



Single-Valued Quadripartitioned Neutrosophic Lie-Ideal of Lie-Algebra

Binod Chandra Tripathy ¹, Suman Das ^{2,*}, Rakhal Das ³, Arghadip Debnath ⁴, Dibajyoti Nag ⁵, Dipa Chakraborty ⁶, Asmita Mukherjee ⁷, and Bimal Shil ⁸

¹ Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; tripathybc@gmail.com,

² Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; sumandas18842@gmail.com,

³ Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; rakhalDas95@gmail.com,

⁴ Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; arghadipdeb99@gmail.com,

⁵ Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; dibajyoti.n@gmail.com,

⁶ Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; dipa2902@gmail.com,

⁷ Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India; asmitamukherjee1999@gmail.com

⁸ Department of Statistics, Tripura University, Agartala, 799022, Tripura, India; bimalshil738@gmail.com

* Correspondence: sumandas18842@gmail.com

Abstract

The main aim of this paper is to procure the concept of Single-Valued Quadripartitioned Neutrosophic Lie algebra (in short SVQN-Lie-Algebra). Besides, we study the Single-Valued Quadripartitioned Neutrosophic Lie ideal (in short SVQN-Lie-Ideal) of SVQN-Lie-Algebra. Further, we formulate several interesting results on SVQN-Lie-Algebra and SVQN-Lie-Ideal.

Keywords: SVNS, SVQN-set, Lie-Ideal, Lie-Algebra, SVQN-Lie-Algebra, SVQN-Lie-Ideal

1. Introduction

In nineteenth century, Sophus Lie grounded the concept of Lie groups. Sophus Lie also discovered the notion of Lie algebra. In 1972, Humphreys [15] presented the representation theory of Lie-Algebra. In 2003, Coelho and Nunes [10] proposed an application of Lie-Algebra to mobile robot control. Till now, the concept of Lie theory has been applied in mathematics, physics, continuum mechanics, cosmology and life sciences. The problems of computer vision can also be solved by using the idea of Lie algebra. In 1965, Zadeh [21] grounded the notion of Fuzzy Set (FS) theory. Afterwards, Yehia [19] presented the concept of Fuzzy Lie-Ideals and Fuzzy Lie-Sub-Algebra of Lie-Algebra in 1996. Later on, Yehia [20] also studied the adjoint representation of Fuzzy Lie-Algebra. In 1998, Kim and Lee [16] further studied the Fuzzy Lie-Ideals and Fuzzy Li-Sub-Algebra. The notion of anti-Fuzzy Lie-Ideals of Lie-Algebra was studied by Akram [1]. Later on, Akram [4] studied the concept of generalized Fuzzy Lie-Sub-Algebra in 2008. The concept of Fuzzy Lie-Ideals of Lie-Algebra with the interval-valued membership function was studied by Akram [5]. In 1986, Atanassov [8] grounded the idea of Intuitionistic Fuzzy Set (IFS) theory by introducing the idea of non-membership of a mathematical expression. Afterwards, Akram and Shum [7] grounded the concept of Lie-Algebra on IFSs. The notion of Intuitionistic (S, T)-Fuzzy Lie-Ideals was studied by Akram [2]. In 2008, Akram [3] further studied Intuitionistic Fuzzy Lie-Ideals of Lie-Algebra. Smarandache [17] grounded the idea of Neutrosophic Set (NS) by introducing the indeterminacy membership function of mathematical expression. Later on, Wang et al. [18] defined Single-Valued Neutrosophic Set (SVNS) as a generalization of FS

and IFS. In 2020, Das et al. [12] proposed a multi-criteria decision making algorithm via SVNS environment. Thereafter, Akram et al. [6] introduced the concept of Single-Valued Neutrosophic Lie-Algebra in 2019. Afterwards, Das et al. [13] grounded the notion of d-ideal on NSs. In 2016, Chatterjee et al. [9] presented the idea of Single-Valued Quadripartitioned Neutrosophic Set (SVQN-set) by extending the notion of SVNS. Recently, Das et al. [11] presented the notion of Pentapartitioned neutrosophic Q-Ideal of Pentapartitioned neutrosophic Q-Algebra under the Pentapartitioned Neutrosophic environment.

In this article, we introduce the notion of Single-Valued Quadripartitioned Neutrosophic Lie-Ideal (SVQN-Lie-Ideal) of Single-Valued Quadripartitioned Neutrosophic Lie-Algebra (SVQN-Lie-Algebra). Further, we formulate several interesting results on SVQN-Lie-Ideal of SVQN-Lie-Algebra.

The remaining part of this article has been split into following sections:

In section-2, we recall some basic definitions and results on SVNS, SVQN-set, Lie-Algebra, Lie-Ideal, Neutrosophic Lie-Algebra and Neutrosophic Lie-Ideal those are relevant to the main results of this article. Section-3 introduces the notion of Single-Valued Quadripartitioned Neutrosophic Lie-Algebra and Single-Valued Quadripartitioned Neutrosophic Lie-Ideal. In section-4, we conclude the work done in this article.

2. Some Relevant Results

In this section, we recall some basic definitions and results that are necessary for developing the main results of this article.

Definition 2.1.[15] Let F be a field. Consider a vector space L over F on which $L \times L \rightarrow L$ is defined by $(q, r) \rightarrow [x, y]$ for all $x, y \in L$. Then, L is called Lie algebra if the following condition holds:

- (i) $[q, r]$ is a bilinear,
- (ii) $[q, q]=0$, for all $q \in L$,
- (iii) $[[q, r], s] + [[r, s], q] + [[s, q], r] = 0$, for all $q, r, s \in L$.

Remark 2.1.[15] The associative property was not hold in a Lie algebra for the multiplication operation, i.e., $[[q, r], s] = [q, [r, s]]$ is not true in general for a Lie algebra. But the Lie algebra is anti-commutative, i.e., $[q, r] = -[r, q]$. A sub-set N of a Lie algebra L , which is closed under $[\cdot, \cdot]$ will be called a Lie sub-algebra.

Definition 2.2.[21] A Fuzzy Set (FS) W over a universe of discourse Π is defined as follows:

$$W = \{(\eta, T_W(\eta)) : \eta \in \Pi\},$$

where $T_W(\eta)$ is the truth-membership value of $\eta \in \Pi$ such that $0 \leq T_W(\eta) \leq 1$.

Definition 2.3.[19] A FS $W = \{(\eta, T_W(\eta)) : \eta \in L\}$ is called a Fuzzy Lie ideal (F-L-Ideal) of a Lie algebra L if and only if the following three conditions hold:

- (i) $T_W(q + r) \geq \min \{T_W(q), T_W(r)\}$;
- (ii) $T_W(\alpha q) \geq T_W(q)$;
- (iii) $T_W([q, r]) \geq T_W(q)$, for all $q, r \in L$ and $\alpha \in F$.

Definition 2.4.[17] A Single-Valued Neutrosophic Set (SVNS) W over Π is defined as follows:

$$W = \{(\eta, T_W(\eta), I_W(\eta), F_W(\eta)) : \eta \in \Pi\},$$

where T_W, I_W, F_W are truth, indeterminacy and falsity membership mappings from W to $[0, 1]$, and so $0 \leq T_W(\eta) + I_W(\eta) + F_W(\eta) \leq 3$, for all $\eta \in \Pi$.

Definition 2.5.[13] Assume that $Y = \{(c, T_Y(c), I_Y(c), F_Y(c)) : c \in \Pi\}$ be a SVNS over Π . Then, the sets $W(T_Y, \alpha) = \{c \in \Pi : T_Y(c) \geq \alpha\}$, $W(I_Y, \alpha) = \{c \in \Pi : I_Y(c) \leq \alpha\}$, $W(F_Y, \alpha) = \{c \in \Pi : F_Y(c) \leq \alpha\}$ are respectively called T-level α -cut, I-level α -cut, F-level α -cut of Y .

Definition 2.6.[6] A SVNS $W = \{(q, T_W(q), I_W(q), F_W(q)) : q \in L\}$ on Lie algebra L is called a Single-Valued Neutrosophic Lie algebra (SVN-Lie-Algebra) if the following condition holds:

- (i) $T_W(q + r) \geq \min \{T_W(q), T_W(r)\}$, $I_W(q + r) \geq \min \{I_W(q), I_W(r)\}$ and $F_W(q + r) \leq \max \{F_W(q), F_W(r)\}$;
- (ii) $T_W(\alpha q) \geq T_W(q)$, $I_W(\alpha q) \geq I_W(q)$ and $F_W(\alpha q) \leq F_W(q)$;
- (iii) $T_W([q, r]) \geq \min \{T_W(q), T_W(r)\}$, $I_W([q, r]) \geq \min \{I_W(q), I_W(r)\}$ and $F_W([q, r]) \leq \max \{F_W(q), F_W(r)\}$, for all $q, r \in L$ and $\alpha \in F$.

Definition 2.7.[6] Suppose that L be a Lie algebra over a field F . A SVNS $W = \{(q, T_W(q), I_W(q), F_W(q)) : q \in L\}$ on L is called a single-valued neutrosophic Lie ideal (SVN-Lie-Ideal) if the following conditions hold:

- (i) $T_w(r + q) \geq \min \{T_w(r), T_w(q)\}$, $I_w(r + q) \geq \min \{I_w(r), I_w(q)\}$ and $F_w(r + q) \leq \max \{F_w(r), F_w(q)\}$;
- (ii) $T_w(\alpha q) \geq T_w(q)$, $I_w(\alpha q) \geq I_w(q)$ and $F_w(\alpha q) \leq F_w(q)$;
- (iii) $T_w([r, q]) \geq T_w(r)$, $I_w([r, q]) \geq I_w(r)$ and $F_w([r, q]) \leq F_w(r)$, for all $r, q \in L$.

Remark 2.2.[6] Let $W = \{(q, T_w(q), I_w(q), F_w(q)) : q \in L\}$ be a SVN-Lie-Algebra on a Lie algebra L. Then,

- (i) $T_w(0) \geq T_w(q)$, $I_w(0) \geq I_w(q)$, $F_w(0) \leq F_w(q)$;
- (ii) $T_w(-q) \geq T_w(q)$, $I_w(-q) \geq I_w(q)$, $F_w(-q) \leq F_w(q)$, for all $q \in L$.

Definition 2.8.[9] Suppose that Π be a universal set. Then, a Single-Valued Quadripartitioned Neutrosophic Set (SVQN-set) W over Π is defined as follows:

$$W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in \Pi\},$$

where $T_w(\eta)$, $C_w(\eta)$, $G_w(\eta)$ and $F_w(\eta)$ ($\in [0, 1]$) are the truth, contradiction, ignorance, and false membership values of $\eta \in \Pi$. So, $0 \leq T_w(\eta) + C_w(\eta) + G_w(\eta) + F_w(\eta) \leq 4$, for all $\eta \in \Pi$.

Definition 2.9.[9] Assume that $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in \Pi\}$ and $E = \{(\eta, T_E(\eta), C_E(\eta), G_E(\eta), F_E(\eta)) : \eta \in \Pi\}$ be two SVQN-sets over a fixed set Π . Then,

- (i) $W \subseteq E$ if and only if $T_w(\eta) \leq T_E(\eta)$, $C_w(\eta) \leq C_E(\eta)$, $G_w(\eta) \geq G_E(\eta)$, $F_w(\eta) \geq F_E(\eta)$, $\forall \eta \in \Pi$.
- (ii) $W \cup E = \{(\eta, \max \{T_w(\eta), T_E(\eta)\}, \max \{C_w(\eta), C_E(\eta)\}, \min \{G_w(\eta), G_E(\eta)\}, \min \{F_w(\eta), F_E(\eta)\}) : \eta \in \Pi\}$.
- (iii) $W^c = \{(\eta, F_w(\eta), G_w(\eta), C_w(\eta), T_w(\eta)) : \eta \in \Pi\}$.
- (iv) $W \cap E = \{(\eta, \min \{T_w(\eta), T_E(\eta)\}, \min \{C_w(\eta), C_E(\eta)\}, \max \{G_w(\eta), G_E(\eta)\}, \max \{F_w(\eta), F_E(\eta)\}) : \eta \in \Pi\}$.

3. Single-valued Quadripartitioned Neutrosophic Lie-Ideal

In this section, we procure the notion of SVQN-Lie-Ideal of SVQN-Lie-Algebra. Besides, we study different properties of SVQN-Lie-Ideal, and formulate several results on it.

Definition 3.1. A SVQN-set $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in \Pi\}$ on Lie algebra L is called a SVQN-Lie-Algebra if the following conditions hold:

- (i) $T_w(\eta + \delta) \geq \min \{T_w(\eta), T_w(\delta)\}$, $C_w(\eta + \delta) \geq \min \{C_w(\eta), C_w(\delta)\}$, $G_w(\eta + \delta) \leq \max \{G_w(\eta), G_w(\delta)\}$ and $F_w(\eta + \delta) \leq \max \{F_w(\eta), F_w(\delta)\}$;
- (ii) $T_w(\alpha\eta) \geq T_w(\eta)$, $C_w(\alpha\eta) \geq C_w(\eta)$, $G_w(\alpha\eta) \leq G_w(\eta)$ and $F_w(\alpha\eta) \leq F_w(\eta)$;
- (iii) $T_w([\eta, \delta]) \geq \min \{T_w(\eta), T_w(\delta)\}$, $C_w([\eta, \delta]) \geq \min \{C_w(\eta), C_w(\delta)\}$, $G_w([\eta, \delta]) \leq \max \{G_w(\eta), G_w(\delta)\}$ and $F_w([\eta, \delta]) \leq \max \{F_w(\eta), F_w(\delta)\}$, for all $\eta, \delta \in L$ and $\alpha \in F$.

Definition 3.2. A SVQN-set $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in \Pi\}$ on a Lie algebra L is called a SVQN-Lie-Ideal if the following conditions hold:

- (i) $T_w(\eta + \delta) \geq \min \{T_w(\eta), T_w(\delta)\}$, $C_w(\eta + \delta) \geq \min \{C_w(\eta), C_w(\delta)\}$, $G_w(\eta + \delta) \leq \max \{G_w(\eta), G_w(\delta)\}$ and $F_w(\eta + \delta) \leq \max \{F_w(\eta), F_w(\delta)\}$;
- (ii) $T_w(\alpha\eta) \geq T_w(\eta)$, $C_w(\alpha\eta) \geq C_w(\eta)$, $G_w(\alpha\eta) \leq G_w(\eta)$ and $F_w(\alpha\eta) \leq F_w(\eta)$;
- (iii) $T_w([\eta, \delta]) \geq T_w(\eta)$, $C_w([\eta, \delta]) \geq C_w(\eta)$, $G_w([\eta, \delta]) \leq G_w(\eta)$ and $F_w([\eta, \delta]) \leq F_w(\eta)$, for all $\eta, \delta \in L$.

Theorem 3.1. Suppose that $\{W_i : i \in \Delta\}$ be the family of SVQN-Lie-Ideals on a Lie-Algebra L. Then, $\bigcap W_i = \{(\eta, \wedge T_{N_i}(\eta), \wedge C_{N_i}(\eta), \vee G_{N_i}(\eta), \vee F_{N_i}(\eta)) : \eta \in L\}$ is also a SVQN-Lie-Ideal of L.

Proof. Suppose that $\{W_i : i \in \Delta\}$ be the family of SVQN-Lie-Ideals on a Lie-Algebra L. It is known that, $\bigcap W_i = \{(\eta, \wedge T_{N_i}(\eta), \wedge C_{N_i}(\eta), \vee G_{N_i}(\eta), \vee F_{N_i}(\eta)) : \eta \in L\}$.

Now, we have

$$\begin{aligned} & (i) \quad \wedge T_{N_i}(\eta + \delta) \\ &= \min \{T_{N_i}(\eta + \delta) : i \in \Delta\} \\ &\geq \min \{\min \{T_{N_i}(\eta), T_{N_i}(\delta)\} : i \in \Delta\} \\ &\geq \min \{\wedge T_{N_i}(\eta), \wedge T_{N_i}(\delta)\}, \\ &\wedge C_{N_i}(\eta + \delta) \\ &= \min \{C_{N_i}(\eta + \delta) : i \in \Delta\} \\ &\geq \min \{\min \{C_{N_i}(\eta), C_{N_i}(\delta)\} : i \in \Delta\} \\ &\geq \min \{\wedge C_{N_i}(\eta), \wedge C_{N_i}(\delta)\}, \\ &\vee G_{N_i}(\eta + \delta) \\ &= \max \{G_{N_i}(\eta + \delta) : i \in \Delta\} \\ &\leq \max \{\max \{G_{N_i}(\eta), G_{N_i}(\delta)\} : i \in \Delta\} \\ &\leq \max \{\vee G_{N_i}(\eta), \vee G_{N_i}(\delta)\}, \end{aligned}$$

$$\begin{aligned} & \vee F_{N_i}(\eta + \delta) \\ &= \max \{F_{N_i}(\eta + \delta) : i \in \Delta\} \\ &\leq \max \{\max \{F_{N_i}(\eta), F_{N_i}(\delta)\} : i \in \Delta\} \\ &\leq \max \{\vee F_{N_i}(\eta), \vee F_{N_i}(\delta)\}. \\ \text{(ii)} \quad & \wedge T_{N_i}(\alpha\eta) = \min \{T_{N_i}(\alpha\eta) : i \in \Delta\} \geq \min \{T_{N_i}(\eta) : i \in \Delta\} \geq \wedge T_{N_i}(\eta), \\ & \wedge C_{N_i}(\alpha\eta) = \min \{C_{N_i}(\alpha\eta) : i \in \Delta\} \geq \min \{C_{N_i}(\eta) : i \in \Delta\} \geq \wedge C_{N_i}(\eta), \\ & \vee G_{N_i}(\alpha\eta) = \max \{G_{N_i}(\alpha\eta) : i \in \Delta\} \leq \max \{G_{N_i}(\eta) : i \in \Delta\} \leq \vee G_{N_i}(\eta), \\ & \vee F_{N_i}(\alpha\eta) = \max \{F_{N_i}(\alpha\eta) : i \in \Delta\} \leq \max \{F_{N_i}(\eta) : i \in \Delta\} \leq \vee F_{N_i}(\eta). \\ \text{(iii)} \quad & \wedge T_{N_i}([\eta, \delta]) = \min \{T_{N_i}([\eta, \delta]) : i \in \Delta\} \geq \min \{T_{N_i}(\eta) : i \in \Delta\} \geq \wedge T_{N_i}(\eta), \\ & \wedge C_{N_i}([\eta, \delta]) = \min \{C_{N_i}([\eta, \delta]) : i \in \Delta\} \geq \min \{C_{N_i}(\eta) : i \in \Delta\} \geq \wedge C_{N_i}(\eta), \\ & \vee G_{N_i}([\eta, \delta]) = \max \{G_{N_i}([\eta, \delta]) : i \in \Delta\} \leq \max \{G_{N_i}(\eta) : i \in \Delta\} \leq \vee G_{N_i}(\eta), \\ & \vee F_{N_i}([\eta, \delta]) = \max \{F_{N_i}([\eta, \delta]) : i \in \Delta\} \leq \max \{F_{N_i}(\eta) : i \in \Delta\} \leq \vee F_{N_i}(\eta). \end{aligned}$$

Therefore, $\cap W_i = \{(\eta, \wedge T_{N_i}(\eta), \wedge C_{N_i}(\eta), \vee G_{N_i}(\eta), \vee F_{N_i}(\eta)) : \eta \in L\}$ is a SVQN-Lie-Ideal of the Lie-Algebra L.

Theorem 3.2. Assume that $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in L\}$ be a SVQN-Lie-Algebra on a Lie algebra L. Then,

- (i) $T_w(0) \geq T_w(\delta), C_w(0) \geq C_w(\delta), G_w(0) \leq G_w(\delta), F_w(0) \leq F_w(\delta);$
- (ii) $T_w(-\delta) \geq T_w(\delta), C_w(-\delta) \geq C_w(\delta), G_w(-\delta) \leq G_w(\delta), F_w(-\delta) \leq F_w(\delta),$ for all $\delta \in L.$

Proof. The proof is so easy, so omitted.

Remark 3.1. Every SVQN-Lie-Ideal is a SVQN-Lie-Algebra.

Theorem 3.3. Suppose that $W = \{(\delta, T_w(\delta), C_w(\delta), G_w(\delta), F_w(\delta)) : \delta \in L\}$ be a SVQN-Lie-Ideal of a Lie-Algebra L. Then,

- (i) $T_w(0) \geq T_w(\delta), C_w(0) \geq C_w(\delta), G_w(0) \leq G_w(\delta), F_w(0) \leq F_w(\delta);$
- (ii) $T_w([\delta, \eta]) \geq \max\{T_w(\delta), T_w(\eta)\};$
- (iii) $C_w([\delta, \eta]) \geq \max\{C_w(\delta), C_w(\eta)\};$
- (iv) $G_w([\delta, \eta]) \leq \min\{G_w(\delta), G_w(\eta)\};$
- (v) $F_w([\delta, \eta]) \leq \min\{F_w(\delta), F_w(\eta)\};$
- (vi) $T_w([\delta, \eta]) = T_w(-[\delta, \delta]) = T_w([\eta, \delta]);$
- (vii) $C_w([\delta, \eta]) = C_w(-[\delta, \delta]) = C_w([\eta, \delta]);$
- (viii) $G_w([\delta, \eta]) = G_w(-[\delta, \delta]) = G_w([\eta, \delta]);$
- (ix) $F_w([\delta, \eta]) = F_w(-[\delta, \delta]) = F_w([\eta, \delta]),$ for all $\delta, \eta \in L.$

Proof. The proofs are straightforward, so omitted.

Definition 3.3. Assume that $W = \{(\eta, T_w(\eta), I_w(\eta), F_w(\eta)) : \eta \in L\}$ be a SVNS over a Lie-Algebra L. Suppose that $\alpha, \beta, \gamma, \delta \in [0, 1].$ Then, the sets $L(T_w, \alpha) = \{\eta \in L : T_w(\eta) \geq \alpha\}, L(C_w, \beta) = \{\eta \in L : C_w(\eta) \geq \beta\}, L(G_w, \gamma) = \{\eta \in L : G_w(\eta) \leq \gamma\}, L(F_w, \delta) = \{\eta \in L : F_w(\eta) \leq \delta\}$ are respectively called T-level α -cut, C-level β -cut, G-level γ -cut, F-level δ -cut of W.

Definition 3.4. Suppose that L be a Lie-Algebra. Assume that $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in L\}$ be a SVQN-set over L. Suppose that $\alpha, \beta, \gamma, \delta \in [0, 1].$ Then, $(\alpha, \beta, \gamma, \delta)$ -level subset of W is defined by:

$$L(\alpha, \beta, \gamma, \delta) = \{\eta \in L : T_w(\eta) \geq \alpha, C_w(\eta) \geq \beta, G_w(\eta) \leq \gamma, F_w(\eta) \leq \delta\}.$$

Remark 3.2. Suppose that L be a Lie-Algebra. If $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in L\}$ be a SVQN-set over L, then $L(\alpha, \beta, \gamma, \delta) = L(T_w, \alpha) \cap L(C_w, \beta) \cap L(G_w, \gamma) \cap L(F_w, \delta).$

Proposition 3.1. Suppose that L be a Lie-Algebra. A SVQN-set $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in L\}$ is a SVQN-Lie-Ideal of L if and only if $L(\alpha, \beta, \gamma, \delta)$ is a Lie-Ideal of L for every $\alpha, \beta, \gamma, \delta \in [0, 1].$

Proof. The proof is straightforward, so omitted.

Theorem 3.4. Let L be a Lie-Algebra. Assume that $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in L\}$ be a SVQN-Lie-Ideal of L. Let $\alpha_1, \beta_1, \gamma_1, \delta_1, \alpha_2, \beta_2, \gamma_2, \delta_2 \in [0, 1].$ Then, $L(\alpha_1, \beta_1, \gamma_1, \delta_1) = L(\alpha_2, \beta_2, \gamma_2, \delta_2)$ if and only if $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2.$

Proof. Suppose that L be a Lie-Algebra. Let $W = \{(\eta, T_w(\eta), C_w(\eta), G_w(\eta), F_w(\eta)) : \eta \in L\}$ be a SVQN-Lie-Ideal of L. Let $\alpha_1, \beta_1, \gamma_1, \delta_1, \alpha_2, \beta_2, \gamma_2, \delta_2 \in [0, 1]$ such that $L(\alpha_1, \beta_1, \gamma_1, \delta_1) = L(\alpha_2, \beta_2, \gamma_2, \delta_2).$ Therefore, $\{\eta \in L : T_w(\eta) \geq \alpha_1, C_w(\eta) \geq \beta_1, G_w(\eta) \leq \gamma_1, F_w(\eta) \leq \delta_1\} = \{\eta \in L : T_w(\eta) \geq \alpha_2, C_w(\eta) \geq \beta_2, G_w(\eta) \leq \gamma_2, F_w(\eta) \leq \delta_2\}.$ This is possible only when $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2.$ Therefore, $L(\alpha_1, \beta_1, \gamma_1, \delta_1) = L(\alpha_2, \beta_2, \gamma_2, \delta_2)$ implies $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2.$

Conversely, let $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2.$

Now, $L(\alpha_1, \beta_1, \gamma_1, \delta_1)$
 $= \{\eta \in L : T_W(\eta) \geq \alpha_1, C_W(\eta) \geq \beta_1, G_W(\eta) \leq \gamma_1, F_W(\eta) \leq \delta_1\}$
 $= \{\eta \in L : T_W(\eta) \geq \alpha_2, C_W(\eta) \geq \beta_2, G_W(\eta) \leq \gamma_2, F_W(\eta) \leq \delta_2\}$
 $= L(\alpha_2, \beta_2, \gamma_2, \delta_2).$

Therefore, $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2$ implies $L(\alpha_1, \beta_1, \gamma_1, \delta_1) = L(\alpha_2, \beta_2, \gamma_2, \delta_2).$

Definition 3.5. Assume that L_1 and L_2 be two Lie-Algebras on a common field F . Suppose that f be a bijective mapping from L_1 to L_2 . If $M = \{(\eta, T_M(\eta), C_M(\eta), G_M(\eta), F_M(\eta)) : \eta \in L\}$ be a SVQN-set in L_2 , then $f^{-1}(M)$ defined by $f^{-1}(M) = \{(\eta, f^{-1}(T_M(\eta)), f^{-1}(C_M(\eta)), f^{-1}(G_M(\eta)), f^{-1}(F_M(\eta))) : \eta \in L\}$ is also a SVQN-set in L_1 .

Theorem 3.5. Assume that L_1 and L_2 be two Lie-Algebras on a common field F . Suppose that f be an onto homomorphism from L_1 to L_2 . If $M = \{(\eta, T_M(\eta), C_M(\eta), G_M(\eta), F_M(\eta)) : \eta \in L\}$ is a SVQN-Lie-Ideal of L_2 , then $f^{-1}(M) = \{(\eta, f^{-1}(T_M(\eta)), f^{-1}(C_M(\eta)), f^{-1}(G_M(\eta)), f^{-1}(F_M(\eta))) : \eta \in L\}$ is also a SVQN-Lie-Ideal of L_1 .

Proof. The proof is so easy, so omitted.

Proposition 3.2. Suppose that L_1 and L_2 be two Lie-Algebras. Let f be an epimorphism from L_1 to L_2 . If $M = \{(\eta, T_M(\eta), C_M(\eta), G_M(\eta), F_M(\eta)) : \eta \in L\}$ be a SVQN-Lie-Ideal of L_2 , then $f^{-1}(M^c) = (f^{-1}(M))^c$ is also a SVQN-Lie-Ideal of L_1 .

Proof. The proof is straightforward, so omitted.

Theorem 3.6. Suppose that L_1 and L_2 be two Lie-Algebras. Let f be an epimorphism from L_1 to L_2 . If $M = \{(\eta, T_M(\eta), C_M(\eta), G_M(\eta), F_M(\eta)) : \eta \in L\}$ be an SVQN-Lie-Ideal of L_2 , then $f^{-1}(M) = \{(\eta, f^{-1}(T_M(\eta)), f^{-1}(C_M(\eta)), f^{-1}(G_M(\eta)), f^{-1}(F_M(\eta))) : \eta \in L\}$ is also a SVQN-Lie-Ideal of L_1 .

Proof. The proof is directly holds from Definitions 3.2 and Definition 3.5.

Definition 3.6. Let us consider two Lie-Algebras L_1 and L_2 . Let f be a mapping from a L_1 to L_2 . If $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), F_W(\eta)) : \eta \in L\}$ be a SVQN-set in L_1 , then the image of $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), F_W(\eta)) : \eta \in L\}$ under f denoted by $f(W)$ is a SVQN-set in L_2 , defined as follows:

$$f(T_W)(r) = \begin{cases} \max_{u \in f^{-1}(r)} T_W(u), & \text{if } f^{-1}(r) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}, \text{ for each } r \in L_2$$

$$f(C_W)(r) = \begin{cases} \max_{u \in f^{-1}(r)} C_W(u), & \text{if } f^{-1}(r) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}, \text{ for each } r \in L_2$$

$$f(G_W)(r) = \begin{cases} \min_{u \in f^{-1}(r)} G_W(u), & \text{if } f^{-1}(r) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}, \text{ for each } r \in L_2,$$

$$f(F_W)(r) = \begin{cases} \min_{u \in f^{-1}(r)} F_W(u), & \text{if } f^{-1}(r) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}, \text{ for each } r \in L_2.$$

Theorem 3.7. Let us consider two Lie-Algebras L_1 and L_2 . Suppose that f be an epimorphism from L_1 to L_2 . If $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), F_W(\eta)) : \eta \in L\}$ is a SVQN-Lie-Ideal in L_1 , then the image of $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), F_W(\eta)) : \eta \in L\}$ i.e., $f(W)$ is also a SVQN-Lie-Ideal in L_2 .

Proof. The proof is directly holds from Definition 3.2 and Definition 3.6.

Definition 3.7. Let us consider two Lie-Algebras L_1 and L_2 . Suppose that f be an onto homomorphism from L_1 to L_2 . Let $M = \{(\eta, T_M(\eta), C_M(\eta), G_M(\eta), F_M(\eta)) : \eta \in L\}$ be a SVQN-set in L_2 . Then, we define $L^f = \{(\eta, T_M^f(\eta), C_M^f(\eta), G_M^f(\eta), F_M^f(\eta)) : \eta \in L_1\}$ in L_1 by $T_M^f(\eta) = T_M(f(\eta)), C_M^f(\eta) = C_M(f(\eta)), G_M^f(\eta) = G_M(f(\eta)), F_M^f(\eta) = F_M(f(\eta))$, for all $\eta \in L_1$. Clearly, L^f is a SVQN-set in L_1 .

Theorem 3.8. Suppose that L_1 and L_2 be two Lie-Algebras on a common field F . Assume that f be an onto homomorphism from L_1 to L_2 . If $M = \{(\eta, T_M(\eta), C_M(\eta), G_M(\eta), F_M(\eta)) : \eta \in L_2\}$ is a SVQN-Lie-Ideal of L_2 , then $L^f = \{(\eta, T_M^f(\eta), C_M^f(\eta), G_M^f(\eta), F_M^f(\eta)) : \eta \in L_1\}$ is also a SVQN-Lie-Ideal of L_1 .

Proof. Suppose that L_1 and L_2 be two Lie-Algebras on a common field F . Assume that $\eta, \delta \in L_1$ and $a \in F$. Then,

$$\begin{aligned} & (i) T_N^f(\eta + \delta) \\ &= T_N(f(\eta + \delta)) \\ &= T_N(f(\eta) + f(\delta)) \\ &\geq \min\{T_N(f(\eta)), T_N(f(\delta))\} \\ &= \min\{T_N^f(\eta), T_N^f(\delta)\}, \\ & C_N^f(\eta + \delta) \\ &= C_N(f(\eta + \delta)) \\ &= C_N(f(\eta) + f(\delta)) \\ &\geq \min\{C_N(f(\eta)), C_N(f(\delta))\} \end{aligned}$$

$$\begin{aligned}
&= \min\{C_N^f(\eta), C_N^f(\delta)\}, \\
&= G_N^f(\eta + \delta) \\
&= G_N(f(\eta) + f(\delta)) \\
&\leq \max\{G_N(f(\eta)), G_N(f(\delta))\} \\
&= \max\{G_N^f(\eta), G_N^f(\delta)\}, \\
&= F_N^f(\eta + \delta) \\
&= F_N(f(\eta) + f(\delta)) \\
&= F_N(f(\eta) + f(\delta)) \\
&\leq \max\{F_N(f(\eta)), F_N(f(\delta))\} \\
&= \max\{F_N^f(\eta), F_N^f(\delta)\},
\end{aligned}$$

$$(ii) T_N^f(a\eta) = T_N(f(a\eta)) = T_N(af(\eta)) \geq T_N(f(\eta)) = T_N^f(\eta),$$

$$C_N^f(a\eta) = C_N(f(a\eta)) = C_N(af(\eta)) \geq C_N(f(\eta)) = C_N^f(\eta),$$

$$G_N^f(a\eta) = G_N(af(\eta)) = G_N(af(\eta)) \leq G_N(f(\eta)) = G_N^f(\eta),$$

$$F_N^f(a\eta) = F_N(af(\eta)) = F_N(af(\eta)) \leq F_N(f(\eta)) = F_N^f(\eta).$$

$$(iii) T_N^f([\eta, \delta]) = T_N(f([\eta, \delta])) = T_N(f(\eta), f(\delta)) \geq T_N(f(\eta)) = T_N^f(\eta),$$

$$C_N^f([\eta, \delta]) = C_N(f([\eta, \delta])) = C_N(f(\eta), f(\delta)) \geq C_N(f(\eta)) = C_N^f(\eta),$$

$$G_N^f([\eta, \delta]) = G_N(f([\eta, \delta])) = G_N(f(\eta), f(\delta)) \leq G_N(f(\eta)) = G_N^f(\eta),$$

$$F_N^f([\eta, \delta]) = F_N(f([\eta, \delta])) = F_N(f(\eta), f(\delta)) \leq F_N(f(\eta)) = F_N^f(\eta).$$

Therefore, $L^f = \{(\eta, T_M^f(\eta), C_M^f(\eta), G_M^f(\eta), F_M^f(\eta)) : \eta \in L_1\}$ satisfies all the conditions for being a SVQN-Lie-Ideal of L_1 . Hence, L^f is a SVQN-Lie-Ideal of L_1 .

Theorem 3.9. Let L_1 and L_2 be two Lie-Algebras on a common field F . Let f be an onto homomorphism from L_1 to L_2 . Then, $L^f = \{(w, T_M^f(w), C_M^f(w), G_M^f(w), F_M^f(w)) : w \in L_1\}$ is a SVQN-Lie-Ideal of L_1 iff $M = \{(w, T_M(w), C_M(w), G_M(w), F_M(w)) : w \in L\}$ is a SVQN-Lie-Ideal of L_2 .

Proof. The sufficiency of this theorem directly follows from the previous theorem.

Now, we just need to prove the necessity part of this theorem. Since, the mapping f is an onto mapping, so for any $w, q \in L_2$ there are $w_1, q_1 \in L_1$ such that $w = f(w_1), q = f(q_1)$. Therefore, $T_N(w) = T_N^f(w_1)$, $T_N(q) = T_N^f(q_1)$, $C_N(w) = C_N^f(w_1)$, $C_N(q) = C_N^f(q_1)$, $G_N(w) = G_N^f(w_1)$, $G_N(q) = G_N^f(q_1)$, $F_N(w) = F_N^f(w_1)$, $F_N(q) = F_N^f(q_1)$.

$$\begin{aligned}
&\text{Now, (i) } T_N(w + q) \\
&= T_N(f(w_1) + f(q_1)) \\
&= T_N(f(w_1 + q_1)) \\
&= T_N^f(w_1 + q_1) \\
&\geq \min\{T_N^f(w_1), T_N^f(q_1)\} \\
&= \min\{T_N(w), T_N(q)\}, \\
&C_N(w + q) \\
&= C_N(f(w_1) + f(q_1)) \\
&= C_N(f(w_1 + q_1)) \\
&= C_N^f(w_1 + q_1) \\
&\geq \min\{C_N^f(w_1), C_N^f(q_1)\} \\
&= \min\{C_N(w), C_N(q)\}, \\
&G_N(w + q) \\
&= G_N(f(w_1) + f(q_1)) \\
&= G_N(f(w_1 + q_1)) \\
&= G_N^f(w_1 + q_1) \\
&\leq \max\{G_N^f(w_1), G_N^f(q_1)\} \\
&= \max\{G_N(w), G_N(q)\}, \\
&F_N(w + q) \\
&= F_N(f(w_1) + f(q_1)) \\
&= F_N(f(w_1 + q_1))
\end{aligned}$$

$$\begin{aligned}
&= F_N^f(w_1 + q_1) \\
&\leq \max\{F_N^f(w_1), F_N^f(q_1)\} \\
&= \max\{F_N(w), F_N(q)\}.
\end{aligned}$$

$$(ii) T_N(\alpha w) = T_N(\alpha f(w_1)) = T_N(f(\alpha w_1)) = T_N^f(f(\alpha w_1)) \geq T_N^f(w_1) = T_N(w),$$

$$C_N(\alpha w) = C_N(\alpha f(w_1)) = C_N(f(\alpha w_1)) = C_N^f(f(\alpha w_1)) \geq C_N^f(w_1) = C_N(w),$$

$$G_N(\alpha w) = G_N(\alpha f(w_1)) = G_N(f(\alpha w_1)) = G_N^f(f(\alpha w_1)) \leq G_N^f(w_1) = G_N(w),$$

$$F_N(\alpha w) = F_N(\alpha f(w_1)) = F_N(f(\alpha w_1)) = F_N^f(f(\alpha w_1)) \leq F_N^f(w_1) = F_N(w).$$

$$(iii) T_N([w, q]) = T_N([f(w_1), f(q_1)]) = T_N(f([w_1, q_1])) = T_N^f([w_1, q_1]) \geq T_N(w_1) = T_N(w),$$

$$C_N([w, q]) = C_N([f(w_1), f(q_1)]) = C_N(f([w_1, q_1])) = C_N^f([w_1, q_1]) \geq C_N(w_1) = C_N(w),$$

$$G_N([w, q]) = G_N([f(w_1), f(q_1)]) = G_N(f([w_1, q_1])) = G_N^f([w_1, q_1]) \leq G_N(w_1) = G_N(w),$$

$$F_N([w, q]) = F_N([f(w_1), f(q_1)]) = F_N(f([w_1, q_1])) = F_N^f([w_1, q_1]) \leq F_N(w_1) = F_N(w),$$

Therefore, $L^f = \{(w, T_M^f(w), C_M^f(w), G_M^f(w), F_M^f(w)) : w \in L_1\}$ satisfies all the conditions for being a SVQN-Lie-Ideal of L_2 .

Conclusions

In this article, we grounded the idea of SVQN-Lie-Ideal of SVQN-Lie-Algebra. Besides, we formulated several interesting results on SVQN-Lie-Ideals and SVQN-Lie-Algebras. In the future, we hope that many new notions namely Single-Valued Quadripartitioned Neutrosophic Anti-Lie-Ideal, Single-Valued Quadripartitioned Neutrosophic Lie-Topology can also be introduced based on the present study of SVQN-Lie-Algebra.

Funding “This research received no external funding”.

Conflicts of Interest “The authors declare no conflict of interest”.

References

1. Akram M. Anti fuzzy Lie ideals of Lie algebras. Quasigroups Related Systems. 2006; 14(2):123-132.
2. Akram M. Intuitionistic (S, T)-fuzzy Lie ideals of Lie algebras. Quasigroups Related Systems. 2007; 15(2):201-218.
3. Akram M. Intuitionistic fuzzy Lie ideals of Lie algebras. J. Fuzzy Math. 2008; 16(4):991-1008.
4. Akram M. Generalized fuzzy Lie subalgebras. J. Gen. Lie Theory Appl. 2008; 2(4):261-268.
5. Akram M. Fuzzy Lie ideals of Lie algebras with interval-valued membership function. Quasigroups Related Systems. 2008; 16(1):1-12.
6. Akram M, Gulzar H and Shum KP. Single-Valued Neutrosophic Lie Algebras. Journal of Mathematical Research with Applications. 2019; 39(2):1-12.
7. Akram M and Shum KP. Intuitionistic fuzzy Lie algebras. Southeast Asian Bull. Math. 2007; 31(5):843-855.
8. Atanassov K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986; 20(1):87-96.
9. Chatterjee R, Majumdar, P and Samanta, SK. On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets. Journal of Intelligent & Fuzzy Systems. 2016; 30(4):2475-2485.
10. Coelho P and Nunes U. Lie algebra application to mobile robot control: a tutorial. Robotica. 2003; 21:483-493.
11. Das S, Das R, Granados C and Mukherjee A. Pentapartitioned Neutrosophic Q-Ideals of Q-Algebra. Neutrosophic Sets and Systems. 2021; 41:52-63.
12. Das S, Das R and Tripathy BC. Multi criteria group decision making model using single-valued neutrosophic set. LogForum. 2020; 16(3):421-429.
13. Das S, Khalid AK and Smarandache F. NEUTROSOPHIC d-IDEAL OF NEUTROSOPHIC d-ALGEBRA. Neutrosophic Sets and Systems, In Press.

14. Das S, Shil B and Tripathy BC. Tangent Similarity Measure Based MADM-Strategy under SVPNS-Environment. *Neutrosophic Sets and Systems*. 2021; 43:93-104.
15. Humphreys JE. *Introduction to Lie Algebras and Representation Theory*. Springer, New York. 1972.
16. Kim CG and Lee DS. Fuzzy Lie ideals and fuzzy Lie subalgebras. *Fuzzy Sets and Systems*. 1998; 94:101-107.
17. Smarandache F. *A unifying field in logics, neutrosophy: neutrosophic probability, set and logic*. Rehoboth, American Research Press. 1998.
18. Wang H, Smarandache F, Zhang YQ and Sunderraman R. Single valued neutrosophic sets. *Multispace and Multistructure*. 2010; 4:410-413.
19. Yehia SE. Fuzzy ideals and fuzzy subalgebras of Lie algebras. *Fuzzy Sets and Systems*. 1996; 80(2):237-244.
20. Yehia SE. The adjoint representation of fuzzy Lie algebras. *Fuzzy Sets and Systems*. 2001; 119(3):409-417.
21. Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8:338-353.