_i, v_j) \in E$ .

The vertex cardinality of an neutrosophic graph is called the order of  $G$  and it is denoted by  $O(G)$ .

The cardinality of the edges in  $G$  is called the size of  $G$  and is denoted by  $S(G)$ .

**DEFINITION 3.3** Let  $D^N$  be a minimum dominating set of a neutrosophic graph  $G$ . If  $V - D^N$  contains a dominating set  $D'^N$  of  $G$ , then  $D'^N$  is called an **inverse dominating set of  $G$**  with respect to  $D^N$ .

The minimum cardinality taken over all inverse dominating sets of  $G$  is called the **inverse dominating number of neutrosophic graph  $G$**  and is denoted by  $\gamma^N(G)$

**THEOREM 3.4** For any neutrosophic graph  $G$  with  $\gamma^N$ , set  $D^N$ ,  $\gamma_G^N + \gamma'^N(G) \leq P^N$ . Also equality holds if  $V - D^N$  is independent and contains inverse dominating set  $D'^N$  with respect to  $D^N$ .

*Proof:*

Let  $D^N$  be a  $\gamma^N$  set of  $G$ .

If  $D'^N$  is an inverse dominating set of  $G$  with respect to  $D^N$ , then  $D'^N \subseteq V - D^N$ .

Therefore,  $|D'^N| \leq |V - D^N|$

Hence,  $\gamma'^N(G) \leq P^N - \gamma^N(G)$ , since  $V - D^N$  is independent and contains an inverse dominating set  $D'^N$  with respect to  $D^N$ .

Therefore,  $V - D^N$  itself is an inverse dominating set of the neutrosophic graph  $G$ .

Hence the proof.

**COROLLARY 3.5** If  $G$  or  $\bar{G}$  contains at least one isolated vertex, then,  $\gamma^N(G) + \gamma^N(\bar{G}) \leq P^N$ .

**THEOREM 3.6** For any neutrosophic graph  $G$  with at least one isolated vertex,  $\gamma^N(G) = 0$ .

*Proof:*

Let  $D^N$  be a  $\gamma^N$  set of  $G$  and  $u \in S$  be an isolated vertex.

Then,  $\mu(uv) = \min\{\sigma(u), \sigma(v)\}$ , for all  $v \in V - D^N$

Hence,  $\gamma^N(G) = 0$ .

**THEOREM 3.7** For any neutrosophic graph  $G$ ,  $\gamma^N(G) \leq \Gamma^N(G)$

*Proof:*

Let  $D^N$  be a  $\gamma^N$  set of  $G$ .

We prove the following three cases:

**Case:(i)**  $V - D^N$  contains no dominating set.

Then,  $\gamma^N(G) = \Gamma^N(G)$  and  $\gamma'^N(G) = 0$ .

**Case:(ii)**  $V - D^N$  contains only one dominating set.

This implies that,  $\gamma'^N(G) = \Gamma^N(G)$

**Case:(iii)**  $V - D^N$  contains at least two dominating set.

Then, minimum dominating set in  $V - D^N$  with minimum fuzzy cardinality is  $\gamma'^N(G)$ .

Hence,  $\gamma'^N(G) \leq \Gamma^N(G)$

**THEOREM 3.8** Let  $p^N$  be a path in a neutrosophic graph  $G$ . Then  $\gamma'^N(P^N) = \Gamma^N(P^N)$

*Proof:*

Since  $P^N$  contains only two dominating sets in  $G$ , the proof follows.

**THEOREM 3.9** For any neutrosophic graph  $G$ ,  $\gamma'^N(G) \leq \beta_0^N(G)$ .

*Proof:*

Let  $D^N$  be a  $\gamma^N$  set of  $G$  and  $S^N$  be a maximal independent set of  $\langle V - D^N \rangle$ .

Then, every vertex in  $V - D^N - S^N$  is adjacent to at least one vertex in  $S^N$ .

If every vertex in  $D^N$  is adjacent to at least one vertex in  $S^N$ , then  $S^N$  is an inverse dominating set.

Otherwise, let  $D'^N \subseteq D^N$  be a set of vertices in  $D^N$  such that no vertex in  $D'^N$  is adjacent the vertices of  $S^N$ .

Since  $D^N$  is a minimum dominating set, every vertex in  $D'^N$  must be adjacent to vertex in  $V - D^N - S^N$ .

Let  $S'^N \subseteq V - D^N - S^N$  such that every vertex of  $D'^N$  is adjacent to at least one vertex in  $S'^N$ .

Then, there exists at least one vertex  $v \in S'^N$  such that both  $N(v) \cap S^N \neq \emptyset$  and  $N(v) \cup S^N \neq \emptyset$ .

Therefore,  $(S^N \cup S'^N - (N(v) \cap S^N))$  is an inverse dominating set of  $G$  and

$$|(S^N \cup S'^N - (N(v) \cap S^N))| \leq \beta_0^N(G)$$

Hence,  $\gamma'^N(G) \leq \beta_0^N(G)$ .

**THEOREM 3.10** For any neutrosophic graph  $G$  with at least one inverse dominating set,

$$\gamma^N(G) \leq \frac{(P^N + \gamma'^N(G))}{3}$$

*Proof:*

For any neutrosophic graph  $G$  with at least one inverse dominating set,  $\gamma^N(G) \leq \gamma'^N(G)$

$$\text{Also, } \gamma^N \leq \frac{P^N}{2}$$

Hence the result.

**THEOREM 3.11** For any neutrosophic graph  $G = (\sigma, \mu)$ ,  $\gamma'^N(G) < P^N$

*Proof:*

We know that, any neutrosophic graph contains at least one  $\gamma^N$  - set with  $\gamma^N(G) > 0$

$$\gamma^N(G) + \gamma'^N(G) \leq P^N .$$

Thus,  $\gamma'^N(G) < P^N$ .

**THEOREM 3.12** An inverse dominating set  $D^N$  of  $G$  is a minimal inverse dominating set iff for each  $d \in D^N$ , one of the following conditions hold:

(i).  $N(d) \cap D^N = \emptyset$

(ii). There is a vertex  $c \in V - D^N$  such that,  $N(c) \cap D^N = \{d\}$

*Proof:*

Let  $D^N$  be a minimal inverse dominating set and  $d \in D^N$ .

Then,  $D_d^N = D^N - d$  is not a dominating set and there exists  $x \in V - D_d^N$  such that  $x$  is not dominated by any element of  $D_d^N$

If  $x = d$  we get condition (i) and  $x \neq d$  we get condition (ii).

Condition (ii) is obvious.

**THEOREM 3.13** If every non end vertex of an neutrosophic tree  $T^N$  is adjacent to at least one end vertex, then,  $\gamma^N(T^N) + \gamma'^N(T^N) = P^N$

*Proof:*

Suppose every non end vertex of an neutrosophic tree  $T^N$  is adjacent to at least two end vertices.

Then, the set of non end vertices  $D^N$  is the only minimum dominating set in neutrosophic tree  $T^N$  and the set of end vertices  $V - D^N$  is the corresponding inverse dominating set in  $T^N$ .

Thus,  $\gamma^N(T^N) + \gamma'^N(T^N) = |D^N| = |V - D^N| = P^N$ .

Suppose, there are non end vertices which are adjacent to exactly one end vertex.

Let  $D^N$  and  $D'^N$  denote the minimum dominating and inverse dominating sets and  $u$  be a non end vertex adjacent to exactly one end vertex.

Clearly, if  $u \in D^N, v \in D^N$  and  $u \in D'^N, v \in D'^N$ .

In any case,  $D^N + D'^N = P^N$

Thus,  $\gamma^N(T^N) + \gamma'^N(T^N) = P^N$ .

### 3.1 Inverse Split and Non-Split Dominating Sest in Neutrosophic Graphs

DEFINITION 3.14 Let  $D'^N$  be a minimum inverse dominating set of neutrosophic graph of  $G$  with respect to  $D^N$ . Then,  $D'^N$  an **inverse split dominating set of  $G$** , if the induced subgraph  $\langle V - D'^N \rangle$  is disconnected. The **inverse split domination number** is denoted by  $\gamma_s^N(G)$  and it is the minimum cardinality taken over all minimal inverse split dominating set of  $G$ .

EXAMPLE 3.15 In figure 1,

$\sigma(v_1) = (0.3, 0.5, 0.7); \sigma(v_2) = (0.5, 0.6, 0.9); \sigma(v_3) = (0.2, 0.4, 0.6);$

$\sigma(v_4) = (0.3, 0.5, 0.8); \sigma(v_5) = (0.2, 0.4, 0.6); \sigma(v_6) = (0.2, 0.3, 0.5);$

Here,  $D^N = \{v_2, v_5\}$  and  $D'^N = \{v_3, v_4\}$

Hence,  $\langle V - D'^N \rangle$  is disconnected.

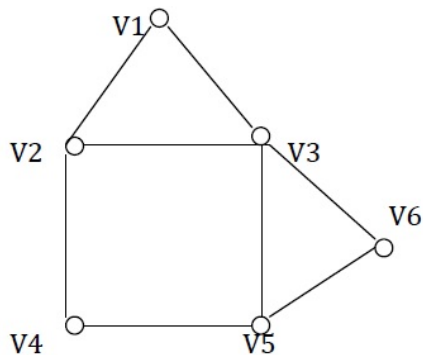


Figure 1: Inverse split dominating set in a neutrosophic graph

DEFINITION 3.16 Let  $D'^N$  be a minimum inverse dominating set of neutrosophic graph  $G$  with respect to  $D^N$ . Then,  $D'^N$  an **inverse non-split dominating set of  $G$** , if the induced subgraph  $\langle V - D'^N \rangle$  is connected. The **inverse non split domination number** is denoted by  $\gamma'_{ns}{}^N(G)$  and it is minimum cardinality non split dominating set of  $G$ .

RESULT 3.17 For any complete neutrosophic graph  $K_n^N$  with  $n \geq 2$  vertices,  $\gamma_s^N K_n^N = 0, \gamma'_{ns}{}^N K_n^N \leq 1$

THEOREM 3.18 For any neutrosophic graph  $G, \gamma^N(G) \leq \gamma_s^N(G)$  and  $\gamma^N(G) \leq \gamma'_{ns}{}^N(G)$ .

Proof:

Since every inverse split dominating set of  $G$  is an inverse dominating set of  $G, \gamma^N(G) \leq \gamma_s^N(G)$ .

Similarly,

$\gamma^N(G) \leq \gamma'_{ns}{}^N(G)$ , since every inverse non split dominating set of  $G$  is an inverse dominating set of  $G$ .

THEOREM 3.19 For any neutrosophic graph  $G, \gamma^N(G) \leq \min\{\gamma_s^N(G), \gamma'_{ns}{}^N(G)\}$ .

Proof:

Since every inverse split dominating set is a non split dominating set of  $G$ , we have,

$\gamma^N(G) \leq \gamma_s^N(G)$  and  $\gamma^N(G) \leq \gamma'_{ns}{}^N(G)$ .

Hence,  $\gamma^N(G) \leq \min\{\gamma_s^N(G), \gamma'_{ns}{}^N(G)\}$ .

**THEOREM 3.20** Let  $T^N$  be a neutrosophic tree such that any two adjacent cut vertices  $u$  and  $v$  with at least one of  $u$  and  $v$  is adjacent to an end vertex then,  $\gamma'^N(T^N) = \gamma'_s(T^N)$ .

*Proof:*

Let  $D'^N$  be a  $\gamma'^N$  set of  $T^N$ , then, we consider the following two cases.

**Case:(i).** Suppose that at least one of  $u, v \in D'^N$ , then  $\langle V - D'^N \rangle$  is disconnected with at least one vertex.

Hence,  $D'^N$  is a  $\gamma'_s(T^N)$  set of  $T^N$ .

Thus the theorem is true.

**Case:(ii).** Suppose  $u, v \in V - D'^N$ . Then there exists an end vertex  $w$  adjacent to either  $u$  or  $v$  say  $u$ .

This implies that  $w \in D'^N$ .

Thus, it follows that  $D'^N = \{w\} \cup \{v\}$  is of  $\gamma'^N$ - set of  $T^N$ .

Hence, by case (i) the theorem is true.

**THEOREM 3.21** For any neutrosophic tree in  $G$ ,  $\gamma'_{ns}(T^N) \leq n - p$ , where  $p$  is the number of vertices adjacent to end vertices.

**THEOREM 3.22** For any neutrosophic graph  $G$ ,  $\gamma'_{ns}(G)n - \delta^N(G)$ , where  $\delta^N(G)$  is the minimum degree among the vertices of  $G$

**THEOREM 3.23** Let  $G$  be a graph which is not a cycle with at least 5 vertices and let  $H$  be a connected spanning subgraph of  $G$ . Then,

i)  $\gamma'_s(G) \leq \gamma'_s(H)$

ii)  $\gamma'_{ns}(G) \leq \gamma'_{ns}(H)$

*Proof:*

Since  $G$  is connected, then any spanning tree  $T^N$  of  $G$  is minimally connected subgraph  $G$  such that,

$$\gamma'_s(G) \leq \gamma'_s(T^N) \leq \gamma'_s(H)$$

$$\text{Similarly, } \gamma'_{ns}(G) \leq \gamma'_{ns}(T^N) \leq \gamma'_{ns}(H)$$

Hence the proof.

**THEOREM 3.24** If  $T^N$  is a neutrosophic tree which is not a star then,

$$\gamma'_{ns}(T^N) \leq n - 2 \text{ for all } n \geq 3.$$

*Proof:*

Since  $T^N$  is not a star, there exists two adjacent cut vertices  $u$  and  $v$  with degree  $u$  and degree  $v \geq 2$ .

This implies that  $V - \{u, v\}$  is an inverse non split dominating set of  $T^N$ .

Hence the theorem.

**THEOREM 3.25** An inverse non split dominating set  $D'^N$  of neutrosophic graph  $G$  is minimal iff for each vertex  $v \in D'^N$  one of the following conditions is satisfied.

i) There exists a vertex  $u \in V - D'^N$  such that,  $N(u) \cap D'^N = \{v\}$

ii)  $v$  is not an isolated vertex in  $\langle D'^N \rangle$

iii)  $u$  is not an isolated vertex in  $\langle V - D'^N \rangle$

*Proof:*

Suppose  $D'^N$  is minimal inverse non split dominating set of  $G$ .

Suppose not (ie), if there exists a vertex  $v \in D'^N$  such that  $v$  does not satisfy any of the given conditions,

then, there exists an inverse dominating set  $D''^N = D'^N - \{v\}$  such that the induced subgraph  $\langle V - D''^N \rangle$  is connected.

This implies that,  $D''^N$  is an inverse non split dominating set of  $G$  contradicting the minimality of  $D'^N$ .

Therefore, the conditions is necessary.

Sufficient follows from the given conditions.

## 4 Conclusion

Neutrosophic set is the generalization of fuzzy set and intuitionistic fuzzy sets. Neutrosophic models in real world applications are flexible and compatibility than fuzzy and intuitionistic fuzzy models. Whenever indeterminacy occurs in some applications, fuzzy graph and intuitionistic graph approaches are failed. In such situations neutrosophic graph gives best results. In this paper, the definition of inverse dominating set, inverse split and non split dominating sets in neutrosophic graph  $G$  are defined with suitable examples and some theorems in inverse domination in neutrosophic graph are developed. Also the bound on inverse

domination number related to the above concepts are studied. In future, the concept of inverse domination in neutrosophic graphs will be extended and applied to real life problems.

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### Conflicts of Interest

Authors declare that they have no conflict of interest.

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