On Linguistic Neutrosophic Semi-irresolute mappings and Semi-homoeomorphism

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Abstract

The purpose of this article is to discuss about linguistic neutrosophic semi-irresolute mapping and linguistic neutrosophic locally semi-irresolute mapping in linguistic neutrosophic topological spaces. It is examined how these mappings relate to other mappings, as well as some of their characteristics. Moreover, a brief introduction and analysis of the linguistic neutrosophic semi-homeomorphism and linguistic neutrosophic semi-c-homeomorphism are presented with appropriate examples.

Keywords: Linguistic Neutrosophic semi-open Mapping; Linguistic Neutrosophic semi-irresolute mapping; Linguistic Neutrosophic locally semi-irresolute mapping; Linguistic Neutrosophic semi-homeomorphism; Linguistic Neutrosophic semi-c-homeomorphism;

1 Introduction

There was a requirement for the indeterminacy membership to represent inconsistent linguistic information even though there exists an intuitionistic linguistic variable made up of degrees of truth and falsity membership. This idea originated from Fang and Ye,[6] who introduced linguistic neutrosophic numbers. Smarandache[9] combined indeterminacy membership with existing membership in intuitionistic fuzzy sets[1] to develop the idea of neutrosophic sets. Gayathri and Helen[7] begot a new concept, by mingling linguistic neutrosophic numbers and topological spaces, named linguistic neutrosophic topological spaces.
Irresolute mappings play a momentous role in the study of topological spaces which was introduced by Crossley.[5] Researchers have examined irresolute mappings in considerable detail. The article provides an analysis of some properties and implications of linguistic neutrosophic semi-irresolute mappings in a novel linguistic neutrosophic topological space. Through linguistic neutrosophic semi-open mappings, a new mapping class referred to as linguistic neutrosophic semi-homomorphism and linguistic neutrosophic semi-c-homeomorphism are instigated.

2 Preambles

Definition 2.1. [9] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is
$T_A : S \to]0^-, 1^+[, I_A : S \to]0^-, 1^+[, F_A : S \to]0^-, 1^+[$
There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [9] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point s in S, $T_A(x), I_A(x), F_A(x) \in$

$[0, 1]$.
When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$.
When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 2.3. [6] Let $S = \{s_\theta | \theta = 0, 1, 2, ..., \tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable.

Definition 2.4. [6] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t + 1$ and $\overline{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,
$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \leq \theta + \psi + \sigma \leq 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5. [6] Let $\alpha = (s_\theta, s_\psi, s_\sigma), \alpha_1 = (s_{\theta 1}, s_{\psi_1}, s_{\sigma 1}), \alpha_2 = (s_{\theta 2}, s_{\psi_2}, s_{\sigma 2})$ be three LSVNNs, then

(1) $\alpha^c = (s_\sigma, s_\psi, s_\theta)$;

(2) $\alpha_1 \cup \alpha_2 = (max(\theta_1, \theta_2), max(\psi_1, \psi_2), min(\sigma_1, \sigma_2))$;

(3) $\alpha_1 \cap \alpha_2 = (min(\theta_1, \theta_2), min(\psi_1, \psi_2), max(\sigma_1, \sigma_2))$;

(4) $\alpha_1 = \alpha_2$ iff $\theta_1 = \theta_2, \psi_1 = \psi_2, \sigma_1 = \sigma_2$;

Definition 2.6. [7] For a linguistic neutrosophic topology τ, the collection of linguistic neutrosophic sets should obey,

1. $0_{LN}, 1_{LN} \in \tau$
2. $K_1 \bigcap K_2 \in \tau$ for any $K_1, K_2 \in \tau$
3. $\bigcup K_i \in \tau, \forall \{K_i : i \in J\} \subseteq \tau$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

Definition 2.7. [7] Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space (LNTS). Then,

- $(S_{LN}, \tau_{LN})^c$ is the dual linguistic neutrosophic topology, whose elements are ${K^C}_{LN}$ for $K_{LN} \in (S_{LN}, \tau_{LN})$.
- Any open set in τ_{LN} is known as linguistic neutrosophic open set(LNOS).
- Any closed set in τ_{LN} is known as linguistic neutrosophic closed set(LNCS) if and only if it's complement is linguistic neutrosophic open set.

3 Linguistic Neutrosophic Semi-irresolute Mappings

Definition 3.1. A function $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is

1. linguistic neutrosophic continuous function if the inverse image of every linguistic neutrosophic open set F_{LN} is linguistic neutrosophic open in S_{LN}.
2. linguistic neutrosophic semi-continuous mapping if the inverse image $(f_{LN})^{-1}(A_{LN})$ is a linguistic neutrosophic semi-open set in S_{LN} for every linguistic neutrosophic open set in T_{LN}.
3. linguistic neutrosophic semi-irresolute if for any linguistic neutrosophic semi-closed set H_{LN} of T_{LN}, the inverse image $(f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-closed in S_{LN}.
4. linguistic neutrosophic perfectly semi-continuous mapping if the inverse image $f_{LN}(E_{LN})$ of every linguistic neutrosophic semi-open set E_{LN} of T_{LN} is linguistic neutrosophic clopen set in S_{LN}.
5. linguistic neutrosophic open mapping if and only if for every linguistic neutrosophic open set K_{LN} of S_{LN}, $f_{LN}(K_{LN})$ is a linguistic neutrosophic open set in T_{LN}.
6. linguistic neutrosophic semi-open mapping if and only if for every linguistic neutrosophic open set K_{LN} of S_{LN}, $f_{LN}(K_{LN})$ is a linguistic neutrosophic semi-open set in T_{LN}.

7. linguistic neutrosophic closed mapping if and only if for every linguistic neutrosophic closed set E_{LN} of S_{LN}, $f_{LN}(E_{LN})$ is a linguistic neutrosophic closed set in T_{LN}.

8. linguistic neutrosophic semi-closed mapping if and only if for every linguistic neutrosophic closed set E_{LN} of S_{LN}, $f_{LN}(E_{LN})$ is a linguistic neutrosophic semi-closed set in T_{LN}.

Proposition 3.2. *A mapping $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi-irresolute if and only if for every linguistic neutrosophic semi-closed set K_{LN} of T_{LN}, $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-closed in S_{LN}.*

Proof:
Necessity Part: If f_{LN} is linguistic neutrosophic semi-irresolute, then for every linguistic neutrosophic semi-open set H_{LN} of T_{LN}, $(f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-open set in S_{LN}.
If K_{LN} is any linguistic neutrosophic semi-closed set of T_{LN}, then the linguistic neutrosophic set $T_{LN} \backslash K_{LN}$ is linguistic neutrosophic semi-open. Thus, $(f_{LN})^{-1}(T_{LN} \backslash K_{LN})$ is linguistic neutrosophic semi-open, but $(f_{LN})^{-1}(T_{LN} \backslash K_{LN}) = S_{LN} \backslash (f_{LN})^{-1}(K_{LN})$ and hence $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-closed.
Sufficiency Part: Let K_{LN} be a linguistic neutrosophic semi-closed set in T_{LN}. Then $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-closed set in S_{LN}. If H_{LN} is any linguistic neutrosophic semi-open set in T_{LN}, then $T_{LN} \backslash H_{LN}$ is linguistic neutrosophic semi-closed. Also, $(f_{LN})^{-1}(T_{LN} \backslash H_{LN}) = S_{LN} \backslash (f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-closed. Thus, $(f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-open. Hence f_{LN} is linguistic neutrosophic semi-irresolute.

Proposition 3.3. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic continuous and linguistic neutrosophic open mapping and let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then $f_{LN}(H_{LN})$ be a linguistic neutrosophic semi-open set in T_{LN}.*

Proof: Let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then there exists a linguistic neutrosophic open set M_{LN} in S_{LN} such that $M_{LN} \subseteq H_{LN} \subseteq LNCl(M_{LN})$. As f_{LN} is linguistic neutrosophic open, $f_{LN}(m) \in T_{LN}$ where $m \in M_{LN}$. And since f_{LN} is linguistic neutrosophic continuous, $f_{LN}(LNCl(M_{LN})) \subseteq LNCl(f_{LN}(M_{LN}))$. Hence $f_{LN}(H_{LN})$ be a linguistic neutrosophic semi-open set in T_{LN}.

Proposition 3.4. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic continuous and linguistic neutrosophic open mapping and let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then f_{LN} is linguistic neutrosophic semi-irresolute.*

Proof: Let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then there exists a linguistic neutrosophic open set M_{LN} in S_{LN} such that $M_{LN} \subseteq H_{LN} \subseteq LNCl(M_{LN})$. It is true that, $(f_{LN})^{-1}(LNCl(M_{LN})) = LNCl((f_{LN})^{-1}(M_{LN}))$. Also, $(f_{LN})^{-1}(M_{LN}) \subseteq (f_{LN})^{-1}(H_{LN}) \subseteq (f_{LN})^{-1}(LNCl(M_{LN}))$
$= LNCl((f_{LN})^{-1}(M_{LN}))$. Since f_{LN} is linguistic neutrosophic continuous, $(f_{LN})^{-1}(M_{LN})$ is a linguistic neutrosophic open set. Thus, $(f_{LN})^{-1}(H_{LN})$ is a linguistic neutrosophic open set. Hence f_{LN} is linguistic neutrosophic semi-irresolute.

Proposition 3.5. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic perfectly semi-continuous function in S_{LN}, then f_{LN} is linguistic neutrosophic semi-irresolute.*

Proof: Let K_{LN} be a linguistic neutrosophic semi-open set in (T_{LN}, η_{LN}). Then, $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic cl-open set. As a linguistic neutrosophic cl-open set is linguistic neutrosophic semi-open set, $(f_{LN})^{-1}(K_{LN})$ is obviously a linguistic neutrosophic semi-open set. The reverse implication need not be true always, thus demonstrating the validity of the counter example.

Counter Example 3.6. Let the universe of discourse be $U = \{u, v, w, x\}$ and let $S_{LN} = \{w\} = T_{LN}$. The set of all linguistic term set be L= {no healing(l_0), deterioting(l_1), chronic(l_2), some what chronic(l_3), extremely chronic(l_4), very ill(l_5), ill(l_6), no healing(l_7), healing(l_8), slowly healing(l_9), fastly healing(l_{10})}.
Let f_{LN} be the mapping from (S_{LN}, τ_{LN}) to (T_{LN}, η_{LN}) defined by $f_{LN}(a) = b, f_{LN}(b) = c, f_{LN}(c) = a$. Now, $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}, \eta_{LN} = \{0_{LN}, 1_{LN}, A_{LN}, B_{LN}\}$ where $K_{LN} = \{\langle w, (l_6, l_2, l_3)\rangle\}$ and $A_{LN} = \{\langle w, (l_2, l_6, l_3)\rangle\}, B_{LN} = \{\langle w, (l_5, l_2, l_0)\rangle\}$. In (T_{LN}, η_{LN}), the set of all linguistic neutrosophic semi-open is, $\{0_{LN}, 1_{LN}, A_{LN}\}$. Thus, the map f_{LN} is linguistic neutrosophic semi-irresolute but not linguistic neutrosophic perfectly semi-continuous.

Proposition 3.7. *A mapping $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi-irresolute then for every linguistic neutrosophic set K_{LN} of S_{LN}, $f_{LN}(LNSCl(K_{LN})) \subseteq LNSCl(f_{LN}(K_{LN}))$.*

Proof: For each linguistic neutrosophic set K_{LN} in S_{LN}, $LNSCl(f_{LN}(K_{LN}))$ is linguistic neutrosophic semi-closed set in T_{LN}. As f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))$ is linguistic neutrosophic semi-closed set in S_{LN}. Since $K_{LN} \subseteq (f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))$, from the definition of linguistic neutrosophic semi-closure, $LNSCl(K_{LN}) \subseteq (f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))$. Obviously, $f_{LN}(LNSCl(K_{LN})) \subseteq f_{LN}((f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))) = LNSCl(f_{LN}(K_{LN}))$.

Proposition 3.8. *A mapping $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi-irresolute if and only if for all K_{LN} in (S_{LN}, τ_{LN}), $LNSCl((f_{LN})^{-1}(K_{LN})) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN}))$.*

Proof:
Necessity Part: Let K_{LN} be a linguistic neutrosophic semi-closed set in T_{LN} and this implies, $(f_{LN})^{-1}(LNSCl(K_{LN}))$ is linguistic neutrosophic semi-closed in S_{LN}. Since $(f_{LN})^{-1}(K_{LN}) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN}))$. And also from the definition of linguistic neutrosophic semi-closure, $LNSCl((f_{LN})^{-1}(K_{LN})) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN}))$. Sufficiency Part : If K_{LN} is linguistic neutrosophic semi-closed in T_{LN}, then $K_{LN} = LNSCl(K_{LN})$. By hypothesis, $(f_{LN})^{-1}(K_{LN}) \subseteq LNSCl((f_{LN})^{-1}(K_{LN})) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN})) = (f_{LN})^{-1}(K_{LN})$.

3.1 Composition of Linguistic Neutrosophic Semi-irresolute Mappings

Proposition 3.9. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ and $g_{LN} : (T_{LN}, \eta_{LN}) \to (P_{LN}, \mu_{LN})$ be linguistic neutrosophic semi-irresolute, then their composition $(g_{LN} \circ f_{LN}) : (S_{LN}, \tau_{LN}) \to (P_{LN}, \mu_{LN})$ is linguistic neutrosophic semi-irresolute.*

Proof: Let K_{LN} be a linguistic neutrosophic semi-open set in (P_{LN}, μ_{LN}), then $(g_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-open in (T_{LN}, η_{LN}) and $(f_{LN})^{-1}((g_{LN})^{-1}(K_{LN}))$ is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}), since f_{LN} and g_{LN} are linguistic neutrosophic semi-irresolute. Therefore, $(f_{LN})^{-1}((g_{LN})^{-1}(K_{LN})) = (g_{LN} \circ f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-open and hence $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi-irresolute.

Proposition 3.10. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be linguistic neutrosophic semi-irresolute and $g_{LN} : (T_{LN}, \eta_{LN}) \to (P_{LN}, \mu_{LN})$ be linguistic neutrosophic semi continuous, then their composition $(g_{LN} \circ f_{LN}) : (S_{LN}, \tau_{LN}) \to (P_{LN}, \mu_{LN})$ is linguistic neutrosophic semi continuous.*

Proof: Let U_{LN} be any linguistic neutrosophic semi closed set in (P_{LN}, μ_{LN}). Since g_{LN} is linguistic neutrosophic semi continuous, $(g_{LN})^{-1}(U_{LN})$ is linguistic neutrosophic semi closed set in (T_{LN}, η_{LN}). Since f_{LN} is linguistic neutrosophic semi irresolute, $(f_{LN})^{-1}(g^{-1}(U_{LN})) = (g_{LN} \circ f_{LN})^{-1}(U_{LN})$ is linguistic neutrosophic semi closed set in (S_{LN}, τ_{LN}). Thus, $(g_{LN} \circ f_{LN}) : (S_{LN}, \tau_{LN}) \to (P_{LN}, \mu_{LN})$ is linguistic neutrosophic semi continuous.

Proposition 3.11. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ and $g_{LN} : (T_{LN}, \eta_{LN}) \to (P_{LN}, \mu_{LN})$, where (S_{LN}, τ_{LN}), (T_{LN}, η_{LN}) and (P_{LN}, μ_{LN}) are linguistic neutrosophic topological spaces. If f_{LN} is linguistic neutrosophic semi irresolute and g_{LN} is linguistic neutrosophic semi continuous, then $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi continuous function.*

Proof: Let K_{LN} be a linguistic neutrosophic semi-open set in (P_{LN}, μ_{LN}), then $(g_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-open in (T_{LN}, η_{LN}) and $(f_{LN})^{-1}((g_{LN})^{-1}(K_{LN}))$ is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}), since f_{LN} is linguistic neutrosophic semi irresolute and g_{LN} is linguistic neutrosophic semi continuous. Thus, $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi continuous function.

Proposition 3.12. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be any linguistic neutrosophic mapping and $g_{LN} : (T_{LN}, \eta_{LN}) \to (P_{LN}, \mu_{LN})$ be linguistic neutrosophic semi irresolute and one to one. If their composition $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi-closed function, then the mapping f_{LN} is linguistic neutrosophic semi-closed.*

Proof: Let V_{LN} be any linguistic neutrosophic closed set in (S_{LN}, τ_{LN}). Since $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi-closed, the linguistic neutrosophic set $(g_{LN} \circ f_{LN})(V_{LN})$ is linguistic neutrosophic semi-closed in (P_{LN}, μ_{LN}). As g_{LN} is linguistic neutrosophic semi irresolute, $(g_{LN})^{-1}((g_{LN} \circ f_{LN})(V_{LN})) = f_{LN}(V_{LN})$ is linguistic neutrosophic semi-closed in (T_{LN}, η_{LN}). Thus, f_{LN} is linguistic neutrosophic semi-closed.

3.2 Linguistic Neutrosophic Locally Semi-irresolute mappings

Definition 3.13. A linguistic neutrosophic function $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ is

1. linguistic neutrosophic locally continuous function if for each $V_{LN} \in LNOS(T_{LN}, \eta_{LN}), f^{-1}(V_{LN})$ is linguistic neutrosophic locally closed set in S_{LN}.

2. linguistic neutrosophic locally semi-continuous function if for each $V_{LN} \in LNOS(T_{LN}, \eta_{LN}), f^{-1}(V_{LN})$ is linguistic neutrosophic locally semi closed set in S_{LN}.

3. linguistic neutrosophic locally irresolute if for each linguistic neutrosophic locally closed set V_{LN} in $T_{LN}, (f_{LN})^{-1}(V_{LN}) \in LNLC(S_{LN})$, where $LNLC(S_{LN})$ is the set of all linguistic neutrosophic locally closed sets in S_{LN}.

4. linguistic neutrosophic locally semi-irresolute if for each linguistic neutrosophic locally semi-closed set V_{LN} in $T_{LN}, (f_{LN})^{-1}(V_{LN}) \in LNLSC(S_{LN})$, where $LNLSC(S_{LN})$ is the set of all linguistic neutrosophic locally semi-closed sets in S_{LN}.

Proposition 3.14. *A linguistic neutrosophic locally irresolute map $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ is linguistic neutrosophic locally continuous function.*

Proof: Let f_{LN} be a linguistic neutrosophic locally irresolute map. And let V_{LN} be a linguistic neutrosophic closed in S_{LN}. Then $(f_{LN})^{-1}(V_{LN})$ is linguistic neutrosophic closed in S_{LN}. As every linguistic neutrosophic closed set is linguistic neutrosophic locally closed, we have, $(f_{LN})^{-1}(V_{LN})$ is linguistic neutrosophic locally closed in S_{LN}. Thus, f_{LN} is linguistic neutrosophic locally continuous function.

Remark 3.15. The reverse implication need not be true always, thus demonstrating the validity of the counter example.

Counter Example 3.16. Let the universe of discourse and the set of all linguistic terms be as in example(3.6). And $S_{LN} = \{y, z\} = T_{LN}, \tau_{LN} = \{0_{LN}, 1_{LN}, A_{LN}, B_{LN}, C_{LN}\}$ defines linguistic neutrosophic topology where $A_{LN} = \{\langle y, (1_3, l_5, l_4)\rangle, \langle z, (l_6, l_2, l_5)\rangle\}$, $B_{LN} = \{\langle y, (l_2, l_6, l_{10})\rangle, \langle z, (l_0, l_3, l_5)\rangle\}$ and $C_{LN} = \{\langle y, (1_4, l_5, l_4)\rangle, \langle z, (l_5, l_2, l_6)\rangle\}$. Here the mapping f_{LN} is linguistic neutrosophic locally continuous but not linguistic neutrosophic locally irresolute.

Proposition 3.17. *A linguistic neutrosophic locally semi-irresolute map $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ is linguistic neutrosophic locally semi-continuous function.*

Proof: Let f_{LN} be a linguistic neutrosophic locally semi-irresolute map. And let V_{LN} be a linguistic neutrosophic semi-closed in S_{LN}. Then $(f_{LN})^{-1}(V_{LN})$ is linguistic neutrosophic semi-closed in S_{LN}. As every linguistic neutrosophic semi-closed set is linguistic neutrosophic locally semi-closed, $(f_{LN})^{-1}(V_{LN})$ is linguistic neutrosophic locally semi-closed in S_{LN}. Thus, f_{LN} is linguistic neutrosophic locally semi-continuous function.

Remark 3.18. The reverse implication need not be true always, thus demonstrating the validity of the counter example.

Counter Example 3.19. Consider the counter example(3.15). Then, any linguistic neutrosophic set $H_{LN} = \{\langle y, (1_3, l_6, l_4)\rangle, \langle z, (l_6, l_3, l_1)\rangle$ is semi-open in (T_{LN}, η_{LN}) whose inverse image is not linguistic neutrosophic locally semi-closed in (S_{LN}, τ_{LN}). Thus, the function f_{LN} is not linguistic neutrosophic semi-irresolute.

4 Linguistic Neutrosophic Semi Homeomorphisms

Definition 4.1. The spaces (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) are linguistic neutrosophic semi-homeomorphic if and only if there exists a linguistic neutrosophic function $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ such that f_{LN} is linguistic neutrosophic injective, linguistic neutrosophic surjective and linguistic neutrosophic continuous.

Definition 4.2. The spaces (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) are linguistic neutrosophic semi-homeomorphic if and only if there exists a linguistic neutrosophic function $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ such that f_{LN} is linguistic neutrosophic injective, linguistic neutrosophic surjective and linguistic neutrosophic semi-continuous.

Proposition 4.3. *If $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ is a linguistic neutrosophic homeomorphism, then the map f_{LN} is a linguistic neutrosophic semi-homeomorphism.*

Proof: Let $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic homeomorphism, (i.e) f_{LN} is linguistic neutrosophic bijective, linguistic neutrosophic continuous and linguistic neutrosophic semi-open mapping. That is, the image and inverse image of linguistic neutrosophic open sets are linguistic neutrosophic open in S_{LN} and T_{LN} respectively. We know that, every linguistic neutrosophic open set is linguistic neutrosophic semi-open. Then the the image and inverse image of linguistic neutrosophic semi-open sets are linguistic neutrosophic semi-open in S_{LN} and T_{LN} respectively. Thus, f_{LN} is a linguistic neutrosophic semi-homeomorphism.

Remark 4.4. A linguistic neutrosophic semi-homeomorphism need not be a linguistic neutrosophic homeomorphism for the most part.

Counter Example 4.5. Let the universe of discourse be $U = \{p, q, r, s, t\}$ and let $S_{LN} = \{q, r\} = T_{LN}$. The set of all linguistic term set be L= {never familiar(l_0), almost never familiar(l_1), slightly familiar(l_2), some what familiar(l_3),occasionally familiar(l_4),moderately familiar(l_5), almost every time familiar(l_6), frequently familiar(l_7), extremely familiar(l_8)}. Let $f : (S_{LN}, \tau_{LN}) \Rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic mapping defined by $f_{LN}(a) = b, f_{LN}(b) = c, f_{LN}(c) = a$, where $\tau_{LN} = \{0_{LN}, 1_{LN}, A_{LN}, B_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$. The linguistic neutrosophic sets A_{LN}, B_{LN} and H_{LN} are given by $A_{LN} = \{\langle p, (l_2, l_0, l_4)\rangle, \langle t, (l_3, l_1, l_5)\rangle\}, B_{LN} = \{\langle p, (l_3, l_2, l_2)\rangle, \langle t, (l_0, l_1, l_3)\rangle\}$ and $H_{LN} = \{\langle p, (l_2, l_3, l_2)\rangle, \langle t, (l_3, l_0, l_1)\rangle\}$ respectively. Here the mapping f_{LN} is linguistic neutrosophic semi-homeomorphism but not a linguistic neutrosophic homeomorphism as f_{LN} is not a linguistic neutrosophic continuous map.

Proposition 4.6. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic one to one, onto and linguistic neutrosophic continuous map, then the following are equivalent.*

(a) the map f_{LN} is linguistic neutrosophic homeomorphism.

(b) the map f_{LN} is linguistic neutrosophic closed map.

(c) the map f_{LN} is linguistic neutrosophic open map.

Proof: $(a) \Rightarrow (b)$: Let f_{LN} be a linguistic neutrosophic homeomorphism. From the definition, it is clear that f_{LN} is linguistic neutrosophic open map. Let A_{LN} be a linguistic neutrosophic closed set in S_{LN}. As f_{LN} is linguistic neutrosophic open map, $f_{LN}(S_{LN} \backslash A_{LN}) = T_{LN} \backslash f_{LN}(A_{LN})$ is linguistic neutrosophic open set in T_{LN}. Thus $f_{LN}(A_{LN})$ is linguistic neutrosophic closed set and the map f_{LN} is linguistic neutrosophic closed.
$(a) \Rightarrow (c)$: Let f_{LN} be a linguistic neutrosophic homeomorphism. From the definition, it is clear that f_{LN} is linguistic neutrosophic open map.
$(b) \Rightarrow (c)$: Proof is obvious.

Proposition 4.7. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic one to one, onto and linguistic neutrosophic continuous map, then the following are equivalent.*

(a) the map f_{LN} is linguistic neutrosophic semi-homeomorphism

(b) the map f_{LN} is linguistic neutrosophic semi-closed map

(c) the map f_{LN} is linguistic neutrosophic semi-open map

Proof: $(a) \Rightarrow (b)$: Let f_{LN} be a linguistic neutrosophic semi-homeomorphism. From the definition, evidently f_{LN} is linguistic neutrosophic semi-open map. Let A_{LN} be a linguistic neutrosophic semi-closed set in S_{LN}. As f_{LN} is linguistic neutrosophic semi-open map, $f_{LN}(S_{LN} \backslash A_{LN}) = T_{LN} \backslash f_{LN}(A_{LN})$ is linguistic neutrosophic semi-open set in T_{LN}. Thus $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-closed set and the map f_{LN} is linguistic neutrosophic semi-closed.
$(a) \Rightarrow (c)$: Let f_{LN} be a linguistic neutrosophic semi-homeomorphism. It is apparent from the definition that f_{LN} is linguistic neutrosophic semi-open map.
$(b) \Rightarrow (c)$: Proof is obvious.

Definition 4.8. A bijective map $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ is said to be linguistic neutrosophic semi-c-homeomorphism if the linguistic neutrosophic map f_{LN} is linguistic neutrosophic irresolute and its inverse $(f_{LN})^{-1}$ is linguistic neutrosophic irresolute. The set of all linguistic neutrosophic semi-c-homeomorphisms is denoted by $LNSCH(S_{LN}, \tau_{LN})$.

Proposition 4.9. *The composition of any two linguistic neutrosophic semi-c-homeomorphism is a linguistic neutrosophic semi-c-homeomorphism.*

Proof: Let $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ and $g_{LN} : (T_{LN}, \eta_{LN}) \rightarrow (P_{LN}, \mu_{LN})$ be two linguistic neutrosophic semi-c-homeomorphisms. Let B_{LN} be a linguistic neutrosophic semi-closed set in (P_{LN}, μ_{LN}). Then, $(g_{LN})^{-1}(B_{LN})$ is linguistic neutrosophic semi-closed in T_{LN}, since g_{LN} is linguistic neutrosophic irresolute. As f_{LN} is linguistic neutrosophic irresolute, $(f_{LN})^{-1}((g_{LN})^{-1}(B_{LN})) = (g_{LN} \circ f_{LN})^{-1}(B_{LN})$ is linguistic neutrosophic semi-closed in S_{LN}. Thus, $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic irresolute.
Let C_{LN} be a linguistic neutrosophic semi-closed set in (S_{LN}, τ_{LN}). As $(f_{LN})^{-1}$ is linguistic neutrosophic semi-irresolute, ${(f_{LN})^{-1}}^{-1}(C_{LN})$ is linguistic neutrosophic semi-closed set in (T_{LN}, η_{LN}), (i.e) $f_{LN}(C_{LN})$ is linguistic neutrosophic semi-closed set in T_{LN}. Since $(g_{LN})^{-1}$ is linguistic neutrosophic semi-irresolute, $((g_{LN})^{-1})^{-1}(f_{LN}(C_{LN})) = g_{LN}(f_{LN}(C_{LN}))$ is linguistic neutrosophic semi-closed in P_{LN}, (i.e) $(g_{LN} \circ f_{LN})(C_{LN})$ is linguistic neutrosophic semi-closed set in (P_{LN}, μ_{LN}). This implies, ${(g_{LN} \circ f_{LN})^{-1}}^{-1}(C_{LN})$ is linguistic neutrosophic semi-closed set in (P_{LN}, μ_{LN}). Thus, $(g_{LN} \circ f_{LN})^{-1} : (P_{LN}, \mu_{LN}) \rightarrow (S_{LN}, \tau_{LN})$ is linguistic neutrosophic semi-irresolute and hence $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi-c-homeomorphism.

Proposition 4.10. *The collection $LNSCH(S_{LN}, \tau_{LN})$ forms an equivalence relation.*

Proof:

- Reflexive: Proof is obvious.
- Symmetric: Proof is obvious.
- Transitive: It is evident from last proposition.

Proposition 4.11. *The set $LNSCH(S_{LN}, \tau_{LN})$ forms a group under the composition of linguistic neutrosophic mappings.*

Proof: Define a binary operation $(*_{LN}) : LNSCH(S_{LN}, \tau_{LN}) \times LNSCH(S_{LN}, \tau_{LN}) \rightarrow LNSCH(S_{LN}, \tau_{LN})$ by $(f_{LN}(*_{LN})g_{LN}) = (g_{LN} \circ f_{LN})$. Composition of two linguistic neutrosophic semi-c-homeomorphism is again a linguistic neutrosophic semi-c-homeomorphism and also it is proved that associative law is satisfied by composition of two linguistic neutrosophic mappings. Thirdly, the identity mapping behaves as an identity element which is also a linguistic neutrosophic semi-c-homeomorphism. If the function $f_{LN} \in LNSCH(S_{LN}, \tau_{LN})$, then $(f_{LN})^{-1} \in LNSCH(S_{LN}, \tau_{LN})$ and by the binary operation $(f_{LN}(*_{LN})(f_{LN})^{-1}) = ((f_{LN})^{-1} \circ f_{LN}) = 1$. Ergo, the collection $LNSCH(S_{LN}, \tau_{LN})$ is a group with respect to the composition of linguistic neutrosophic mappings.

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