



Bipolar neutrosophic soft contra generalized pre-continuous and contra generalized α -continuous mappings

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Abstract

In this paper, we introduce and investigate the classes of continuous mappings in bipolar neutrosophic soft topological spaces such as bipolar neutrosophic soft contra generalized pre-continuous mappings and bipolar neutrosophic soft contra generalized α -continuous mappings. Further, we investigate some of its properties via theorems.

Keywords: Bipolar neutrosophic soft set; BNSCGP-continuous; BNSCG-closed set; BNSCG α -continuous; BNSS-topology.)

1 Introduction

Zadeh⁵ introduced the concept of fuzzy sets in 1965. Initially, fuzzy sets were used in decision making problems; but nowadays it is applied in many science and engineering fields. Later, fuzzy topological spaces was introduced by Cheng⁹ in 1965. Atanassov¹⁰ developed some concepts in fuzzy sets and proposed intuitionistic fuzzy sets which is the extension of conventional Fuzzy sets. Smarandache^{1,2} introduced a theory of neutrosophic sets as the extension and development of fuzzy sets which includes three membership degrees namely, truth, indeterminacy and falsify memberships. Deli et al.⁸ extended the neutrosophic set and proposed bipolar neutrosophic sets. Later, many researchers were investigated and proposed various neutrosophic sets.^{4,7} Molodtsov³ introduced soft set theory in 1999. A.A.Salama et al.¹³ developed a new concept neutrosophic topology, i.e. the topology concepts were introduced for neutrosophic sets. Norman Levine^{11,12} introduced generalized closed set and some continuity mappings in point set topology in 1970. In this study, we have proposed bipolar neutrosophic soft contra generalized pre-continuous mapping and bipolar neutrosophic soft contra generalized α -continuous mappings.

This paper is organized as follows: Section 2 consists the required preliminary definitions for main concept. Section 3 consists bipolar neutrosophic soft generalized homeomorphism and pre-homeomorphism. Section 4 consists bipolar neutrosophic soft contra generalized pre-continuous mapping. Section 5 consists bipolar neutrosophic soft contra generalized α -continuous mapping. Each section deals with theorems related to the proposed mappings and examples. Section 6 concludes the proposed work.

2 Preliminaries

Definition 2.1. ¹ Let X be a universe set. For every $x \in X$, the components $u(x)$, $v(x)$ and $w(x)$ are truth, indeterminate and false degrees of x . Then the Neutrosophic set (NS) over X be defined as

follows.

$$N = \{u(x), v(x), w(x) : x \in X\}$$

Here, $u(x), v(x), w(x)$ ranges in the non-standard interval $]^{-}0, 1^{+}[$ and their sum $^{-}0 \leq u + v + w \leq 3^{+}$. For scientific problems, we prefer standard interval $[0, 1]$ instead of non-standard interval and it is called single-valued neutrosophic set.

Definition 2.2. ⁴ For the universe set X and positive member values $u^{+}, v^{+}, w^{+} : E \rightarrow [0, 1]$, negative member values $u^{-}, v^{-}, w^{-} : E \rightarrow [-1, 0]$, A bipolar neutrosophic set (BNS) is defined by

$$B = \left\{ \langle x, u^{+}(x), v^{+}(x), w^{+}(x), u^{-}(x), v^{-}(x), w^{-}(x) \rangle : x \in X \right\}$$

Definition 2.3. ³ A sot set is a function which maps a parameter set to the power set of X . It is denoted by (f, E) and is defined by

$$f : E \rightarrow P(X)$$

Each member of X is parametrized with the parameter set E by the function f .

Definition 2.4. ⁴ A bipolar neutrosophic soft set (BNSS) is the fusion of soft set and bipolar neutrosophic set and is defined as follows.

$$BNS = (f_A, E) = \{ \langle e, f_A(x) \rangle : e \in A \subset E, f_A(x) \in BNS(X) \}$$

$$\text{Here } f_A(x) = \left\{ \left\langle x, u_{f_A(e)}^{+}(x), v_{f_A(e)}^{+}(x), w_{f_A(e)}^{+}(x), u_{f_A(e)}^{-}(x), v_{f_A(e)}^{-}(x), w_{f_A(e)}^{-}(x) \right\rangle : x \in X \right\}.$$

Definition 2.5. ⁴ Let B_1 and B_2 be two BNSSs. Then for every $x \in X, e \in E$,

(i). (subset) $B_1 \subseteq B_2$ if and only if

$$\{B_1 \subseteq B_2\} = \left\{ \begin{array}{l} [u_{B_1}^{+}(x) \leq u_{B_2}^{+}(x)], [v_{B_1}^{+}(x) \geq v_{B_2}^{+}(x)], [w_{B_1}^{+}(x) \geq w_{B_2}^{+}(x)] \\ [u_{B_1}^{-}(x) \geq u_{B_2}^{-}(x)], [v_{B_1}^{-}(x) \leq v_{B_2}^{-}(x)], [w_{B_1}^{-}(x) \leq w_{B_2}^{-}(x)] \end{array} \right\}$$

(ii). (equal) $B_1 = B_2$ if and only if $B_1 \subseteq B_2$ and $B_2 \subseteq B_1$.

(iii). (complement)

$$B^c = \left\{ \left\langle e, w_f^{+}(e), 1 - v_f^{+}(e), u_f^{+}(e), w_f^{-}(e), -1 - v_f^{-}(e), u_f^{-}(e) \right\rangle \right\}$$

(iv). (union) $B_1 \cup B_2 = \left\{ \langle e, \cup_i f^{(i)}(e) \rangle \right\}$. Here

$$\bigcup_i f^{(i)}(e) = \left\{ \left\langle x, \max [u_{f_i}^{+}(e)(x)], \min [v_{f_i}^{+}(e)(x)], \min [w_{f_i}^{+}(e)(x)], \right. \right. \\ \left. \left. \min [u_{f_i}^{-}(e)(x)], \max [v_{f_i}^{-}(e)(x)], \max [w_{f_i}^{-}(e)(x)] \right\rangle \right\}$$

(v). (intersection) $B_1 \cap B_2 = \left\{ \langle e, \cap_i f^{(i)}(e) \rangle \right\}$. Here

$$\bigcap_i f^{(i)}(e) = \left\{ \left\langle x, \min [u_{f_i}^{+}(e)(x)], \max [v_{f_i}^{+}(e)(x)], \max [w_{f_i}^{+}(e)(x)], \right. \right. \\ \left. \left. \max [u_{f_i}^{-}(e)(x)], \min [v_{f_i}^{-}(e)(x)], \min [w_{f_i}^{-}(e)(x)] \right\rangle \right\}$$

(vi). (null) $\phi_{\mathbb{B}} = \{ \langle e_i, \{x_i, 0, 1, 1, 0, -1, -1\} \rangle : x \in X, e \in E \}$

(vii). (complete) $1_{\mathbb{B}} = \{ \langle e_i, \{x_i, 1, 0, 0, -1, 0, 0\} \rangle : x \in X, e \in E \}$

Definition 2.6. A bipolar neutrosophic soft topology (BNST) on X is a collection $\tau_{\mathbb{B}}$ of bipolar neutrosophic soft sets (BNSS) in X satisfying the following conditions:

- 1). $\phi_{\mathbb{B}}, 1_{\mathbb{B}} \in \tau_{\mathbb{B}}$
- 2). $\bigcup_{i \in n} \mathbb{B}_i \in \tau_{\mathbb{B}}$ for each $\mathbb{B}_i \in \tau_{\mathbb{B}}$
- 3). $\mathbb{B}_i \cap \mathbb{B}_j \in \tau_{\mathbb{B}}$ for any $\mathbb{B}_i, \mathbb{B}_j \in \tau_{\mathbb{B}}$

The pair $(X, \tau_{\mathbb{B}})$ is called BNSS-topological space. The members of $\tau_{\mathbb{B}}$ are called bipolar neutrosophic soft open sets (BNOS) and their complements are called bipolar neutrosophic soft closed sets (BNCS).

Definition 2.7. Let $(X, \tau_{\mathbb{B}})$ be a BNST and $B = \{ \langle e, f(x) \rangle : e \in E, f(x) \in BNS(X) \}$ be BNSS in X . Then the bipolar neutrosophic soft interior and bipolar neutrosophic soft closure are defined by

$$BNint(B) = \bigcup \left\{ U : U \text{ is a BNOS in } U \subseteq B \right\}$$

$$BNcl(B) = \bigcap \left\{ V : V \text{ is a BNCS in } B \subseteq V \right\}$$

Remark 2.8. Let B be BNS of a BNTS (X, τ) . Then

1. $BN\alpha cl(B) = B \cup BNcl(BNint(BNcl(B)))$
2. $BN\alpha int(B) = B \cap BNint(BNcl(BNint(B)))$

Remark 2.9. Following relations hold for any BNS set $B \in (X, \tau)$.

1. $BNcl(B^c) = (BNint(B))^c$ and $BNint(B^c) = (BNcl(B))^c$.
2. $BNcl(B)$ is a BNCS and $BNint(B)$ is a BNOS in X .
3. B is BNCS in X if and only if $BNcl(B) = B$.
4. B is BNOS in X if and only if $BNint(B) = B$.

Definition 2.10. A BNSset B in BNSTS (X, τ) is said to be

- 1). Bipolar neutrosophic soft semi closed set (BNSCS) if $BNint(BNcl(B)) \subseteq B$,
- 2). Bipolar neutrosophic soft pre-closed set (BNPCS) if $BNcl(BNint(B)) \subseteq B$,
- 3). Bipolar neutrosophic soft α -closed set (BN α CS) if $BNcl(BNint(BNcl(B))) \subseteq B$,
- 4). Bipolar neutrosophic soft regular closed set (BNRCS) if $B = BNcl(BNint(B))$

Definition 2.11. Let $(X, \tau_{\mathbb{B}}^1, E)$ and $(Y, \tau_{\mathbb{B}}^2, E')$ be two bipolar neutrosophic soft topological spaces. For every bipolar neutrosophic closed set B of $(Y, \tau_{\mathbb{B}}^2, E')$, a map $\psi : ((X, \tau_{\mathbb{B}}^1, E) \rightarrow (Y, \tau_{\mathbb{B}}^2, E'))$ is said to be,

1. Bipolar neutrosophic soft continuous (BNS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft closed set in $(X, \tau_{\mathbb{B}}^1, E)$.
2. Bipolar neutrosophic soft semi-continuous (BNSS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft semi-closed set in $(X, \tau_{\mathbb{B}}^1, E)$.

3. Bipolar neutrosophic soft pre-continuous (BNSP-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft pre-closed set in $(X, \tau_{\mathbb{B}}^1, E)$.
4. Bipolar neutrosophic soft α -continuous (BNS α -continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft α -closed set in $(X, \tau_{\mathbb{B}}^1, E)$.
5. Bipolar neutrosophic soft regular continuous (BNSR-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft regular closed set in $(X, \tau_{\mathbb{B}}^1, E)$.
6. Bipolar neutrosophic soft generalized continuous (BNSG-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized closed set in $(X, \tau_{\mathbb{B}}^1, E)$.
7. Bipolar neutrosophic soft generalized semi-continuous (BNSGS-continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized semi-closed set in $(X, \tau_{\mathbb{B}}^1, E)$.
8. Bipolar neutrosophic soft generalized α -continuous (BNS α -continuous) if $\psi^{-1}(B)$ is bipolar neutrosophic soft generalized α -closed set in $(X, \tau_{\mathbb{B}}^1, E)$.

Definition 2.12. A map $\psi : ((X, \tau_{\mathbb{B}}^1, E) \rightarrow (Y, \tau_{\mathbb{B}}^2, E'))$ is said to be bipolar neutrosophic soft generalized pre irresolute (BNSGP-irresolute) mapping if $\psi^{-1}(B)$ is a BNSGPCS in $(X, \tau_{\mathbb{B}}^1, E)$ for every BNSGPCS B in $(Y, \tau_{\mathbb{B}}^2, E)$.

Definition 2.13. Let $(X, \tau_{\mathbb{B}}, E)$ be a BNSGT. The bipolar neutrosophic soft generalized pre closure ($BNSGPcl(B)$) for any BNS B is defined as follows.

$$BNSGPcl(B) = \cap \{K \mid K \text{ is a BNSGPCS in } X \text{ and } B \in K\}.$$

If B is BNSGPCS, then $BNSGPcl(B) = B$.

Definition 2.14. Let (X, τ_1) and (Y, τ_2) be any two topological spaces. A map $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be contra continuous if $f^{-1}(V)$ is closed set in (X, τ_1) for every open set V in (Y, τ_2) .

Definition 2.15. Let f be a bijective mapping from a topological space (X, τ_1) into a topological space (Y, τ_2) . Then f is said to be generalized homeomorphism if f and f^{-1} are generalized continuous mappings.

Definition 2.16. A map $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be *generalized pre closed* if $f(V)$ is generalized pre closed set in (Y, τ_2) for every closed set V in (X, τ_1) .

3 Bipolar neutrosophic soft generalized homeomorphism and pre homeomorphism in topological spaces

Definition 3.1. Let ψ be a bijective mapping from a BNST (X, τ_1, E) into a BNST (Y, τ_2, E) . Then ψ is said to be

1. Bipolar neutrosophic soft homeomorphism (BNS-homeomorphism) if ψ and ψ^{-1} are BNS-continuous mapping.
2. Bipolar neutrosophic soft pre homeomorphism (BNSP-homeomorphism) if ψ and ψ^{-1} are BNSP-continuous mapping.
3. Bipolar neutrosophic soft generalized homeomorphism (BNSG-homeomorphism) if ψ and ψ^{-1} are BNSG-continuous mapping.

Definition 3.2. A map $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be Bipolar neutrosophic soft generalized pre closed if $\psi(V)$ is bipolar neutrosophic soft generalized pre closed set in (Y, τ_2, E) for every BNS-closed set V in (X, τ_1, E) .

Definition 3.3. A bijection mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is called a Bipolar neutrosophic soft generalized pre-homeomorphism (BNSGP-homeomorphism) if ψ and ψ^{-1} are BNSGP-continuous mapping.

Example 3.4. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle \} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$ with . Then ψ is a BNSGP-continuous mapping and ψ^{-1} is also a BNSGP-continuous mapping. Therefore ψ is a BNSGP homeomorphism.

Theorem 3.5. Every BNS-homeomorphism is a BNSGP-homeomorphism but not conversely.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNS-homeomorphism. Then ψ and ψ^{-1} are BNS-continuous mapping. This gives, ψ and ψ^{-1} are BNSGP-continuous mapping. Therefore, ψ is a BNSGP-homeomorphism. □

Example 3.6. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle \} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSGP-homeomorphism but not BNS-homeomorphism since ψ and ψ^{-1} are not BNS-continuous mapping.

Theorem 3.7. Every BNSP-homeomorphism is a BNSGP-homeomorphism but not conversely.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSP-homeomorphism. Then ψ and ψ^{-1} are BNSP-continuous mapping. This gives, ψ and ψ^{-1} are BNSGP-continuous mapping. Therefore, ψ is a BNSGP-homeomorphism. □

Example 3.8. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle\} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSGP-homeomorphism but not BNSP-homeomorphism since ψ and ψ^{-1} are not BNSP-continuous mapping.

Theorem 3.9. *Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a bijective mapping. If ψ is a BNSGP continuous mapping, then the following statements are equivalent.*

1. ψ is a BNSGP closed mapping.
2. ψ is a BNSGP open mapping.
3. ψ is a BNSGP homeomorphism.

Proof. (1) \rightarrow (2): Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a bijective mapping and let ψ be a BNSGP-closed mapping. This gives, $\psi^{-1} : (Y, \tau_2, E) \rightarrow (X, \tau_1, E)$ is a BNSGP-continuous mapping. This means, every BNOS in X is a BNSGPOS in Y . Hence ψ is a BNSGP-open mapping.

(2) \rightarrow (3): Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a bijective mapping and let ψ be a BNSGP-open mapping. This gives, $\psi^{-1} : (Y, \tau_2, E) \rightarrow (X, \tau_1, E)$ is a BNSGP-continuous mapping. Hence ψ and ψ^{-1} are BNSGP-homeomorphism.

(3) \rightarrow (1): Let ψ is a BNSGP homeomorphism. This means, ψ and ψ^{-1} are BNSGP continuous mapping. Since every BNCS in X is a BNSGPCS in Y , then ψ is a BNSGP-closed mapping. \square

Remark 3.10. The composition of two BNSGP homeomorphism need not be a BNSGP homeomorphism in general.

Example 3.11. Let $X = \{a, b\}, Y = \{u, v\}, Z = \{p, q\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1, 1, 0, 1, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle\} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle\} \right\rangle \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{l} \left\langle e_1, \{\langle x_1, 1, 0, 0.7, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.7, 0, -0.3 \rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1, 0.4, 0.5, 0.3, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.1, -0.3, -0.5, -0.3 \rangle\} \right\rangle \end{array} \right\}$$

Let $\tau_1 = \{0, B_1, 1\}, \tau_2 = \{0, B_2, 1\}$ and $\tau_3 = \{0, B_3, 1\}$ are BNSTs on X, Y and Z respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$ and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ by $\phi(x) = p$ and $\phi(y) = q$. Then ψ and ψ^{-1} are BNSGP-continuous mappings, also ϕ and ϕ^{-1} are BNSGP-continuous mappings. Hence ψ and ϕ are BNSGP-homeomorphism. But the mapping $\psi \circ \phi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is not a BNSGP-homeomorphism since $\psi \circ \phi$ is not a BNSGP-continuous mapping.

4 Bipolar neutrosophic soft contra generalized pre continuous mappings

Definition 4.1. A map $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be a Bipolar neutrosophic soft contra generalized pre-continuous (BNSCGP-continuous) mapping if $\psi^{-1}(B)$ is a BNSGPCS in (X, τ_1, E) for every BNOS B in (Y, τ_2, E) .

Example 4.2. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle \} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping.

Theorem 4.3. Every BNSC-continuous mapping is a BNSCGP-continuous mapping but not conversely.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSC-continuous mapping. Let B be a BNOS in Y . Then $\psi^{-1}(B)$ is a BNCS in X . Since every BNCS is a BNSGPCS, $\psi^{-1}(B)$ is a BNSGPCS in X . Hence, ψ is a BNSCGP continuous mapping. □

Example 4.4. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle \} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping but not a BNSC-continuous mapping.

Theorem 4.5. Every BNSC α -continuous mapping is a BNSCGP continuous mapping but not conversely.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSC α -continuous mapping. Let B be a BNOS in Y . Then $\psi^{-1}(B)$ is a BNS α CS in X . Since every BNS α CS is a BNGPCS, $\psi^{-1}(B)$ is a BNGPCS in X . Hence, ψ is a BNSCGP-continuous mapping. □

Example 4.6. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle \} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping but not a $BNSC\alpha$ -continuous mapping.

Theorem 4.7. Every BNSCP-continuous mapping is a BNSCGP continuous mapping but not conversely.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSCP-continuous mapping. Let B be a BNOS in Y . Then $\psi^{-1}(B)$ is a BNSPCS in X . Since every BNSPCS is a BNSGPCS, $\psi^{-1}(B)$ is a BNSGPCS in X . Hence, ψ is a BNSCGP-continuous mapping. \square

Example 4.8. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.5, 0.6, 0.8, -1, 0, 0 \rangle, \langle x_2, 0.5, 0.2, 0.4, -0.5, -0.4, -0.3 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.4, 0.6, 0.3, -0.4, -0.7, -0.2 \rangle, \langle x_2, 0.7, 0.2, 0.1, -0.3, -0.5, -0.7 \rangle \} \right\rangle \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{l} \left\langle e_1, \{ \langle x_1, 0.3, 0.1, 0.7, -0.5, -0.6, -0.3 \rangle, \langle x_2, 0, 1, 1, -0.7, 0, -1 \rangle \} \right\rangle, \\ \left\langle e_2, \{ \langle x_1, 0.2, 0.5, 0.7, -1, 0, -0.2 \rangle, \langle x_2, 0.9, 0.1, 0.3, -0.1, -0.6, -0.3 \rangle \} \right\rangle \end{array} \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCGP-continuous mapping but not a BNSCP-continuous mapping.

Theorem 4.9. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a mapping. Then the following statements are equivalent.

1. ψ is a BNSCGP continuous mapping.
2. $\psi^{-1}(B)$ is a BNSGPOS in X for every BNCS B in Y .

Proof. (1) \rightarrow (2): Let B be a BNCS in Y . Then B^c is a BNOS in Y . By statement, $\psi^{-1}(B^c)$ is a BNSGPCS in X . Hence $\psi^{-1}(B)$ is a BNSGPOS in X .

(2) \rightarrow (1): Let B be a BNOS in Y . Then B^c is a BNCS in Y . By statement, $\psi^{-1}(B^c)$ is a BNSGPOS in X . Hence $\psi^{-1}(B)$ is a BNSGPCS in X . Thus ψ is a BNSCGP-continuous mapping. \square

Theorem 4.10. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a mapping. If one of the following properties hold:

1. $\psi(BNSPcl(A)) \subseteq BNSint(\psi(A))$ for each A in X .
2. $BNSPcl(\psi^{-1}(B)) \subseteq \psi^{-1}(BNSint(B))$ for each B in Y .

3. $\psi^{-1}(BN\text{Scl}(B)) \subseteq BNSP\text{int}(\psi^{-1}(B))$ for each B in Y .

Then ψ is a BNSCGP-continuous mapping.

Proof. (1) \rightarrow (2): Let B be a BNS in Y . Then $\psi^{-1}(B)$ is a BNS in X . By statement, we have $\psi(BNSP\text{cl}(\psi^{-1}(B))) \subseteq BNS\text{int}(\psi(\psi^{-1}(B))) \subseteq BNS\text{int}(B)$. Now $BNSP\text{cl}(\psi^{-1}(B)) \subseteq \psi^{-1}(\psi(BNSP\text{cl}(\psi^{-1}(B)))) \subseteq \psi^{-1}(BNS\text{int}(B))$.

(2) \rightarrow (1): By taking the complement in (2). Suppose that (3) holds. Let B be a BNCS in Y . Then $BN\text{cl}(B) = B$. By the assumption, $\psi^{-1}(B) = \psi^{-1}(BN\text{cl}(B)) \subseteq BNSP\text{int}(\psi^{-1}(B))$. But $BNSP\text{int}(\psi^{-1}(B)) \subseteq \psi^{-1}(B)$, hence $BNSP\text{int}(\psi^{-1}(B)) = \psi^{-1}(B)$. This implies $\psi^{-1}(B)$ is a BNSPOS in X and hence $\psi^{-1}(B)$ is a BNSGPOS in X . Thus ψ is a BNSCGP-continuous mapping. \square

Theorem 4.11. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a bijective mapping. If one of the following properties hold:

1. $\psi^{-1}(BN\text{cl}(B)) \subseteq BN\text{int}(BNSP\text{cl}(\psi^{-1}(B)))$ for each B in Y .
2. $BN\text{cl}(BNSP\text{int}(\psi^{-1}(B))) \subseteq \psi^{-1}(BN\text{int}(B))$ for each B in Y .
3. $\psi(BN\text{cl}(BNSP\text{int}(A))) \subseteq BN\text{int}(\psi(A))$ for each A in X .
4. $\psi(BN\text{cl}(A)) \subseteq BN\text{int}(\psi(A))$ for each BNPOS A in X .

Then ψ is a BNSCGP-continuous mapping.

Proof. (1) \rightarrow (2): It is obvious, by taking the complement in (1).

(2) \rightarrow (3): Let A be a BNS in X . Put $B = \psi(A)$ in Y . This implies $A = \psi^{-1}(\psi(A)) = \psi^{-1}(B)$ in X . Now $BN\text{cl}(BNSP\text{int}(A)) = BN\text{cl}(BNSP\text{int}(\psi^{-1}(B))) \subseteq \psi^{-1}(BN\text{int}(B))$ by the hypothesis. Therefore, $\psi(BN\text{cl}(BNSP\text{int}(A))) \subseteq \psi(\psi^{-1}(BN\text{int}(B))) = BN\text{int}(B) = BN\text{int}(\psi(A))$.

(3) \rightarrow (4): Let A be a BNSPOS in X . Then $BNSP\text{int}(A) = A$. By hypothesis, $\psi(BN\text{cl}(BNSP\text{int}(A))) \subseteq BN\text{int}(\psi(A))$.

Therefore, $\psi(BN\text{cl}(A)) = \psi(BN\text{cl}(BNSP\text{int}(A))) \subseteq BN\text{int}(\psi(A))$.

Suppose (4) holds: Let A be a BNOS in Y . Then $\psi^{-1}(A)$ is a BNOS in X and $BNSP\text{int}(\psi^{-1}(A))$ is a BNSPOS in X . Hence, by hypothesis,

$\psi(BN\text{cl}(BNSP\text{int}(\psi^{-1}(A)))) \subseteq BN\text{int}(\psi(BNSP\text{int}(\psi^{-1}(A)))) \subseteq BN\text{int}(\psi(\psi^{-1}(A))) = BN\text{int}(A) \subseteq A$.

Therefore $BN\text{cl}(BNSP\text{int}(\psi^{-1}(A))) = \psi^{-1}(\psi(BN\text{cl}(BNSP\text{int}(\psi^{-1}(A)))) \subseteq \psi^{-1}(A)$. Now, $BN\text{cl}(BN\text{int}(\psi^{-1}(A))) \subseteq BN\text{cl}(BNSP\text{int}(\psi^{-1}(A))) \subseteq \psi^{-1}(A)$. This implies $\psi^{-1}(A)$ is a BNSPCS in X and hence a BNSGPCS in X . Thus ψ is a BNSCGP-continuous mapping. \square

Theorem 4.12. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a bijective mapping. Then ψ is a BNSCGP continuous mapping if $BN\text{cl}(\psi(A)) \subseteq \psi(BNSP\text{int}(A))$ for every BNS A in X .

Proof. Let A be a BNCS in Y . Then $BN\text{cl}(A) = A$ and $\psi^{-1}(A)$ is a BNS in X . By hypothesis, $BN\text{cl}(\psi(\psi^{-1}(A))) \subseteq \psi(BNSP\text{int}(\psi^{-1}(A)))$. Since ψ is onto, $\psi(\psi^{-1}(A)) = A$. Therefore $A = BN\text{cl}(A) = BN\text{cl}(\psi(\psi^{-1}(A))) \subseteq \psi(BNSP\text{int}(\psi^{-1}(A)))$.

Now $\psi^{-1}(A) \subseteq \psi^{-1}(\psi(BNSP\text{int}(\psi^{-1}(A)))) = BNSP\text{int}(\psi^{-1}(A)) \subseteq \psi^{-1}(A)$. Hence $\psi^{-1}(A)$ is a BNSPOS in X and hence BNSGPOS in X . Thus ψ is a BNSCGP-continuous mapping. \square

Theorem 4.13. 1. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCGP-continuous mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a BNSCGP-continuous mapping.

2. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCGP-continuous mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a BNSC-continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a BNSGP-continuous mapping.

3. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSGP-irresolute mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a BNSCGP-continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a BNSCGP-continuous mapping.

Proof. 1. Let A be BNOS in Z . Then $\phi^{-1}(A)$ is a BNOS in Y , by hypothesis. Since ψ is a BNSCGP-continuous mapping, $\psi^{-1}(\phi^{-1}(A))$ is a BNSGPCS in X . Hence $\phi \circ \psi$ is a BNSCGP-continuous mapping.

2. Let A be BNOS in Z . Then $\phi^{-1}(A)$ is a BNCS in Y , by hypothesis. Since ψ is a BNSCGP-continuous mapping, $\psi^{-1}(\phi^{-1}(A))$ is a BNSGPOS in X . Hence $\phi \circ \psi$ is a BNSGP-continuous mapping.

3. Let A be BNOS in Z . Then $\phi^{-1}(A)$ is a BNSGPCS in Y , by hypothesis. Since ψ is a BNSGP-continuous mapping, $\psi^{-1}(\phi^{-1}(A))$ is a BNSGPCS in X . Hence $\phi \circ \psi$ is a BNSCGP-continuous mapping. □

Theorem 4.14. A mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCGP-continuous mapping if $\psi^{-1}(BNSPcl(B)) \subseteq BNint(\psi^{-1}(B))$ for every BNS B in Y .

Proof. Let B be a BNCS in Y . Then $BNcl(B) = B$. Since every BNCS is a BNSPCS, this implies $BNSPcl(B) = B$. By hypothesis, $\psi^{-1}(B) = \psi^{-1}(BNSPcl(B)) \subseteq BNint(\psi^{-1}(B)) \subseteq \psi^{-1}(B)$. This implies $\psi^{-1}(B)$ is a BNOS in X . Therefore ψ is a BNSC-continuous mapping, since every BNSC-continuous mapping is a BNSCGP-continuous mapping, ψ is a BNSCGP-continuous mapping. □

Theorem 4.15. A BNS-continuous mapping, $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCGP continuous mapping if $BNSGPO(X) = BNSGPC(X)$.

Proof. Let B be a BNOS in Y . By hypothesis, $\psi^{-1}(B)$ is a BNOS in X and hence is a BNSGPOS in X . Since $BNSGPO(X) = BNSGPC(X)$, $\psi^{-1}(B)$ is a BNSGPCS in X . Therefore, ψ is a BNSCGP-continuous mapping. □

5 Bipolar neutrosophic soft contra generalized α -continuous mapping

in this section we introduce bipolar neutrosophic soft contra generalized α -continuous mapping and investigate some of its properties.

Definition 5.1. A bipolar neutrosophic soft set B in (X, τ, E) is said to be a bipolar neutrosophic soft generalized α -closed set (BNSG α CS) if $BN\alpha cl(B) \subseteq U$ whenever $B \subseteq U$ is a BN α OS in (X, τ, E) .

Definition 5.2. A mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be a bipolar neutrosophic soft contra generalized α -continuous (BNSCG α -continuous) if $\psi^{-1}(B)$ is a bipolar neutrosophic soft generalized α -closed set in (x, τ_1, E) for every bipolar neutrosophic soft open set B in (Y, τ_2, E) .

Definition 5.3. A mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be a bipolar neutrosophic soft strongly generalized α -continuous (BNSSG α -continuous) if $\psi^{-1}(B)$ is a bipolar neutrosophic soft open set in (x, τ_1, E) for every bipolar neutrosophic soft generalized α -open set B in (Y, τ_2, E) .

Definition 5.4. A mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be a bipolar neutrosophic soft contra strongly generalized α -continuous (BNCSG α -continuous) if $\psi^{-1}(B)$ is a bipolar neutrosophic soft closed set in (x, τ_1, E) for every bipolar neutrosophic soft generalized α -open set B in (Y, τ_2, E) .

Definition 5.5. A mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is said to be a bipolar neutrosophic soft contra generalized α -irresolute if $\psi^{-1}(B)$ is a bipolar neutrosophic soft generalized α -closed set in (x, τ_1, E) for every bipolar neutrosophic soft generalized α -open set B in (Y, τ_2, E) .

Theorem 5.6. Every BNSC-continuous mapping is a BNCSG α -continuous mapping but converse not true.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSC-continuous mapping and let B be BNOS in Y . Then $\psi^{-1}(B)$ is a BNCS in X . Since every BNCS is a BNSG α CS, $\psi^{-1}(B)$ is a BNSG α CS in (X, τ_1, E) . Hence ψ is a BNSG α -continuous mapping. \square

Example 5.7. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \left\langle e_1, \{ \langle x_1, 0.6, 0.6, 0.6, -0.5, -0.4, -0.6 \rangle, \langle x_2, 0.4, 0.4, 0.4, -0.2, -0.4, -0.3 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.7, 0.5, 0.4, -0.6, -0.7, -0.2 \rangle, \langle x_2, 0.6, 0.2, 0.5, -0.3, -0.5, -0.5 \rangle \} \right\rangle \right\}$$

$$B_2 = \left\{ \left\langle e_1, \{ \langle x_1, 0.2, 0.2, 0.3, -0.4, -0.2, -0.6 \rangle, \langle x_2, 0.6, 0.4, 0.6, -0.7, -0.4, -0.3 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.7, 0.7, 0.8, -0.4, -0.5, -0.2 \rangle, \langle x_2, 0.6, 0.7, 0.3, -0.4, -0.3, -0.2 \rangle \} \right\rangle \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNCSG α -continuous mapping but not a BNSC-continuous mapping.

Theorem 5.8. Every BNSC α -continuous mapping is a BNSG α -continuous mapping but converse not true.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSC α -continuous mapping and let B be BNOS in Y . Then $\psi^{-1}(B)$ is a BN α CS in X . Since every BN α CS is a BNSG α CS, $\psi^{-1}(B)$ is a BNSG α CS in (X, τ_1, E) . Hence ψ is a BNSG α -continuous mapping. \square

Example 5.9. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \left\langle e_1, \{ \langle x_1, 0.5, 0.5, 0.5, -0.4, -0.4, -0.4 \rangle, \langle x_2, 0.4, 0.4, 0.4, -0.5, -0.5, -0.5 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.4, 0.4, 0.4, -0.5, -0.5, -0.5 \rangle, \langle x_2, 0.5, 0.5, 0.5, -0.4, -0.4, -0.4 \rangle \} \right\rangle \right\}$$

$$B_2 = \left\{ \left\langle e_1, \{ \langle x_1, 0.6, 0.6, 0.6, -0.4, -0.4, -0.4 \rangle, \langle x_2, 0.4, 0.4, 0.4, -0.6, -0.6, -0.6 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.4, 0.4, 0.4, -0.6, -0.6, -0.6 \rangle, \langle x_2, 0.6, 0.6, 0.6, -0.4, -0.4, -0.4 \rangle \} \right\rangle \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSG α -continuous mapping but not a BNSC α -continuous mapping.

Theorem 5.10. Every BNSCSG α -continuous mapping is a BNSCG α -continuous mapping but converse not true.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSCSG α -continuous mapping and let B be BNOS in Y . Since every BNOS is a BNSG α OS, B is a BNSG α OS in Y . Also, since ψ is a BNSCSG α -continuous mapping, $\psi^{-1}(B)$ is a BNCS in X . Since every BNCS is a BNSG α CS, $\psi^{-1}(B)$ is a BNSG α CS in (X, τ_1, E) . Hence ψ is a BNSG α -continuous mapping. \square

Example 5.11. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \left\langle e_1, \{ \langle x_1, 0.3, 0.3, 0.3, -0.4, -0.4, -0.4 \rangle, \langle x_2, 0.4, 0.4, 0.4, -0.5, -0.5, -0.5 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.2, 0.2, 0.4, -0.3, -0.3, -0.5 \rangle, \langle x_2, 0.3, 0.3, 0.3, -0.4, -0.4, -0.5 \rangle \} \right\rangle \right\}$$

$$B_2 = \left\{ \left\langle e_1, \{ \langle x_1, 0.8, 0.8, 0.7, -0.3, -0.3, -0.4 \rangle, \langle x_2, 0.6, 0.6, 0.4, -0.7, -0.7, -0.4 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.4, 0.4, 0.4, -0.6, -0.6, -0.6 \rangle, \langle x_2, 0.6, 0.6, 0.6, -0.4, -0.4, -0.4 \rangle \} \right\rangle \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSCG α -continuous mapping but not a BNSCSG α -continuous mapping.

Theorem 5.12. Every BNSCSG α -continuous mapping is a BNSC-continuous mapping but converse not true.

Proof. Let $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a BNSCSG α -continuous mapping and let B be BNOS in Y . Since every BNOS is a BNSG α OS, B is a BNSG α OS in Y . Also, since ψ is a BNSCSG α -continuous mapping, $\psi^{-1}(B)$ is a BNCS in X . Hence ψ is a BNSC-continuous mapping. \square

Example 5.13. Let $X = \{a, b\}, Y = \{u, v\}, E = \{e_1, e_2\}$. Then

$$B_1 = \left\{ \left\langle e_1, \{ \langle x_1, 0.3, 0.3, 0.3, -0.4, -0.4, -0.4 \rangle, \langle x_2, 0.4, 0.4, 0.4, -0.5, -0.5, -0.5 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.2, 0.2, 0.4, -0.3, -0.3, -0.5 \rangle, \langle x_2, 0.3, 0.3, 0.3, -0.4, -0.4, -0.5 \rangle \} \right\rangle \right\}$$

$$B_2 = \left\{ \left\langle e_1, \{ \langle x_1, 0.8, 0.8, 0.7, -0.3, -0.3, -0.4 \rangle, \langle x_2, 0.6, 0.6, 0.4, -0.7, -0.7, -0.4 \rangle \} \right\rangle, \left\langle e_2, \{ \langle x_1, 0.4, 0.4, 0.4, -0.6, -0.6, -0.6 \rangle, \langle x_2, 0.6, 0.6, 0.6, -0.4, -0.4, -0.4 \rangle \} \right\rangle \right\}$$

Then $\tau_1 = \{0, B_1, 1\}$ and $\tau_2 = \{0, B_2, 1\}$ are BNSTs on X and Y respectively. Now we define a bijective mapping $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ by $\psi(a) = u$ and $\psi(b) = v$. Then ψ is a BNSC-continuous mapping but not a BNSCSG α -continuous mapping.

Theorem 5.14. 1. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCG α -continuous mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a BNSC-continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a BNSCG α -continuous mapping.

2. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCG α -continuous mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a BNSC α -continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a BNSG α -continuous mapping.

3. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a BNSCG α -irresolute mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a BNSCG α -continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a BNSG α -continuous mapping.

4. If $\psi : (X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ is a $BNSG\alpha$ -irresolute mapping and $\phi : (Y, \tau_2, E) \rightarrow (Z, \tau_3, E)$ is a $BNSCG\alpha$ -continuous mapping, then $\phi \circ \psi : (X, \tau_1, E) \rightarrow (Z, \tau_3, E)$ is a $BNSCG\alpha$ -continuous mapping.

Proof. 1. Let B be a BNOS in Z . Since ϕ is a BNS-continuous mapping, $\phi^{-1}(B)$ is BNOS in Y . Since ψ is a $BNSCG\alpha$ -continuous mapping, $\psi^{-1}(\phi^{-1}(B))$ is a $BNSG\alpha$ CS in X . Hence, $\phi \circ \psi$ is a $BNSCG\alpha$ -continuous mapping.

2. Let B be a BNOS in Z . Since ϕ is a BNSC-continuous mapping, $\phi^{-1}(B)$ is BNCS in Y . Since ψ is a $BNSCG\alpha$ -continuous mapping, $\psi^{-1}(\phi^{-1}(B))$ is a $BNSG\alpha$ OS in X . Hence, $\phi \circ \psi$ is a $BNSG\alpha$ -continuous mapping.

3. Let B be a BNOS in Z . Since ϕ is a $BNSCG\alpha$ -continuous mapping, $\phi^{-1}(B)$ is $BNSG\alpha$ CS in Y . Since ψ is a $BNSCG\alpha$ -irresolute mapping, $\psi^{-1}(\phi^{-1}(B))$ is a $BNSG\alpha$ OS in X . Hence, $\phi \circ \psi$ is a $BNSG\alpha$ -continuous mapping.

4. Let B be a BNOS in Z . Since ϕ is a $BNSCG\alpha$ -continuous mapping, $\phi^{-1}(B)$ is $BNSG\alpha$ CS in Y . Since ψ is a $BNSG\alpha$ -irresolute mapping, $\psi^{-1}(\phi^{-1}(B))$ is a $BNSG\alpha$ CS in X . Hence, $\phi \circ \psi$ is a $BNSCG\alpha$ -continuous mapping. □

6 Conclusion

In this paper, we have introduced bipolar neutrosophic soft contra generalized pre-continuous mapping and bipolar neutrosophic soft contra generalized α -continuous mapping. Results in this paper shows that preservation of topological structures such as closeness and openness by various continuity mappings.

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