New Operators Using Neutrosophic δ-Open Set

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Abstract

In this paper, we introduce some new operators called neutrosophic δ frontier, neutrosophic δ border and neutrosophic δ exterior with the help of neutrosophic δ-open sets in neutrosophic topological space. Also, we discuss the important properties of them and the relations between them.

Keywords: neutrosophic open set, neutrosophic δ-open set, neutrosophic δ frontier, neutrosophic δ border, neutrosophic δ exterior

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1 Introduction

In 1965, the idea of fuzzy set (briefly, fs) gives a degree of membership function was first introduced by Zadeh.[19] In 1968, the concept of fuzzy topological space (briefly, fts) was introduced by Chang.[4] In 1983, the next stage of fuzzy set was developed by Atanassov[3] which gives a degree of membership and a degree of non-membership functions named as intutionistic fuzzy set (briefly, Ifs). In 1997, Coker[5] introduced the concept of intutionistic fuzzy topological space (briefly, $Ifts$) in intutionistic fuzzy set. In 2005, the concept of neutrosophic crisp set and neutrosophic set (briefly, $N_s s$) was investigated by Smaradache.[9,14,15] After the introduction of neutrosophic set, there are many fields of mathematics and various applications.[1,7,8,13] In 2012, Salama and Alblowi[10] defined neutrosophic topological space (briefly, $N_s ts$) and many of its applications in.[11,12] The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama et al.[12] in 2014. Saha[16] defined δ-open sets in topological spaces. Vadivel et al. in[17,18] introduced δ-open sets and their maps in a neutrosophic topological space. P. Iswarya and K. Bageerathi,[6] studied neutrosophic frontier and semi-frontier in neutrosophic topological spaces.

In this paper we introduce neutrosophic δ frontier, neutrosophic δ border and neutrosophic δ exterior and discuss their properties in $N_s ts$'s.

2 Preliminaries

Definition 2.1. [10] Let Y be a non-empty set. A neutrosophic set (briefly, $N_s s$) L is an object having the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\sigma_L \to [0,1]$ denote the degree of indeterminacy function and $\nu_L \to [0,1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set L and $0 \leq \mu_L(y) + \sigma_L(y) + \nu_L(y) \leq 3$ for each $y \in Y$.

Remark 2.2. [10] A $N_s s$ $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ can be identified to an ordered triple $\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle$ in $[0,1]$ on Y.

Definition 2.3. [10] Let Y be a non-empty set and the $N_s s$'s L and M in the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$, $M = \{\langle y, \mu_M(y), \sigma_M(y), \nu_M(y)\rangle : y \in Y\}$, then

(i) $0_N = \langle y, 0, 0, 1\rangle$ and $1_N = \langle y, 1, 1, 0\rangle$,

(ii) $L \subseteq M$ iff $\mu_L(y) \leq \mu_M(y)$, $\sigma_L(y) \leq \sigma_M(y)$ & $\nu_L(y) \geq \nu_M(y) : y \in Y$,

(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,

(iv) $1_N - L = \{\langle y, \nu_L(y), 1 - \sigma_L(y), \mu_L(y)\rangle : y \in Y\} = L^c$,

(v) $L \cup M = \{\langle y, \max(\mu_L(y), \mu_M(y)), \max(\sigma_L(y), \sigma_M(y)), \min(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$,

(vi) $L \cap M = \{\langle y, \min(\mu_L(y), \mu_M(y)), \min(\sigma_L(y), \sigma_M(y)), \max(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$.

Definition 2.4. [10] A neutrosophic topology (briefly, $N_s t$) on a non-empty set Y is a family Ψ_N of neutrosophic subsets of Y satisfying

(i) $0_N, 1_N \in \Psi_N$.

(ii) $L_1 \cap L_2 \in \Psi_N$ for any $L_1, L_2 \in \Psi_N$.

(iii) $\bigcup L_x \in \Psi_N, \forall\ L_x : x \in X \subseteq \Psi_N$.

Then (Y, Ψ_N) is called a neutrosophic topological space (briefly, $N_s ts$) in Y. The Ψ_N elements are called neutrosophic open sets (briefly, $N_s os$) in Y. A $N_s s$ C is called a neutrosophic closed sets (briefly, $N_s cs$) iff its complement C^c is $N_s os$.

Definition 2.5. [10] Let (Y, Ψ_N) be $N_s ts$ on Y and L be an $N_s s$ on Y, then the neutrosophic interior of L (briefly, $N_s int(L)$) and the neutrosophic closure of L (briefly, $N_s cl(L)$) are defined as

$$N_s int(L) = \bigcup\{I : I \subseteq L \text{ and } I \text{ is a } N_s os \text{ in } Y\}$$

$$N_s cl(L) = \bigcap\{J : L \subseteq J \text{ and } J \text{ is a } N_s cs \text{ in } Y\}.$$

Definition 2.6. [2] Let (Y, Ψ_N) be $N_s ts$ on Y and L be an $N_s s$ on Y. Then L is said to be a neutrosophic regular open set (briefly, $N_s ros$) if $L = N_s int(N_s cl(L))$.

The complement of a $N_s ros$ is called a neutrosophic regular closed set (briefly, $N_s rcs$) in Y.

Definition 2.7. [17] A set K is said to be a neutrosophic

(i) δ interior of G (briefly, $N_s\delta int(K)$) is defined by $N_s\delta int(K) = \bigcup\{B : B \subseteq K$ and B is a $N_s ros$ in $Y\}$.

(ii) δ closure of K (briefly, $N_s\delta cl(K)$) is defined by $N_s\delta cl(K) = \bigcap\{J : K \subseteq J$ and J is a $N_s rcs$ in $Y\}$.

Definition 2.8. [17] A set L is said to be a neutrosophic δ-open set (briefly, $N_s\delta os$) if $L = N_s\delta int(L)$.

The complement of an $N_s\delta os$ is called a neutrosophic δ closed set (briefly, $N_s\delta cs$) in Y.

Proposition 2.9. [17] The Neutrosophic δ-interior operator satisfies

(i) $N_s\delta int(K) \subseteq K$.

(ii) $K \subseteq M \Rightarrow N_s\delta int(K) \subseteq N_s\delta int(M)$.

(iii) $N_s\delta int(K \cap M) = N_s\delta int(K) \cap N_s\delta int(M)$.

(iv) $N_s\delta int(K)$ is the greatest $N_s\delta os$ containing K.

(v) $N_s\delta int(K) = K$ iff K is an $N_s\delta os$.

(vi) $N_s\delta int(N_s\delta int(K)) = N_s\delta int(K)$.

(vii) $(1_{N_s} - N_s\delta int(K)) = N_s\delta cl(1_{N_s} - K)$.

Proposition 2.10. [17] The Neutrosophic δ-closure operator satisfies

(i) $K \subseteq N_s\delta cl(K)$.

(ii) $K \subseteq M \Rightarrow N_s\delta cl(K) \subseteq N_s\delta cl(M)$.

(iii) $N_s\delta cl(K \cup M) = N_s\delta cl(K) \cup N_s\delta cl(M)$.

(iv) $N_s\delta cl(K)$ is the smallest $N_s\delta cs \subseteq K$.

(v) $N_s\delta cl(K) = K$ iff K is an $N_s\delta c$ set.

(vi) $N_s\delta cl(N_s\delta cl(K)) = N_s\delta cl(K)$.

(vii) $(1_{N_s} - N_s\delta cl(K)) = N_s\delta int(1_{N_s} - K)$.

(viii) $y \in N_s\delta cl(K)$ iff $K \cap C \neq \emptyset$ for every $N_s\delta os$ C containing y.

(ix) $N_s\delta cl(S \cap T) \subseteq N_s\delta cl(S) \cap N_s\delta cl(T)$.

Proposition 2.11. [17] The statements are hold for N_sts. Every $N_s\delta os$ (resp. $N_s\delta cs$) is a N_sos (resp. N_scs). But not converse.

3 Neutrosophic δ frontier

In this section, we introduce neutrosophic δ frontier and discuss their properties in neutrosophic topological spaces.

Definition 3.1. Let (Y, Ψ_N) be a N_sts. Let A be a neutrosophic subset of Y. Then the neutrosophic (resp. δ) frontier of a neutrosophic subset A were denoted by $N_sFr(A)$ (resp. $N_s\delta Fr(A)$) and were defined by $N_sFr(A) = N_scl(A) \cap N_scl(A^c)$ (resp. $N_s\delta Fr(A) = N_s\delta cl(A) \cap N_s\delta cl(A^c)$).

Example 3.2. Let $Y = \{l, m, n\}$ and define Ns's Y_1, Y_2 & Y_3 in Y are

$$Y_1 = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle,$$
$$Y_2 = \langle Y, (\frac{\mu_l}{0.1}, \frac{\mu_m}{0.1}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.9}, \frac{\nu_m}{0.9}, \frac{\nu_n}{0.6})\rangle.$$

Then we have $\tau_N = \{0_N, Y_1, Y_2, 1_N\}$. Let $A = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6})\rangle$, then

(i) $N_sFr(A) = \langle Y, (\frac{\mu_l}{0.8}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.2}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.4})\rangle$.

(ii) $N_s\delta Fr(A) = \langle Y, (\frac{\mu_l}{0.8}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.2}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.4})\rangle$.

Remark 3.3. For a neutrosophic subset A of Y, $N_sFr(A)$ (resp. $N_s\delta Fr(A)$) is N_sc (resp. $N_s\delta c$).

Theorem 3.4. For a neutrosophic subset A in N_sts (Y, Ψ_N),

(i) $N_sFr(A) = N_sFr(A^c)$.

(ii) $N_s\delta Fr(A) = N_s\delta Fr(A^c)$.

Proof. (i) Let A be a neutrosophic subset in N_sts (Y, Ψ_N). Then by Definition 3.1 $N_sFr(A) = N_scl(A) \cap N_scl(A^c) = N_scl(A^c) \cap N_scl(A) = N_scl(A^c) \cap (N_scl(A^c)^c)$. Again by Definition 3.1 this is equal to $N_sFr(A^c)$. Hence $N_sFr(A) = N_sFr(A^c)$.

(ii) Let A be a neutrosophic subset in N_sts (Y, Ψ_N). Then by Definition 3.1 $N_s\delta Fr(A) = N_s\delta cl(A) \cap N_s\delta cl(A^c) = N_s\delta cl(A^c) \cap N_s\delta cl(A) = N_s\delta cl(A^c) \cap (N_s\delta cl(A^c)^c)$. Again by Definition 3.1 this is equal to $N_s\delta Fr(A^c)$. Hence $N_s\delta Fr(A) = N_s\delta Fr(A^c)$. □

Theorem 3.5. Let A be a neutrosophic subset in N_sts (Y, Ψ_N). Then

(i) $N_sFr(A) = N_scl(A) - N_sint(A)$.

(ii) $N_s\delta Fr(A) = N_s\delta cl(A) - N_s\delta int(A)$.

Proof. (i) Let A be a neutrosophic subset in N_sts (Y, Ψ_N). By Theorem 2.10 (vii), $(N_scl(A^c))^c = N_sint(A)$ and by Definition 3.1, $N_sFr(A) = N_scl(A) \cap (N_scl(A^c)) = N_scl(A) \cap (N_s\ int(A^c))^c$. By using $A - B = A \cap B^c$, $N_sFr(A) = N_scl(A) - N_sint(A)$. Hence $N_sFr(A) = N_scl(A) - N_sint(A)$.

(ii) Let A be a neutrosophic subset in N_sts (Y, Ψ_N). By Theorem 2.10 (vii), $(N_s\delta cl(A^c))^c = N_s\delta int(A)$ and by Definition 3.1, $N_s\delta Fr(A) = N_s\delta cl(A) \cap (N_s\delta cl(A^c)) = N_s\delta cl(A) \cap (N_s\delta\ int(A^c))^c$. By using $A - B = A \cap B^c$, $N_s\delta Fr(A) = N_s\delta cl(A) - N_s\delta int(A)$. Hence $N_s\delta Fr(A) = N_s\delta cl(A) - N_s\delta int(A)$. □

Theorem 3.6. A neutrosophic subset A is N_sc (resp. $N_s\delta c$) set in Y if and only if $N_sFr(A) \subseteq A$ (resp. $N_s\delta Fr(A) \subseteq A$).

Proof. Let A be a $N_s\delta c$ set in the N_sts (Y, Ψ_N). Then by Definition 3.1, $N_s\delta Fr(A) = N_s\delta cl(A) \cap N_s\delta cl(A^c) \subseteq N_s\delta cl(A)$. By using Theorem 2.10 (v), $N_s\delta cl(A) = A$. Hence $N_s\delta Fr(A) \subseteq A$, if A is $N_s\delta c$ in Y.

Conversely, Assume that, $N_s\delta Fr(A) \subseteq A$. Then $N_s\delta cl(A) - N_s\delta int(A) \subseteq A$. Since $N_s\delta int(A) \subseteq A$, we conclude that $N_s\delta cl(A) = A$ and hence A is $N_s\delta c$.

The proof of the other is similar. □

Theorem 3.7. If A is a N_so (resp. $N_s\delta o$) set in Y, then $N_sFr(A) \subseteq A^c$ (resp. $N_s\delta Fr(A) \subseteq A^c$).

Proof. Let A be a $N_s\delta o$ set in the N_sts (Y, Ψ_N). By Definition 2.8, A^c is $N_s\delta c$ set in Y. By Theorem 3.6, $N_s\delta Fr(A^c) \subseteq A^c$ and by Theorem 3.6, we get $N_s\delta Fr(A) \subseteq A^c$.

The proof of the other is similar. □

Theorem 3.8. Let $A \subseteq B$ and B be any N_sc (resp. $N_s\delta c$) set in Y. Then $N_sFr(A) \subseteq B$ (resp. $N_s\delta Fr(A) \subseteq B$).

Proof. By Theorem 2.10 (ii), $A \subseteq B$, $N_s\delta cl(A) \subseteq N_s\delta cl(B)$. By Definition 3.1, $N_s\delta Fr(A) = N_s\delta cl(A) \cap N_s\delta cl(A^c) \subseteq N_s\delta cl(B) \cap N_s\delta cl(A^c) \subseteq N_s\delta cl(B)$. Then by Proposition 2.10 (v), this is equal to B. Hence $N_s\delta Fr(A) \subseteq B$.

The proof of the other is similar. □

Theorem 3.9. Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Then $(N_sFr(A))^c = N_sint(A) \cup N_sint(A^c)$ (resp. $(N_s\delta Fr(A))^c = N_s\delta int(A) \cup N_s\delta int(A^c)$).

Proof. Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Then by Definition 3.1, $(N_s\delta Fr(A))^c = (N_s\delta cl(A) \cap N_s\delta cl(A^c))^c = ((N_s\delta cl(A))^c \cup (N_s\delta cl(A^c))^c$. By Theorem 2.10 (vii), which is equal to $N_s\delta int(A^c) \cup N_s\delta int(A)$. Hence $(N_s\delta Fr(A))^c = N_s\delta int(A) \cup N_s\delta int(A^c)$.

The proof of the other is similar. □

Theorem 3.10. For a neutrosophic subset A in the N_sts (Y, Ψ_N), then $N_s\delta Fr(A) \subseteq N_sFr(A)$.

Proof. Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Then by Proposition 2.11, $N_s\delta cl(A) \supseteq N_scl(A)$ and $N_scl(A^c) \subseteq N_scl(A^c)$. By Definition 3.1, $N_s\delta Fr(A) = N_s\delta cl(A) \cap N_s\delta cl(A^c) \subseteq N_scl(A) \cap N_scl(A^c)$, this is equal to $N_sFr(A)$. Hence $N_s\delta Fr(A) \subseteq N_sFr(A)$. □

Theorem 3.11. For a neutrosophic subset A in the N_sts (Y, Ψ_N), $N_scl(N_sFr(A)) \subseteq N_sFr(A)$ (resp. $N_s\delta cl(N_s\delta Fr(A)) \subseteq N_s\delta Fr(A)$).

Proof. Let A be the neutrosophic subset in the N_sts (Y, Ψ_N). Then by Definition 3.1, $N_s\delta cl(N_s\delta Fr(A)) = N_s\delta cl(N_s\delta cl(A) \cap (N_s\delta cl(A^c))) \subseteq (N_s\delta cl(N_s\delta cl(A))) \cap (N_s\delta cl(N_s\delta cl(A^c)))$. By Theorem 2.10 (vi), $N_s\delta cl(N_s\delta Fr(A)) = N_s\delta cl(A) \cap (N_s\delta cl(A^c))$. By Definition 3.1, this is equal to $N_s\delta Fr(A)$.

The proof of the other is similar. □

Theorem 3.12. For a neutrosophic subset A in the N_sts (Y, Ψ_N), $N_sFr(N_sint(A)) \subseteq N_sFr(A)$ (resp. $N_s\delta Fr(N_s\delta int(A)) \subseteq N_s\delta Fr(A)$).

Proof. Let A be the neutrosophic subset in the N_sts (Y, Ψ_N). Then

$$\begin{aligned} N_s\delta Fr(N_s\delta int(A)) =& N_s\delta cl(N_sint(A)) \cap (N_s\delta cl(N_s\delta int(A))^c)[\text{by Definition 3.1}] \\ =& N_s\delta cl(N_s\delta int(A)) \cap (N_s\delta cl(N_s\delta cl(A^c)))[\text{by Theorem 2.9 (vii)}] \\ =& N_s\delta cl(N_s\delta int(A)) \cap (N_s\delta cl(A^c))[\text{ by Theorem 2.10 (vi)}] \\ \subseteq& N_s\delta cl(A) \cap N_s\delta cl(A^c)[\text{ by Theorem 2.9 (i)}] \\ =& N_s\delta Fr(A)[\text{ by Definition 3.1}]. \end{aligned}$$

Hence $N_s\delta Fr(N_s\delta int(A)) \subseteq (N_s\delta Fr(A))$.

The proof of the other is similar. □

Theorem 3.13. For a neutrosophic subset A in the N_sts (Y, Ψ_N), then $N_sFr(N_scl(A)) \subseteq N_sFr(A)$ (resp. $N_s\delta Fr(N_s\delta cl(A)) \subseteq N_s\delta Fr(A)$).

Proof. Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Then

$$\begin{aligned} N_s\delta Fr(N_s\delta cl(A)) =& N_s\delta cl(N_s\delta cl(A)) \cap (N_s\delta cl(N_s\delta cl(A))^c)[\text{by Definition 3.1}] \\ =& N_s\delta cl(A) \cap (N_s\delta cl(N_s\delta int(A^c)))[\text{by Theorems 2.10 (vii) and 2.10 (ii) \& (vi)}] \\ \subseteq& N_s\delta cl(A) \cap N_s\delta cl(A^c)[\text{by Theorem 2.9 (i)}] \\ =& N_s\delta Fr(A)[\text{by Definition 3.1}] \end{aligned}$$

Hence $N_s\delta Fr(N_s\delta cl(A)) \subseteq N_s\delta Fr(A)$.
The proof of the other is similar. □

Theorem 3.14. Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Then $N_sint(A) \subseteq A - N_sFr(A)$ (resp. $N_s\delta int(A) \subseteq A - N_s\delta Fr(A)$).

Proof. Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Now by Definition 3.1,

$$\begin{aligned} A - N_s\delta Fr(A) =& A \cap (N_s\delta Fr(A))^c \\ =& A \cap [N_s\delta cl(A) \cap N_s\delta cl(A^c)]^c \\ =& A \cap [N_s\delta int(A^c) \cup N_s\delta int(A)] \\ =& [A \cap N_s\delta int(A^c)] \cup [A \cap N_s\delta int(A)] \\ =& [A \cap N_s\delta int(A^c)] \cup N_s\delta int(A) \supseteq N_s\delta int(A) \end{aligned}$$

Hence $N_s\delta int(A) \subseteq A - N_s\delta Fr(A)$.
The proof of the other is similar. □

Remark 3.15. In general topology, the following conditions are hold:

(i) $N_sFr(A) \cap N_sint(A) = 0_N$ (resp. $N_s\delta Fr(A) \cap N_s\delta int(A) = 0_N$),

(ii) $N_sint(A) \cup N_sFr(A) = N_scl(A)$ (resp. $N_s\delta int(A) \cup N_s\delta Fr(A) = N_s\delta cl(A)$),

(iii) $N_sint(A) \cup N_sint(A^c) \cup N_sFr(A) = 1_N$ (resp. $N_s\delta int(A) \cup N_s\delta int(A^c) \cup N_s\delta Fr(A) = 1_N$).

But the neutrosophic topology, we give counter-examples to show that the condition of neutrosophic subset of the above remark may not be hold in general.

Example 3.16. In Example 3.2, Let $A = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6})\rangle$, then

(i) $N_s\delta Fr(A) = \langle Y, (\frac{\mu_l}{0.8}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.2}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.4})\rangle$,
$N_s\delta int(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$N_s\delta Fr(A) \cap N_s\delta int(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$\implies N_s\delta Fr(A) \cap N_s\delta int(A) \neq 0_N$.

(ii) $N_s\delta int(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$,
$N_s\delta int(A^c) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$,
$N_s\delta Fr(A) = \langle Y, (\frac{\mu_l}{0.8}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.2}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.4})\rangle$.
$\implies N_s\delta int(A) \cup N_s\delta int(A^c) \cup N_s\delta Fr(A) \neq 1_N$.

Example 3.17. In Example 3.2, Let $A = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.7}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.3})\rangle$, then
$N_s\delta int(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$,
$N_s\delta Fr(A) = \langle Y, (\frac{\mu_l}{0.8}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.6}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.2}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.4})\rangle$,
$N_s\delta cl(A) = 1_N$.
$\implies N_s\delta int(A) \cup N_s\delta Fr(A) \neq N_s\delta cl(A)$

Theorem 3.18. Let A and B be neutrosophic subsets in the N_sts (Y, Ψ_N). Then $N_sFr(A \cup B) \subseteq N_sFr(A) \cup N_sFr(B)$ (resp. $N_s\delta Fr(A \cup B) \subseteq N_s\delta Fr(A) \cup N_s\delta Fr(B)$).

Proof. Let A and B be neutrosophic subsets in the N_sts (Y, Ψ_N). Then

$$\begin{aligned} N_s\delta Fr(A \cup B) =& N_s\delta cl(A \cup B) \cap N_s\delta cl(A \cup B)^c \text{[by Definition 3.1]} \\ =& N_s\delta cl(A \cup B) \cap N_s\delta cl(A^c \cap B^c) \\ \subseteq& (N_s\delta cl(A) \cup N_s\delta cl(B) \cap ((N_s\delta cl(A^c))) \cap (N_s\delta cl(B^c))\text{[by Theorem 2.10 (iii) \& (ix)]} \\ =& [(N_s\delta cl(A) \cup (N_s\delta cl(B)) \cap (N_s\delta cl(A^c)))] \cap [(N_s\delta cl(A) \cup (N_s\delta cl(B)) \cap (N_s\delta cl(B^c)))] \\ =& [(N_s\delta cl(A) \cap N_s\delta cl(A^c)) \cup ((N_s\delta cl(B) \cap (N_s\delta cl(A^c))))] \cap [(N_s\delta cl(A) \cap (N_s\delta cl(B^c))) \\ & \cup ((N_s\delta cl(B) \cap (N_s\delta cl(B^c))))] \\ =& [N_s\delta Fr(A) \cup (N_s\delta cl(B)) \cap (N_s\delta cl(A^c))] \cap [(N_s\delta cl(A) \cap (N_s\delta cl(B^c))) \cup (N_s\delta Fr(B))] \\ & \text{[by Definition 3.1]} \\ =& (N_s\delta Fr(A) \cup N_s\delta Fr(B)) \cap [(N_s\delta cl(B) \cap (N_s\delta cl(A^c))) \cup ((N_s\delta cl(A) \cap N_s\delta cl(B^c)))] \\ \subseteq& N_s\delta Fr(A) \cup N_s\delta Fr(B). \end{aligned}$$

Hence, $N_s\delta Fr(A \cup B) \subseteq N_s\delta Fr(A) \cup N_s\delta Fr(B)$.
The proof of the other is similar. □

Note 1. The following example shows that

(i) $N_sFr(A \cap B) \nsubseteq N_sFr(A) \cup N_sFr(B)$ and $N_sFr(A) \cap N_sF(B) \nsubseteq N_sFr(A \cap B)$.

(ii) $N_s\delta Fr(A \cap B) \nsubseteq N_s\delta Fr(A) \cup N_s\delta Fr(B)$ and $N_s\delta Fr(A) \cap N_s\delta F(B) \nsubseteq N_s\delta Fr(A \cap B)$.

Theorem 3.19. For any neutrosophic subsets A and B in the N_sts (Y, Ψ_N), $N_sFr(A \cap B) \subseteq (N_sFr(A) \cap (N_scl(B))) \cup (N_sFr(B) \cap N_scl(A))$ (resp. $N_s\delta Fr(A \cap B) \subseteq (N_s\delta Fr(A) \cap (N_s\delta cl(B))) \cup (N_s\delta Fr(B) \cap N_s\delta cl(A))$).

Proof. Let A and B be neutrosophic subsets in the N_sts (Y, Ψ_N). Then

$$\begin{aligned} N_s\delta Fr(A \cap B) =& N_s\delta cl(A \cap B) \cap (N_s\delta cl(A \cap B)^c)\text{[by Definition 3.1]} \\ =& N_s\delta cl(A \cap B) \cap (N_s\delta cl(A^c \cup B^c)) \\ \subseteq& (N_s\delta cl(A) \cap N_s\delta cl(B)) \cap (N_s\delta cl(A^c) \cup N_s\delta cl(B^c))\text{[by Theorem 2.10 (iii) \& (ix)]} \\ =& [(N_s\delta cl(A) \cap N_s\delta cl(B)) \cap N_s\delta cl(A^c)] \cup [(N_s\delta cl(A) \cap N_s\delta cl(B)) \cap N_s\delta cl(B^c)] \\ =& (N_s\delta Fr(A) \cap N_s\delta cl(B)) \cup (N_s\delta Fr(B) \cap N_s\delta cl(A))\text{[by Definition 3.1]}. \end{aligned}$$

Hence $N_s\delta Fr(A \cap B) \subseteq ((N_s\delta Fr(A) \cap (N_s\delta cl(B))) \cup (N_s\delta Fr(B) \cap (N_s\delta cl(A))))$.
The proof of the other is similar. □

Corollary 3.20. For any neutrosophic subsets A and B in the N_sts (Y, Ψ_N), $N_sFr(A \cap B) \subseteq N_sFr(A) \cup N_sFr(B)$ (resp. $N_s\delta Fr(A \cap B) \subseteq N_s\delta Fr(A) \cup N_s\delta Fr(B)$).

Proof. Let A and B be neutrosophic subsets in the N_sts (Y, Ψ_N). Then

$$\begin{aligned} N_s\delta Fr(A \cap B) =& N_s\delta cl(A \cap B) \cap (N_s\delta cl(A \cap B)^c)\text{[by Definition 3.1]} \\ =& N_s\delta cl(A \cap B) \cap (N_s\delta cl(A^c \cup B^c) \\ \subseteq& (N_s\delta cl(A) \cap N_s\delta cl(B)) \cap (N_s\delta cl(A^c) \cup N_s\delta cl(B^c))\text{[by Theorem 2.10 (iii) \& (ix)]} \\ =& (N_s\delta cl(A) \cap N_s\delta cl(B)) \cap (N_s\delta cl(A^c) \cup (N_s\delta cl(A) \cap N_s\delta cl(B)) \cap (N_s\delta cl(B^c))) \\ =& (N_s\delta Fr(A) \cap N_s\delta cl(B)) \cup (N_s\delta cl(A) \cap N_s\delta Fr(B))\text{[by Definition 3.1]} \\ \subseteq& N_s\delta Fr(A) \cup (N_s\delta Fr(B). \end{aligned}$$

Hence $N_s\delta Fr(A \cap B) \subseteq N_s\delta Fr(A) \cup N_s\delta Fr(B)$.
The proof of the other is similar. □

Theorem 3.21. For any neutrosophic subset A in the N_sts (Y, Ψ_N),

(i) (a) $N_sFr(N_sFr(A)) \subseteq N_sFr(A)$,

(b) $N_sFr(N_sFr(N_sFr(A))) \subseteq N_sFr(N_s\delta Fr(A))$.

(ii) (a) $N_s\delta Fr(N_s\delta Fr(A)) \subseteq N_s\delta Fr(A)$,

(b) $N_s\delta Fr(N_s\delta Fr(N_s\delta Fr(A))) \subseteq N_s\delta Fr(N_s\delta Fr(A))$.

Proof. (ii) (a) Let A be a neutrosophic subset in the N_sts (Y, Ψ_N). Then

$$\begin{aligned}
N_s\delta Fr(N_s\delta Fr(A)) =& N_s\delta cl(N_s\delta Fr(A)) \cap N_s\delta cl(N_s\delta Fr(A)^c) \text{by Definition 3.1} \\
=& N_s\delta cl(N_s\delta cl(A) \cap (N_s\delta cl(A^c)) \cap (N_s\delta cl(N_s\delta cl(A)) \cap (N_s\delta cl(A^c))^c)) \text{by Definition 3.1} \\
\subseteq& (N_s\delta cl(N_s\delta cl(A)) \cap (N_s\delta cl(N_s\delta cl(A^c))) \cap (N_s\delta cl(N_s\delta int(A^c))) \cup (N_s\delta int(A))) \\
& \text{by Theorem 2.10 (vi) \& (ix)} \\
=& (N_s\delta cl(A) \cap (N_s\delta cl(A^c)) \cap (N_s\delta cl(N_s\delta int(A) \cup N_s\delta int(A)))) \text{by Theorem 2.10 (vi)} \\
\subseteq& N_s\delta cl(A) \cap N_s\delta cl(A^c) \\
=& N_s\delta Fr(A) \text{by Definition 3.1.}
\end{aligned}$$

Therefore $N_s\delta Fr(N_s\delta Fr(A)) \subseteq N_s\delta Fr(A)$.

(b) Again, $N_s\delta Fr(N_s\delta Fr(N_s\delta Fr(A))) \subseteq N_s\delta Fr(N_s\delta Fr(A))$.

The proof of the other is similar. □

4 Neutrosophic δ border and neutrosophic δ exterior

In this section, we introduce the neutrosophic δ border, neutrosophic δ exterior using neutrosophic δ open sets and their properties are discussed in N_sts's.

Definition 4.1. *Let A be a neutrosophic subset of N_sts (Y, Ψ_N). Then the set $N_sBr(A) = A \cap N_sint(A)$ (resp. $N_s\delta Br(A) = A \cap N_s\delta int(A)$) is called the neutrosophic (resp. δ) border of A (briefly, $N_sBr(A)$ (resp. $N_s\delta Br(A)$))*

Example 4.2. *In Example 3.2, Let $A = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6})\rangle$, then*

(i) $N_sint(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$A \cap N_sint(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$\implies N_sBr(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.

(ii) $N_s\delta int(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$A \cap N_s\delta int(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$\implies N_s\delta Br(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.

Theorem 4.3. If a subset A of Y is N_sc (resp. $N_s\delta c$), then $N_sBr(A) = N_sFr(A)$ (resp. $N_s\delta Br(A) = N_s\delta Fr(A)$).

Proof. Let A be a $N_s\delta c$ subset of Y. Then by Theorem 2.10 (vii), $N_s\delta cl(A) = A$. Now, $N_s\delta Fr(A) = N_s\delta cl(A) - N_s\delta int(A) = A - N_s\delta int(A) = N_s\delta Br(A)$.

The proof of the other is similar. □

Theorem 4.4. For a neutrosophic subset A of Y, $A = N_sint(A) \cup N_sBr(A)$ (resp. $A = N_s\delta int(A) \cup N_s\delta Br(A)$).

Proof. Let $x_{(\alpha,\beta,\gamma)} \in A$. If $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A)$, then the result is obvious. If $x_{(\alpha,\beta,\gamma)} \notin N_s\delta int(A)$, then by the definition of $N_s\delta Br(A), x_{(\alpha,\beta,\gamma)} \in N_s\delta Br(A)$. Hence $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A) \cup N_s\delta Br(A)$ and so $A \subseteq N_s\delta int(A) \cup N_s\delta Br(A)$. On the other hand, since $N_s\delta int(A) \subseteq A$ and $N_s\delta Br(A) \subseteq A$, we have $N_s\delta int(A) \cup N_s\delta Br(A) \subseteq A$.

The proof of the other is similar. □

Theorem 4.5. For a neutrosophic subset A of Y, $N_sint(A) \cap N_sBr(A) = 0_N$ (resp. $N_s\delta int(A) \cap N_s\delta Br(A) = 0_N$).

Proof. Suppose $N_s\delta int(A) \cap N_s\delta Br(A) \neq 0_N$. Let $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A) \cap N_s\delta Br(A)$. Then $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A)$ and $x_{(\alpha,\beta,\gamma)} \in N_s\delta Br(A)$. Since $N_s\delta Br(A) = A - N_s\delta int(A)$, then $x_{(\alpha,\beta,\gamma)} \in A$. But $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A), x_{(\alpha,\beta,\gamma)} \in A$. There is a contradiction. Hence $N_s\delta int(A) \cap N_s\delta Br(A) = 0_N$.

The proof of the other is similar. □

Theorem 4.6. For a neutrosophic subset A of Y, A is a N_so (resp. $N_s\delta o$) set if and only if $N_sBr(A) = 0_N$ (resp. $N_s\delta Br(A) = 0_N$).

Proof. **Necessity:** Suppose A is $N_s\delta o$. Then by Theorem 2.9 (v), $N_s\delta int(A) = A$. Now, $N_s\delta Br(A) = A - N_s\delta int(A) = A - A = 0_N$.

Sufficiency: Suppose $N_s\delta Br(A) = 0_N$. This implies, $A - N_s\delta int(A) = 0_N$. Therefore $A = N_s\delta int(A)$ and hence A is $N_s\delta o$.

The proof of the other is similar. □

Corollary 4.7. For a N_sts, $N_sBr(0_N) = 0_N$ and $N_sBr(1_N) = 0_N$ (resp. $N_s\delta Br(0_N) = 0_N$ and $N_s\delta Br(1_N) = 0_N$).

Proof. Since 0_N and 1_N are $N_s\delta o$, by Theorem 4.6, $N_s\delta Br(0_N) = 0_N$ and $N_s\delta Br(1_N) = 0_N$.

The proof of the other is similar. □

Theorem 4.8. For a neutrosophic subset A of Y, $N_sBr(N_sint(A)) = 0_N$ (resp. $N_s\delta Br(N_s\delta int(A)) = 0_N$).

Proof. By the definition of $N_s\delta$ border, $N_s\delta Br(N_s\delta int(A)) = N_s\delta int(A) - N_s\delta int(N_s\delta int(A))$. By Theorem 2.9 (vi), $N_s\delta int(N_s\delta int(A)) = N_s\delta int(A)$ and hence $N_s\delta Br(N_s\delta int(A)) = 0_N$.

The proof of the other is similar. □

Theorem 4.9. For a neutrosophic subset A of Y, $N_sint(N_sBr(A)) = 0_N$ (resp. $N_s\delta int(N_s\delta Br(A)) = 0_N$).

Proof. Let $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(N_s\delta Br(A))$. Since $N_s\delta Br(A) \subseteq A$, by Theorem 2.9 (i), $N_s\delta int(N_s\delta Br(A)) \subseteq N_s\delta int(A)$. Hence $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A)$. Since $N_s\delta int(N_s\delta Br(A)) \subseteq N_s\delta Br(A), x_{(\alpha,\beta,\gamma)} \in N_s\delta Br(A)$. Therefore $x_{(\alpha,\beta,\gamma)} \in N_s\delta int(A) \cap N_s\delta Br(A), x_{(\alpha,\beta,\gamma)} = 0_N$.

The proof of the other is similar. □

Theorem 4.10. For a neutrosophic subset A of Y, $N_sBr(N_sBr(A)) = N_sBr(A)$ (resp. $N_s\delta Br(N_s\delta Br(A)) = N_s\delta Br(A)$).

Proof. By the definition of $N_s\delta$ border, $N_s\delta Br(N_s\delta Br(A)) = N_s\delta Br(A) - N_s\delta int(N_s\delta Br(A))$. By Theorem 4.9 $N_s\delta int(N_s\delta Br(A)) = 0_N$ and hence $N_s\delta Br(N_s\delta Br(A)) = N_s\delta Br(A)$.

The proof of the other is similar. □

Theorem 4.11. For a subset A of a space Y, the following statements are equivalent

(i) A is N_so (resp. $N_s\delta o$).

(ii) $A = N_sint(A)$ (resp. $A = N_s\delta int(A)$).

(iii) $N_sBr(A) = 0_N$ (resp. $N_s\delta Br(A) = 0_N$).

Proof. (i) → (ii) Obvious from Theorem 2.9.

(ii) → (iii). Suppose that $A = N_s\delta int(A)$. Then by Definition, $N_s\delta Br(A) = N_s\delta int(A) - N_s\delta int(A) = 0_N$.

(iii) → (i). Let $N_s\delta Br(A) = 0_N$. Then by Definition 4.1, $A - N_s\delta int(A) = 0_N$ and hence $A = N_s\delta int(A)$.

The proof of the other is similar. □

Theorem 4.12. Let A be a neutrosophic subset of Y. Then, $N_sBr(A) = A \cap N_scl(A^c)$ (resp. $N_s\delta Br(A) = A \cap N_s\delta cl(A^c)$).

Proof. Since $N_s\delta Br(A) = A - N_s\delta int(A)$ and by Theorem 2.10, $N_s\delta Br(A) = A - (N_s\delta cl(A^c))^c = A \cap (N_s\delta cl\ (A^c)^c) = A \cap N_s\delta cl(A^c)$.

The proof of the other is similar. □

Theorem 4.13. For a neutrosophic subset A of Y, $N_sBr(A) \subseteq N_sFr(A)$ (resp. $N_s\delta Br(A) \subseteq N_s\delta Fr(A)$).

Proof. Since $A \subseteq N_s\delta cl(A)$, $A - N_s\delta int(A) \subseteq N_s\delta cl(A) - N_s\delta int(A)$. That implies, $N_s\delta Br(A) \subseteq N_s\delta Fr(A)$.

The proof of the other is similar. □

Definition 4.14. *Let A be a neutrosophic subset of a N_sts (Y, Ψ_N). The neutrosophic (resp. δ) interior of A^c is called the neutrosophic (resp. δ) exterior of A and it is denoted by $N_sExt(A)$ (resp. $N_s\delta Ext(A)$). That is, $N_sExt(A) = N_sint(A^c)$ (resp. $N_s\delta Ext(A) = N_s\delta int(A^c)$).*

Example 4.15. *In Example 3.2, Let* $A = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.4}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.6}, \frac{\nu_n}{0.6})\rangle$*, then*

(i) $N_s int(A^c) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$\implies N_s Ext(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.

(ii) $N_s \delta int(A^c) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.
$\implies N_s \delta Ext(A) = \langle Y, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.6})\rangle$.

Theorem 4.16. For a neutrosophic subset A of Y, $N_s Ext(A) = (N_s cl(A))^c$ (resp. $N_s \delta Ext(A) = (N_s \delta cl(A))^c$).

Proof. We know that, $1_N - N_s \delta cl(A) = N_s \delta int(A^c)$, then $N_s \delta Ext(A) = N_s \delta int(A^c) = (N_s \delta cl(A))^c$.
The proof of the other is similar. □

Theorem 4.17. For a neutrosophic subset A of Y, $N_s Ext(N_s Ext(A)) = N_s int(N_s cl(A)) \supseteq N_s int(A)$ (resp. $N_s \delta Ext(N_s \delta Ext(A)) = N_s \delta int(N_s \delta cl(A)) \supseteq N_s \delta int(A)$).

Proof. Now, $N_s \delta Ext(N_s \delta Ext(A)) = N_s \delta Ext(N_s \delta int(A^c)) = N_s \delta int((N_s \delta int(A^c))^c) = N_s \delta int(N_s \delta cl(A)) \supseteq N_s \delta int(A)$.
The proof of the other is similar. □

Theorem 4.18. For a neutrosophic subset A of Y, If $A \subseteq B$, then $N_s Ext(B) \subseteq N_s Ext(A)$ (resp. $N_s \delta Ext(B) \subseteq N_s \delta Ext(A)$).

Proof. Suppose $A \subseteq B$. Now, $N_s \delta Ext(B) = N_s \delta int(B^c) \subseteq N_s \delta int(A^c) = N_s \delta Ext(A)$.
The proof of the other is similar. □

Theorem 4.19. For a neutrosophic subset A of Y, $N_s Ext(1_N) = 0_N$ and $N_s Ext(0_N) = 1_N$ (resp. $N_s \delta Ext(1_N) = 0_N$ and $N_s \delta Ext(0_N) = 1_N$).

Proof. Now, $N_s \delta Ext(1_N) = N_s \delta int((1_N)^c) = N_s \delta int(0_N)$ and $N_s \delta Ext(0_N) = N_s \delta int((0_N)^c) = N_s \delta int(1_N)$. Since 0_N and 1_N are $N_s \delta o$ sets, then $N_s \delta int(0_N) = 0_N$ and $N_s \delta int(1_N) = 1_N$. Hence $N_s \delta Ext(0_N) = 1_N$ and $N_s \delta Ext(1_N) = 0_N$.
The proof of the other is similar. □

Theorem 4.20. For a neutrosophic subset A of Y, $N_s Ext(A) = N_s Ext((N_s Ext(A))^c)$ (resp. $N_s \delta Ext(A) = N_s \delta Ext((N_s \delta Ext(A))^c)$).

Proof. Now, $N_s \delta Ext((N_s \delta Ext(A))^c) = N_s \delta Ext((N_s \delta int(A^c))^c) = N_s \delta int((((N_s \delta int(A^c))^c))^c) = N_s \delta int(N_s \delta int(A^c)) = N_s \delta int(A^c) = N_s \delta Ext(A)$.
The proof of the other is similar. □

Theorem 4.21. For a sub sets A and B of Y, the followings are valid.

(i) $N_s Ext(A \cup B) \subseteq N_s Ext(A) \cap N_s Ext(B)$ (resp. $N_s \delta Ext(A \cup B) \subseteq N_s \delta Ext(A) \cap N_s \delta Ext(B)$).

(ii) $N_s Ext(A \cap B) \supseteq N_s Ext(A) \cup N_s Ext(B)$ (resp. $N_s \delta Ext(A \cap B) \supseteq N_s \delta Ext(A) \cup N_s \delta Ext(B)$).

Proof. (i) $N_s \delta Ext(A \cup B) = N_s \delta int((A \cup B)^c) = N_s \delta int((A^c) \cap (B^c)) \subseteq N_s \delta cl(A^c) \cap N_s \delta cl(B^c) = N_s \delta Ext(A) \cap N_s \delta Ext(B)$.
(ii) $N_s \delta Ext(A \cap B) = N_s \delta int((A \cap B)^c) = N_s \delta int((A^c) \cup (B^c)) \supseteq N_s \delta cl(A^c) \cup N_s \delta cl(B^c) = N_s \delta Ext(A) \cup N_s \delta Ext(B)$.
The proof of the other is similar. □

5 Conclusions

So far, we have studied some new operators called neutrosophic δ frontier respective border and exterior with the help of neutrosophic δ-open sets in neutrosophic topological space. Also, we discussed the important properties of them and the relations between them. This can be extended to some weaker forms of neutrosophic open sets, in future.

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