Linguistic Neutrosophic Semi-Connectedness and Semi-Compactness

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Abstract

The notions of semi-connectedness and semi-compactness in linguistic neutrosophic topological space are presented and some of their properties discussed in this study. Further the idea of linguistic neutrosophic semi-compact space is instigated and investigated.

Keywords: Linguistic neutrosophic semi-connectedness, Linguistic neutrosophic extremely disconnected, Linguistic neutrosophic super semi-connected, Linguistic neutrosophic semi-compactness, Linguistic neutrosophic semi-compact space.

1 Preliminaries

Definition 1.1. [6] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is
$T_A : S \to]0^-, 1^+[, I_A : S \to]0^-, 1^+[, F_A : S \to]0^-, 1^+[$
There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 1.2. [6] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point s in S, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.
When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$.
When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 1.3. [3] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t + 1$ and $\overline{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,
$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \leq \theta + \psi + \sigma \leq 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 1.4. [4] For a linguistic neutrosophic topology τ_{LN}, the collection of linguistic neutrosophic sets should obey,

1. $0_{LN}, 1_{LN} \in \tau_{LN}$
2. $K_1 \bigcap K_2 \in \tau_{LN}$ for any $K_1, K_2 \in \tau_{LN}$
3. $\bigcup K_i \in \tau_{LN}, \forall \{K_i : i \in J\} \subseteq \tau_{LN}$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

2 Linguistic Neutrosophic Semi-Connectedness

Definition 2.1. Let A_{LN} be a linguistic neutrosophic set of (S_{LN}, τ_{LN}). Then A_{LN} is said to be

1. linguistic neutrosophic semi-open set if there exists a linguistic neutrosophic open set $B_{LN} \in \tau_{LN}$ such that $B_{LN} \subseteq A_{LN} \subseteq LNCl(B_{LN})$.

2. linguistic neutrosophic semi-closed set if there exists a linguistic neutrosophic closed set $B_{LN} \in \tau_{LN}$ such that $LNInt(B_{LN}) \subseteq A_{LN} \subseteq B_{LN}$. ι

Definition 2.2. Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space. A linguistic neutrosophic semi separation on S_{LN} is a pair E_{LN} and F_{LN} of non void linguistic neutrosophic semi open sets such that $S_{LN} = E_{LN} \cup F_{LN}$, where $E_{LN} \cap F_{LN} = \phi$.

Definition 2.3. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is said to be a linguistic neutrosophic semi connected space if there exists no linguistic neutrosophic semi separations in S_{LN}. Suppose if S_{LN} has such linguistic neutrosophic semi separation, then (S_{LN}, τ_{LN}) is a linguistic neutrosophic semi disconnected space.

Theorem 2.4. *Let A_{LN} and B_{LN} be linguistic neutrosophic semi separations in a linguistic neutrosophic topological space (S_{LN}, τ_{LN}). If K_{LN} is a linguistic neutrosophic semi connected subspace of S_{LN}, then one has either $K_{LN} \subseteq A_{LN}$ or $K_{LN} \subseteq B_{LN}$.*

Proof:
As A_{LN} and B_{LN} are linguistic neutrosophic semi open sets, we have $K_{LN} \cap A_{LN}$ and $K_{LN} \cap B_{LN}$ are also linguistic neutrosophic semi open sets. Thus, $K_{LN} \cap A_{LN}$ and $K_{LN} \cap B_{LN}$ are linguistic neutrosophic semi separations of K_{LN}, which is a contradiction. Therefore, either $K_{LN} \cap A_{LN}$ or $K_{LN} \cap B_{LN}$ is an empty set and hence we have either $K_{LN} \subseteq A_{LN}$ or $K_{LN} \subseteq B_{LN}$.

Theorem 2.5. *Let K_{LN} is a linguistic neutrosophic semi connected subspace of a linguistic neutrosophic topological space (S_{LN}, τ_{LN}). If $K_{LN} \subseteq M_{LN} \subseteq LNSCl(K_{LN})$, then M_{LN} is linguistic neutrosophic semi connected.*

Proof:
Suppose M_{LN} is not linguistic neutrosophic semi connected, then there exists non void linguistic neutrosophic semi open sets E_{LN} and F_{LN} such that these sets form a linguistic neutrosophic semi separation of M_{LN}. Then, we have either $K_{LN} \subseteq E_{LN}$ or $K_{LN} \subseteq F_{LN}$. Suppose $K_{LN} \subseteq E_{LN}$, then $M_{LN} \subseteq LNSCl(K_{LN}) \subseteq LNSCl(E_{LN})$. So, $M_{LN} \cap B_{LN} \subseteq LNSCl(E_{LN}) \cap B_{LN} = E_{LN} \cap F_{LN} = \phi$. Hence, $E_{LN} \subseteq M_{LN} = \phi$, a contradiction. Thus, M_{LN} is linguistic neutrosophic semi connected.

Theorem 2.6. *A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected if and only if both linguistic neutrosophic semi open and linguistic neutrosophic semi closed sets are only ϕ and S_{LN}.*

Proof:
Necessity Part: Let the linguistic neutrosophic topological space (S_{LN}, τ_{LN}) be linguistic neutrosophic semi connected. Suppose if K_{LN} is both linguistic neutrosophic semi open and linguistic neutrosophic semi closed set in (S_{LN}, τ_{LN}) that is different from ϕ and S_{LN}. Then, $(K_{LN})^c$ is also a linguistic neutrosophic semi open set. Thus,K_{LN} and $(K_{LN})^c$ forms a linguistic neutrosophic semi separation of S_{LN}, which is a contradiction. Therefore, both linguistic neutrosophic semi open and linguistic neutrosophic semi closed sets are only ϕ and S_{LN}.
Sufficiency Part: Let K_{LN} and J_{LN} be a linguistic neutrosophic semi separation of S_{LN} and $K_{LN} \neq S_{LN}$. Since $S_{LN} = K_{LN} \cup J_{LN}, K_{LN} = (J_{LN})^c$. This shows that K_{LN} is both linguistic neutrosophic semi open and linguistic neutrosophic semi closed set in S_{LN} that is different from ϕ and S_{LN}, which is a contradiction. So, (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected.

Theorem 2.7. *Let (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) be two linguistic neutrosophic topological spaces. And $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi continuous function. Then if (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected then, (T_{LN}, η_{LN}) is linguistic neutrosophic semi connected.*

Proof:
Suppose (T_{LN}, η_{LN}) is not linguistic neutrosophic semi connected, then there exists a linguistic neutrosophic semi separation K_{LN} and J_{LN} of (T_{LN}, η_{LN}) . So, $S_{LN} = (f_{LN})^{-1}(K_{LN} \cap J_{LN}) = (f_{LN})^{-1}(K_{LN}) \cap$

$(f_{LN})^{-1}(J_{LN})$. And $(f_{LN})^{-1}(K_{LN}) \cap (f_{LN})^{-1}(J_{LN}) = \phi$.
Clearly, K_{LN} and J_{LN} are different from ϕ, and so $(f_{LN})^{-1}(K_{LN})$ and $(f_{LN})^{-1}(J_{LN})$ forms a linguistic neutrosophic semi separation of S_{LN}, which is a contradiction. Therefore, (T_{LN}, η_{LN}) is linguistic neutrosophic semi-connected.

Theorem 2.8. *Let (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) be two linguistic neutrosophic topological spaces. And f_{LN} : $(S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi-irresolute and onto function and if (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-connected then, (T_{LN}, η_{LN}) is linguistic neutrosophic semi-connected.*

Proof:
Let suppose (T_{LN}, η_{LN}) is not linguistic neutrosophic semi-connected, then there lies a proper linguistic neutrosophic clopen set B_{LN} in (T_{LN}, η_{LN}). As the map f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}(B_{LN}) \in LNSO(S_{LN}, \tau_{LN})$ and $(f_{LN})^{-1}(B_{LN}) \in LNSC(S_{LN}, \tau_{LN})$, which is contrariety.

Definition 2.9. A linguistic neutrosophic semi-open set in a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic regular semi-open set if $K_{LN} = LNSInt(LNSCl(K_{LN}))$. The complement of a linguistic neutrosophic regular semi-open set is a linguistic neutrosophic regular semi-closed set.

Definition 2.10. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic super semi-connected space if there lies no proper linguistic neutrosophic regular semi-open set in (S_{LN}, τ_{LN}).

Theorem 2.11. *Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space, then the results are equivalent.*

1. *(S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.*
2. *for every non-zero linguistic neutrosophic regular semi-open set $B_{LN}, LNSCl(B_{LN}) = 1_{LN}$.*
3. *for every non-zero linguistic neutrosophic regular semi-closed set $A_{LN} \neq 1_{LN}, LNSInt(A_{LN}) = 0_{LN}$.*
4. *there lies no linguistic neutrosophic regular semi-open sets A_{LN} and B_{LN} in $A_{LN} \neq 0_{LN} \neq B_{LN}, A_{LN} \subseteq (B_{LN})^c$.*
5. *there lies no linguistic neutrosophic regular semi-open sets A_{LN} and B_{LN} in $A_{LN} \neq 0_{LN} \neq B_{LN}, B_{LN} = (LNSCl(A_{LN}))^c, A_{LN} = (LNSCl(B_{LN}))^c$.*
6. *there lies no linguistic neutrosophic regular semi-closed sets A_{LN} and B_{LN} in $A_{LN} \neq 1_{LN} \neq B_{LN}, B_{LN} = (LNSInt(A_{LN}))^c, A_{LN} = (LNSInt(B_{LN}))^c$.*

Proof:
$(1) \Rightarrow (2)$: Assume that there lies a $A_{LN} \neq 0_{LN}$ with $A_{LN} \in LNRSO(S_{LN}, \tau_{LN})$ and $LNSCl(A_{LN}) \neq 1_{LN}$. Let $B_{LN} = LNSInt(LNSCl(A_{LN}))^c$, then B_{LN} is a proper linguistic neutrosophic regular semi-open set, which is a contrariety. Thus, $LNSCl(A_{LN}) = 1_{LN}$.
$(2) \Rightarrow (3)$: Let $A_{LN} \neq 0_{LN}$ be a linguistic neutrosophic regular semi-closed set in (S_{LN}, τ_{LN}). If $(0_{LN} \neq B_{LN}) = (A_{LN})^c$, then B_{LN} is linguistic neutrosophic regular semi-open set. From the hypothesis, $LNSCl(B_{LN}) = 1_{LN}$ which shows that $(LNSCl(B_{LN}))^c = 0_{LN}$. Now, $LNSInt((B_{LN})^c) = LNSInt(A_{LN}) = 0_{LN}$.
$(3) \Rightarrow (4)$: Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $A_{LN} \neq 0_{LN} \neq B_{LN}, A_{LN} \subseteq (B_{LN})^c$. As $(B_{LN})^c$ is linguistic neutrosophic regular semi-closed set, we get $(B_{LN})^c = LNSCl(LNSInt((B_{LN})^c))$ and $LNSInt((B_{LN})^c) = 0_{LN}$. Now, $0_{LN} \neq A_{LN} = LNSInt(LNSCl(A_{LN})) \subseteq LNSInt(LNSCl((B_{LN})^c)) = LNSInt(LNSCl(LNSCl(LNSInt((B_{LN})^c)))) = LNSInt(LNSCl(LNSInt((B_{LN})^c))) = LNSInt((B_{LN})^c) = 0_{LN}$, which results in a contradiction.
$(4) \Rightarrow (1)$: Proof is as above.
$(1) \Rightarrow (5)$: Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $A_{LN} \neq 0_{LN} \neq B_{LN}, B_{LN} = (LNSCl(A_{LN}))^c$ and $A_{LN} = (LNSCl(B_{LN}))^c$. Now, $LNSInt(LNSCl(A_{LN})) = LNSInt((B_{LN})^c) = (LNSCl(B_{LN}))^c = A_{LN}, A_{LN} \neq 0_{LN}$ and $A_{LN} \neq 1_{LN}$. Suppose if $A_{LN} = 1_{LN}$, then $1_{LN} = (LNSCl(B_{LN}))^c \Rightarrow LNSCl(B_{LN}) = 0_{LN} \Rightarrow B_{LN} = 0_{LN}$. But $B_{LN} \neq 0_{LN}$. Due to the conflict, it is concluded that the statement (5) holds true.
$(5) \Rightarrow (1)$: Let A_{LN} be a linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $0_{LN} \neq A_{LN} \neq 1_{LN}$. Put $B_{LN} = (LNSCl(A_{LN}))^c$. Now, $B_{LN} \neq 0_{LN}$ and B_{LN} is a linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}). Consider $B_{LN} = (LNSCl(A_{LN}))^c \Rightarrow (LNSCl(B_{LN}))^c = (LNSCl((LNSCl(A_{LN}))^c))^c = LNSInt((LNSCl(A_{LN}))^c)^c = LNSInt(LNSCl(A_{LN})) = A_{LN}$, which results in a contradiction. Ergo,

(S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.
$(5) \Rightarrow (6)$: Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}), with $A_{LN} \neq 1_{LN} \neq B_{LN}$ and $B_{LN} = (LNSInt(A_{LN}))^c, A_{LN} = (LNSInt(B_{LN}))^c$. Take $K_{LN} = (A_{LN})^c$ and $H_{LN} = (B_{LN})^c$, then K_{LN} and H_{LN} are non-empty linguistic neutrosophic regular semi-open sets. Now, $H_{LN} = ((LNSInt(A_{LN}))^c)^c = LNSInt((K_{LN})^c) = (LNSCl(K_{LN}))^c = (LNSCl(H_{LN}))^c$, which is a contradiction. Thus, (S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.
$(6) \Rightarrow (5)$: Proof is same as above.

3 Linguistic Neutrosophic Semi-Compactness

The concept which depends solely upon linguistic neutrosophic semi open sets is linguistic neutrosophic semi-compactness. In this section, linguistic neutrosophic compactness is discussed with some characterizations.

Definition 3.1. A linguistic neutrosophic cover is defined as the collection in which every member is a linguistic neutrosophic semi-open set.

Definition 3.2. A collection of linguistic neutrosophic subsets of S_{LN} has the finite intersection property if for each finite collection $\{A^1{}_{LN}, A^2{}_{LN}, A^3{}_{LN}, ..., A^k{}_{LN}\}$ the common intersection $\bigcap_{r=1}^{n} A^r{}_{LN}$ is non-empty.

Definition 3.3. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact if each linguistic neutrosophic cover of (S_{LN}, τ_{LN}) by semi open sets has a finite linguistic neutrosophic sub-cover, or for every collection A_{LN} of S_{LN}, $\bigcap_{r=1}^{n} A^r{}_{LN} \neq \phi$.

Remark 3.4. Every linguistic neutrosophic compact space is linguistic neutrosophic semi-compact space.

Definition 3.5. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-Lindelof space if each linguistic neutrosophic cover of (S_{LN}, τ_{LN}) by linguistic neutrosophic semi-open sets has a countable sub-cover of S_{LN}.

Definition 3.6. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic countably semi-compact space relative to (S_{LN}, τ_{LN}) if each linguistic neutrosophic countable semi-open cover has a finite linguistic neutrosophic sub-cover.

Theorem 3.7. *Let H_{LN} be a linguistic neutrosophic pre-open set and $K_{LN} \subset H_{LN}$. ThenK_{LN} is linguistic neutrosophic semi-compact if and only if K_{LN} is linguistic neutrosophic semi-compact in H_{LN}.*

Proof:
Necessity part: Let $\mathcal{K}_{LN} = \{(K_{LN})_i, \imath \in \mathcal{I}\}$ be a linguistic neutrosophic semi-open cover of K_{LN} by the linguistic neutrosophic semi-open sets in H_{LN}. Now, $(K_{LN})_i = (M_{LN})_i \cap H_{LN}$ for each $i \in \mathcal{I}$, where $(M_{LN})_i$ is linguistic neutrosophic semi-open. Thus, $\mathcal{M}_{LN} = \{(M_{LN})_i, \imath \in \mathcal{I}\}$ is a cover of K_{LN} by linguistic neutrosophic semi-open sets in (S_{LN}, τ_{LN}). As K_{LN} is linguistic neutrosophic semi-compact, we can find $i_1, i_2, i_3, ...i_n \in \mathcal{I}$ with $K_{LN} \subset \bigcup_{j=1}^{n} ((M_{LN})_i)_j$ and we have $K_{LN} \subset \bigcup_{j=1}^{n} (((M_{LN})_i)_j \cap H_{LN}) = \bigcup_{j=1}^{n} ((K_{LN})_i)_j$. Therefore, K_{LN} is linguistic neutrosophic semi-compact in H_{LN}.
Sufficiency Part: Let $\mathcal{M}_{LN} = \{(M_{LN})_i, \imath \in \mathcal{I}\}$ is a cover of K_{LN} by linguistic neutrosophic semi-open sets in (S_{LN}, τ_{LN}). Then $\mathcal{K_{LN}} = \{(M_{LN})_i \cap H_{LN}, \imath \in \mathcal{I}\}$ is a cover of K_{LN}. Since $\{(M_{LN})_i, \imath \in \mathcal{I}\}$ is is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}) and H_{LN} is linguistic neutrosophic pre-open, for each $i \in \mathcal{I}$, $(M_{LN})_i \cap H_{LN}$ is linguistic neutrosophic semi-open in H_{LN}. As K_{LN} is linguistic neutrosophic semi-compact in H_{LN}, we can find $i_1, i_2, i_3, ...i_n \in \mathcal{I}$ with $K_{LN} \subset \bigcup_{j=1}^{n} (((M_{LN})_i)_j \cap H_{LN}) \subset \bigcup_{j=1}^{n} ((M_{LN})_i)_j$. Ergo, K_{LN} is linguistic neutrosophic semi-compact in S_{LN}.

Theorem 3.8. *Let H_{LN} be a linguistic neutrosophic pre-open set and $K_{LN} \subset H_{LN}$. ThenK_{LN} is linguistic neutrosophic semi-Lindelof if and only if K_{LN} is linguistic neutrosophic semi-Lindelof in H_{LN}.*

Proof: Proof is straight forward.

Theorem 3.9. *Let $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi-irresolute mapping. Then the succeeding results are equivalent.*

1. *If K_{LN} is linguistic neutrosophic semi-Lindelof in (S_{LN}, τ_{LN}), then $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-Lindelof in (T_{LN}, η_{LN}).*

2. *If K_{LN} is linguistic neutrosophic semi-compact in (S_{LN}, τ_{LN}), then $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-compact in (T_{LN}, η_{LN}).*

Proof:
(1): Let $\mathcal{M}_{LN} = \{(M_{LN})_i, 1 \in \mathcal{I}\}$ is a cover of $f_{LN}(K_{LN}$ by linguistic neutrosophic semi-open sets in (T_{LN}, η_{LN}). Then, $\mathcal{K_{LN}} = \{(f_{LN})^{-1}(K_{LN})_i, i \in \mathcal{I}\}$ is a cover of K_{LN}. As f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}((M_{LN})_i)$ is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}). As K_{LN} is linguistic neutrosophic semi-Lindelof, there lies $i_1, i_2, i_3, ...i_n \in \mathcal{I}$ with $K_{LN} \subset \bigcup_{j=1}^{\infty} (f_{LN})^{-1}((M_{LN})_i)_j$. Therefore, $f_{LN}(K_{LN}) \subset \bigcup_{j=1}^{\infty}((M_{LN})_i)_j$, which shows that $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-compact in (T_{LN}, η_{LN}).
(2): Proof is same as of (1).

Definition 3.10. A linguistic neutrosophic topological space is extremely disconnected if the linguistic neutrosophic closure of each linguistic neutrosophic open subset of (S_{LN}, τ_{LN}) is linguistic neutrosophic open.

Definition 3.11. A linguistic neutrosophic topological space is linguistic neutrosophic semi-compact space if any linguistic neutrosophic subset of (S_{LN}, τ_{LN}) which is linguistic neutrosophic semi-compact is linguistic neutrosophic semi-closed in (S_{LN}, τ_{LN}).

Theorem 3.12. *A linguistic neutrosophic topological space is extremely disconnected if the intersection of any two linguistic neutrosophic semi-open subsets of (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-open.*

Proof: Proof is direct.

Theorem 3.13. *Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic semi-T_2 extremely disconnected, then the space is linguistic neutrosophic semi-compact.*

Proof: Let A_{LN} be a linguistic neutrosophic semi-compact subset of (S_{LN}, τ_{LN}) and let $s \notin A_{LN}$. For each $p \in A_{LN}$, there lie two different linguistic neutrosophic semi-open sets B_{LN} and D_{LN} containing s and t respectively. Then we can find $p_1, p_2, ..., p_n \in A_{LN}$ with $A_{LN} \subset \bigcup_{i=1}^{n}(D_{LN})_{p_i}$. Let $B_{LN} = \bigcap_{i=1}^{n}(B_{LN})_{p_i}$. Then B_{LN} is a linguistic neutrosophic semi-open set that contains s and distinct from A_{LN}. Thus, $s \notin LNSCl(A_{LN})$, which results that A_{LN} is linguistic neutrosophic semi-closed.

Theorem 3.14. *Let f_{LN} be a linguistic neutrosophic semi-irresolute and one to one mapping from a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) into a linguistic neutrosophic semi-compact space (T_{LN}, η_{LN}), then the space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact.*

Proof: Let A_{LN} be a linguistic neutrosophic semi-compact set in S_{LN} then $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-compact set in T_{LN}. As the space (T_{LN}, η_{LN}) is linguistic neutrosophic semi-compact, $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-closed in T_{LN}. Since f_{LN} is one to one and irresolute, $(f_{LN})^{-1}(f(A_{LN}))$ is linguistic neutrosophic semi-closed in S_{LN}. Hence, (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact.

References

[1] Atanassov, K.T. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 1986; 20, pp. 87-96.

[2] Chang, C.L. Fuzzy topological spaces, J Math.Anal.Appl. 1968; 24, pp. 182-190.

[3] Coker, D. An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 1997; 88, pp. 81-89.

[4] Engelking.R. General Topology, Translated from the Polish by the author, Second edition Heldermann, Berlin, 1989.

[5] FleischmanW.M. A new extension of countable compactness, Fund. Math. 67 (1970), 1–9.

[6] Gayathri.N, Helen.M. Linguistic Neutrosophic Topology, Neutrosophic Sets and Systems, 46, 2021, 254-267.

[7] Fang, Zebo and Te, Jun. Multiple Attribute Group Decision-Making Method Based on Linguistic Neutrosophic Numbers.Symmetry,9(7), 2017. 111; https://doi.org/10.3390/sTm9070111.

[8] Munkres, James R. Topology: a First Course. Englewood Cliffs, N.J.: Prentice-Hall, 1974.

[9] Nour.T. M. Totally semi-continuous functions, Indian J. Pure Appl.Math., 26(7) (1995), 675 - 678.

[10] Salama.A.A,Alblowi.S.A. Neutrosophic set and Neutrosophic topological spaces, IOSR journal of Mathematics, 3(4), 2012, 31-35.

[11] Smarandache F. A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic. American Research Press, Rehoboth, 1999.

[12] Smarandache F. Neutrosophic set - a generalization of the intuitionistic fuzzy sets, International Journal of pure and applied mathematics, 24, 2005, 287-297.

[13] Zadeh, L.A. Fuzzy Sets. Information and Control, 1965; 8,pp. 338-353.

[14] Zadeh, L.A. The concept of a linguistic variable and its application to approximate reasoning Part I. Inf. Sci. 1975, 8, 199-249.