



Linguistic Neutrosophic Semi-Connectedness and Semi-Compactness

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Abstract

The notions of semi-connectedness and semi-compactness in linguistic neutrosophic topological space are presented and some of their properties discussed in this study. Further the idea of linguistic neutrosophic semi-compact space is instigated and investigated.

Keywords: Linguistic neutrosophic semi-connectedness, Linguistic neutrosophic extremely disconnected, Linguistic neutrosophic super semi-connected, Linguistic neutrosophic semi-compactness, Linguistic neutrosophic semi-compact space.

1 Preliminaries

Definition 1.1. ⁶ Let S be a space of points (objects), with a generic element in x denoted by S . A neutrosophic set A in S is characterized by a truth-membership function T_A , an indeterminacy membership function I_A and a falsity-membership function F_A . $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is

$$T_A : S \rightarrow]0^-, 1^+[, I_A : S \rightarrow]0^-, 1^+[, F_A : S \rightarrow]0^-, 1^+[$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 1.2. ⁶ Let S be a space of points (objects), with a generic element in x denoted by S . A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A , indeterminacy-membership function I_A and falsity-membership function F_A . For each point s in S , $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x) \rangle / x \in S$.

When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i) \rangle / x_i \in S$.

Definition 1.3. ³ Let $Q = \{s_0, s_1, s_2, \dots, s_t\}$ be a linguistic term set (LTS) with odd cardinality $t + 1$ and $\bar{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,

$A = \{ \langle x, s_\theta(x), s_\psi(x), s_\sigma(x) \rangle | x \in S \}$, where $s_\theta(x)$, $s_\psi(x)$, $s_\sigma(x) \in \bar{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A , respectively, with condition $0 \leq \theta + \psi + \sigma \leq 3t$.

This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 1.4. ⁴ For a linguistic neutrosophic topology τ_{LN} , the collection of linguistic neutrosophic sets should obey,

1. $0_{LN}, 1_{LN} \in \tau_{LN}$
2. $K_1 \cap K_2 \in \tau_{LN}$ for any $K_1, K_2 \in \tau_{LN}$
3. $\bigcup K_i \in \tau_{LN}, \forall \{K_i : i \in J\} \subseteq \tau_{LN}$

We call, the pair (S_{LN}, τ_{LN}) , a linguistic neutrosophic topological space.

2 Linguistic Neutrosophic Semi-Connectedness

Definition 2.1. Let A_{LN} be a linguistic neutrosophic set of (S_{LN}, τ_{LN}) . Then A_{LN} is said to be

1. linguistic neutrosophic semi-open set if there exists a linguistic neutrosophic open set $B_{LN} \in \tau_{LN}$ such that $B_{LN} \subseteq A_{LN} \subseteq LNCl(B_{LN})$.
2. linguistic neutrosophic semi-closed set if there exists a linguistic neutrosophic closed set $B_{LN} \in \tau_{LN}$ such that $LNInt(B_{LN}) \subseteq A_{LN} \subseteq B_{LN}$.

Definition 2.2. Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space. A linguistic neutrosophic semi separation on S_{LN} is a pair E_{LN} and F_{LN} of non void linguistic neutrosophic semi open sets such that $S_{LN} = E_{LN} \cup F_{LN}$, where $E_{LN} \cap F_{LN} = \phi$.

Definition 2.3. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is said to be a linguistic neutrosophic semi connected space if there exists no linguistic neutrosophic semi separations in S_{LN} . Suppose if S_{LN} has such linguistic neutrosophic semi separation, then (S_{LN}, τ_{LN}) is a linguistic neutrosophic semi disconnected space.

Theorem 2.4. Let A_{LN} and B_{LN} be linguistic neutrosophic semi separations in a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) . If K_{LN} is a linguistic neutrosophic semi connected subspace of S_{LN} , then one has either $K_{LN} \subseteq A_{LN}$ or $K_{LN} \subseteq B_{LN}$.

Proof:

As A_{LN} and B_{LN} are linguistic neutrosophic semi open sets, we have $K_{LN} \cap A_{LN}$ and $K_{LN} \cap B_{LN}$ are also linguistic neutrosophic semi open sets. Thus, $K_{LN} \cap A_{LN}$ and $K_{LN} \cap B_{LN}$ are linguistic neutrosophic semi separations of K_{LN} , which is a contradiction. Therefore, either $K_{LN} \cap A_{LN}$ or $K_{LN} \cap B_{LN}$ is an empty set and hence we have either $K_{LN} \subseteq A_{LN}$ or $K_{LN} \subseteq B_{LN}$.

Theorem 2.5. Let K_{LN} is a linguistic neutrosophic semi connected subspace of a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) . If $K_{LN} \subseteq M_{LN} \subseteq LNSCl(K_{LN})$, then M_{LN} is linguistic neutrosophic semi connected.

Proof:

Suppose M_{LN} is not linguistic neutrosophic semi connected, then there exists non void linguistic neutrosophic semi open sets E_{LN} and F_{LN} such that these sets form a linguistic neutrosophic semi separation of M_{LN} . Then, we have either $K_{LN} \subseteq E_{LN}$ or $K_{LN} \subseteq F_{LN}$. Suppose $K_{LN} \subseteq E_{LN}$, then $M_{LN} \subseteq LNSCl(K_{LN}) \subseteq LNSCl(E_{LN})$. So, $M_{LN} \cap B_{LN} \subseteq LNSCl(E_{LN}) \cap B_{LN} = E_{LN} \cap F_{LN} = \phi$. Hence, $E_{LN} \subseteq M_{LN} = \phi$, a contradiction. Thus, M_{LN} is linguistic neutrosophic semi connected.

Theorem 2.6. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected if and only if both linguistic neutrosophic semi open and linguistic neutrosophic semi closed sets are only ϕ and S_{LN} .

Proof:

Necessity Part: Let the linguistic neutrosophic topological space (S_{LN}, τ_{LN}) be linguistic neutrosophic semi connected. Suppose if K_{LN} is both linguistic neutrosophic semi open and linguistic neutrosophic semi closed set in (S_{LN}, τ_{LN}) that is different from ϕ and S_{LN} . Then, $(K_{LN})^c$ is also a linguistic neutrosophic semi open set. Thus, K_{LN} and $(K_{LN})^c$ forms a linguistic neutrosophic semi separation of S_{LN} , which is a contradiction. Therefore, both linguistic neutrosophic semi open and linguistic neutrosophic semi closed sets are only ϕ and S_{LN} .

Sufficiency Part: Let K_{LN} and J_{LN} be a linguistic neutrosophic semi separation of S_{LN} and $K_{LN} \neq S_{LN}$. Since $S_{LN} = K_{LN} \cup J_{LN}$, $K_{LN} = (J_{LN})^c$. This shows that K_{LN} is both linguistic neutrosophic semi open and linguistic neutrosophic semi closed set in S_{LN} that is different from ϕ and S_{LN} , which is a contradiction. So, (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected.

Theorem 2.7. Let (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) be two linguistic neutrosophic topological spaces. And $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi continuous function. Then if (S_{LN}, τ_{LN}) is linguistic neutrosophic semi connected then, (T_{LN}, η_{LN}) is linguistic neutrosophic semi connected.

Proof:

Suppose (T_{LN}, η_{LN}) is not linguistic neutrosophic semi connected, then there exists a linguistic neutrosophic semi separation K_{LN} and J_{LN} of (T_{LN}, η_{LN}) . So, $S_{LN} = (f_{LN})^{-1}(K_{LN} \cap J_{LN}) = (f_{LN})^{-1}(K_{LN}) \cap$

$(f_{LN})^{-1}(J_{LN})$. And $(f_{LN})^{-1}(K_{LN}) \cap (f_{LN})^{-1}(J_{LN}) = \phi$.

Clearly, K_{LN} and J_{LN} are different from ϕ , and so $(f_{LN})^{-1}(K_{LN})$ and $(f_{LN})^{-1}(J_{LN})$ forms a linguistic neutrosophic semi separation of S_{LN} , which is a contradiction. Therefore, (T_{LN}, η_{LN}) is linguistic neutrosophic semi-connected.

Theorem 2.8. Let (S_{LN}, τ_{LN}) and (T_{LN}, η_{LN}) be two linguistic neutrosophic topological spaces. And $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi-irresolute and onto function and if (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-connected then, (T_{LN}, η_{LN}) is linguistic neutrosophic semi-connected.

Proof:

Let suppose (T_{LN}, η_{LN}) is not linguistic neutrosophic semi-connected, then there lies a proper linguistic neutrosophic clopen set B_{LN} in (T_{LN}, η_{LN}) . As the map f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}(B_{LN}) \in LNSO(S_{LN}, \tau_{LN})$ and $(f_{LN})^{-1}(B_{LN}) \in LNSC(S_{LN}, \tau_{LN})$, which is contrariety.

Definition 2.9. A linguistic neutrosophic semi-open set in a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic regular semi-open set if $K_{LN} = LNSInt(LNSCl(K_{LN}))$. The complement of a linguistic neutrosophic regular semi-open set is a linguistic neutrosophic regular semi-closed set.

Definition 2.10. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic super semi-connected space if there lies no proper linguistic neutrosophic regular semi-open set in (S_{LN}, τ_{LN}) .

Theorem 2.11. Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space, then the results are equivalent.

1. (S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.
2. for every non-zero linguistic neutrosophic regular semi-open set B_{LN} , $LNSCl(B_{LN}) = 1_{LN}$.
3. for every non-zero linguistic neutrosophic regular semi-closed set $A_{LN} \neq 1_{LN}$, $LNSInt(A_{LN}) = 0_{LN}$.
4. there lies no linguistic neutrosophic regular semi-open sets A_{LN} and B_{LN} in $A_{LN} \neq 0_{LN} \neq B_{LN}$, $A_{LN} \subseteq (B_{LN})^c$.
5. there lies no linguistic neutrosophic regular semi-open sets A_{LN} and B_{LN} in $A_{LN} \neq 0_{LN} \neq B_{LN}$, $B_{LN} = (LNSCl(A_{LN}))^c$, $A_{LN} = (LNSCl(B_{LN}))^c$.
6. there lies no linguistic neutrosophic regular semi-closed sets A_{LN} and B_{LN} in $A_{LN} \neq 1_{LN} \neq B_{LN}$, $B_{LN} = (LNSInt(A_{LN}))^c$, $A_{LN} = (LNSInt(B_{LN}))^c$.

Proof:

(1) \Rightarrow (2) : Assume that there lies a $A_{LN} \neq 0_{LN}$ with $A_{LN} \in LNRSO(S_{LN}, \tau_{LN})$ and $LNSCl(A_{LN}) \neq 1_{LN}$. Let $B_{LN} = LNSInt(LNSCl(A_{LN}))^c$, then B_{LN} is a proper linguistic neutrosophic regular semi-open set, which is a contrariety. Thus, $LNSCl(A_{LN}) = 1_{LN}$.

(2) \Rightarrow (3) : Let $A_{LN} \neq 0_{LN}$ be a linguistic neutrosophic regular semi-closed set in (S_{LN}, τ_{LN}) . If $(0_{LN} \neq B_{LN}) = (A_{LN})^c$, then B_{LN} is linguistic neutrosophic regular semi-open set. From the hypothesis, $LNSCl(B_{LN}) = 1_{LN}$ which shows that $(LNSCl(B_{LN}))^c = 0_{LN}$. Now, $LNSInt((B_{LN})^c) = LNSInt(A_{LN}) = 0_{LN}$.

(3) \Rightarrow (4) : Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}) , with $A_{LN} \neq 0_{LN} \neq B_{LN}$, $A_{LN} \subseteq (B_{LN})^c$. As $(B_{LN})^c$ is linguistic neutrosophic regular semi-closed set, we get $(B_{LN})^c = LNSCl(LNSInt((B_{LN})^c))$ and $LNSInt((B_{LN})^c) = 0_{LN}$. Now, $0_{LN} \neq A_{LN} = LNSInt(LNSCl(A_{LN})) \subseteq LNSInt(LNSCl((B_{LN})^c)) = LNSInt(LNSCl(LNSInt((B_{LN})^c))) = LNSInt(LNSCl(LNSInt((B_{LN})^c))) = LNSInt((B_{LN})^c) = 0_{LN}$, which results in a contradiction.

(4) \Rightarrow (1) : Proof is as above.

(1) \Rightarrow (5) : Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}) , with $A_{LN} \neq 0_{LN} \neq B_{LN}$, $B_{LN} = (LNSCl(A_{LN}))^c$ and $A_{LN} = (LNSCl(B_{LN}))^c$. Now, $LNSInt(LNSCl(A_{LN})) = LNSInt((B_{LN})^c) = (LNSCl(B_{LN}))^c = A_{LN}$, $A_{LN} \neq 0_{LN}$ and $A_{LN} \neq 1_{LN}$. Suppose if $A_{LN} = 1_{LN}$, then $1_{LN} = (LNSCl(B_{LN}))^c \Rightarrow LNSCl(B_{LN}) = 0_{LN} \Rightarrow B_{LN} = 0_{LN}$. But $B_{LN} \neq 0_{LN}$. Due to the conflict, it is concluded that the statement (5) holds true.

(5) \Rightarrow (1) : Let A_{LN} be a linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}) , with $0_{LN} \neq A_{LN} \neq 1_{LN}$. Put $B_{LN} = (LNSCl(A_{LN}))^c$. Now, $B_{LN} \neq 0_{LN}$ and B_{LN} is a linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}) . Consider $B_{LN} = (LNSCl(A_{LN}))^c \Rightarrow (LNSCl(B_{LN}))^c = (LNSCl((LNSCl(A_{LN}))^c))^c = LNSInt((LNSCl(A_{LN}))^c) = LNSInt(LNSCl(A_{LN})) = A_{LN}$, which results in a contradiction. Ergo,

(S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.

(5) \Rightarrow (6) : Let the sets A_{LN} and B_{LN} be linguistic neutrosophic regular semi-open in (S_{LN}, τ_{LN}) , with $A_{LN} \neq 1_{LN} \neq B_{LN}$ and $B_{LN} = (LNSInt(A_{LN}))^c$, $A_{LN} = (LNSInt(B_{LN}))^c$. Take $K_{LN} = (A_{LN})^c$ and $H_{LN} = (B_{LN})^c$, then K_{LN} and H_{LN} are non-empty linguistic neutrosophic regular semi-open sets. Now, $H_{LN} = ((LNSInt(A_{LN}))^c)^c = LNSInt((K_{LN})^c) = (LNSCl(K_{LN}))^c = (LNSCl(H_{LN}))^c$, which is a contradiction. Thus, (S_{LN}, τ_{LN}) is a linguistic neutrosophic super semi-connected space.

(6) \Rightarrow (5) : Proof is same as above.

3 Linguistic Neutrosophic Semi-Compactness

The concept which depends solely upon linguistic neutrosophic semi open sets is linguistic neutrosophic semi-compactness. In this section, linguistic neutrosophic compactness is discussed with some characterizations.

Definition 3.1. A linguistic neutrosophic cover is defined as the collection in which every member is a linguistic neutrosophic semi-open set.

Definition 3.2. A collection of linguistic neutrosophic subsets of S_{LN} has the finite intersection property if for each finite collection $\{A^1_{LN}, A^2_{LN}, A^3_{LN}, \dots, A^k_{LN}\}$ the common intersection $\bigcap_{r=1}^n A^r_{LN}$ is non-empty.

Definition 3.3. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact if each linguistic neutrosophic cover of (S_{LN}, τ_{LN}) by semi open sets has a finite linguistic neutrosophic sub-cover, or for every collection A_{LN} of S_{LN} , $\bigcap_{r=1}^n A^r_{LN} \neq \phi$.

Remark 3.4. Every linguistic neutrosophic compact space is linguistic neutrosophic semi-compact space.

Definition 3.5. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-Lindelof space if each linguistic neutrosophic cover of (S_{LN}, τ_{LN}) by linguistic neutrosophic semi-open sets has a countable sub-cover of S_{LN} .

Definition 3.6. A linguistic neutrosophic topological space (S_{LN}, τ_{LN}) is linguistic neutrosophic countably semi-compact space relative to (S_{LN}, τ_{LN}) if each linguistic neutrosophic countable semi-open cover has a finite linguistic neutrosophic sub-cover.

Theorem 3.7. Let H_{LN} be a linguistic neutrosophic pre-open set and $K_{LN} \subset H_{LN}$. Then K_{LN} is linguistic neutrosophic semi-compact if and only if K_{LN} is linguistic neutrosophic semi-compact in H_{LN} .

Proof:

Necessity part: Let $\mathcal{K}_{LN} = \{(K_{LN})_{i,1} \in \mathcal{I}\}$ be a linguistic neutrosophic semi-open cover of K_{LN} by the linguistic neutrosophic semi-open sets in H_{LN} . Now, $(K_{LN})_i = (M_{LN})_i \cap H_{LN}$ for each $i \in \mathcal{I}$, where $(M_{LN})_i$ is linguistic neutrosophic semi-open. Thus, $\mathcal{M}_{LN} = \{(M_{LN})_{i,1} \in \mathcal{I}\}$ is a cover of K_{LN} by linguistic neutrosophic semi-open sets in (S_{LN}, τ_{LN}) . As K_{LN} is linguistic neutrosophic semi-compact, we can find $i_1, i_2, i_3, \dots, i_n \in \mathcal{I}$ with $K_{LN} \subset \bigcup_{j=1}^n ((M_{LN})_{i_j})_j$ and we have $K_{LN} \subset \bigcup_{j=1}^n (((M_{LN})_{i_j})_j \cap H_{LN}) = \bigcup_{j=1}^n ((K_{LN})_{i_j})_j$. Therefore, K_{LN} is linguistic neutrosophic semi-compact in H_{LN} .

Sufficiency Part: Let $\mathcal{M}_{LN} = \{(M_{LN})_{i,1} \in \mathcal{I}\}$ is a cover of K_{LN} by linguistic neutrosophic semi-open sets in (S_{LN}, τ_{LN}) . Then $\mathcal{K}_{LN} = \{(M_{LN})_i \cap H_{LN}, 1 \in \mathcal{I}\}$ is a cover of K_{LN} . Since $\{(M_{LN})_{i,1} \in \mathcal{I}\}$ is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}) and H_{LN} is linguistic neutrosophic pre-open, for each $i \in \mathcal{I}$, $(M_{LN})_i \cap H_{LN}$ is linguistic neutrosophic semi-open in H_{LN} . As K_{LN} is linguistic neutrosophic semi-compact in H_{LN} , we can find $i_1, i_2, i_3, \dots, i_n \in \mathcal{I}$ with $K_{LN} \subset \bigcup_{j=1}^n (((M_{LN})_{i_j})_j \cap H_{LN}) \subset \bigcup_{j=1}^n ((M_{LN})_{i_j})_j$. Ergo, K_{LN} is linguistic neutrosophic semi-compact in S_{LN} .

Theorem 3.8. Let H_{LN} be a linguistic neutrosophic pre-open set and $K_{LN} \subset H_{LN}$. Then K_{LN} is linguistic neutrosophic semi-Lindelof if and only if K_{LN} is linguistic neutrosophic semi-Lindelof in H_{LN} .

Proof: Proof is straight forward.

Theorem 3.9. Let $f_{LN} : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic semi-irresolute mapping. Then the succeeding results are equivalent.

1. If K_{LN} is linguistic neutrosophic semi-Lindelof in (S_{LN}, τ_{LN}) , then $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-Lindelof in (T_{LN}, η_{LN}) .

2. If K_{LN} is linguistic neutrosophic semi-compact in (S_{LN}, τ_{LN}) , then $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-compact in (T_{LN}, η_{LN}) .

Proof:

(1): Let $\mathcal{M}_{LN} = \{(M_{LN})_{i,1} \in \mathcal{I}\}$ is a cover of $f_{LN}(K_{LN})$ by linguistic neutrosophic semi-open sets in (T_{LN}, η_{LN}) . Then, $\mathcal{K}_{LN} = \{(f_{LN})^{-1}(K_{LN})_i, i \in \mathcal{I}\}$ is a cover of K_{LN} . As f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}((M_{LN})_i)$ is linguistic neutrosophic semi-open in (S_{LN}, τ_{LN}) . As K_{LN} is linguistic neutrosophic semi-Lindelof, there lies $i_1, i_2, i_3, \dots, i_n \in \mathcal{I}$ with $K_{LN} \subset \bigcup_{j=1}^{\infty} (f_{LN})^{-1}((M_{LN})_{i_j})$. Therefore, $f_{LN}(K_{LN}) \subset \bigcup_{j=1}^{\infty} ((M_{LN})_{i_j})$, which shows that $f_{LN}(K_{LN})$ is linguistic neutrosophic semi-compact in (T_{LN}, η_{LN}) .

(2): Proof is same as of (1).

Definition 3.10. A linguistic neutrosophic topological space is extremely disconnected if the linguistic neutrosophic closure of each linguistic neutrosophic open subset of (S_{LN}, τ_{LN}) is linguistic neutrosophic open.

Definition 3.11. A linguistic neutrosophic topological space is linguistic neutrosophic semi-compact space if any linguistic neutrosophic subset of (S_{LN}, τ_{LN}) which is linguistic neutrosophic semi-compact is linguistic neutrosophic semi-closed in (S_{LN}, τ_{LN}) .

Theorem 3.12. A linguistic neutrosophic topological space is extremely disconnected if the intersection of any two linguistic neutrosophic semi-open subsets of (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-open.

Proof: Proof is direct.

Theorem 3.13. Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic semi- T_2 extremely disconnected, then the space is linguistic neutrosophic semi-compact.

Proof: Let A_{LN} be a linguistic neutrosophic semi-compact subset of (S_{LN}, τ_{LN}) and let $s \notin A_{LN}$. For each $p \in A_{LN}$, there lie two different linguistic neutrosophic semi-open sets B_{LN} and D_{LN} containing s and t respectively. Then we can find $p_1, p_2, \dots, p_n \in A_{LN}$ with $A_{LN} \subset \bigcup_{i=1}^n (D_{LN})_{p_i}$. Let $B_{LN} = \bigcap_{i=1}^n (B_{LN})_{p_i}$. Then B_{LN} is a linguistic neutrosophic semi-open set that contains s and distinct from A_{LN} . Thus, $s \notin LN\mathcal{S}Cl(A_{LN})$, which results that A_{LN} is linguistic neutrosophic semi-closed.

Theorem 3.14. Let f_{LN} be a linguistic neutrosophic semi-irresolute and one to one mapping from a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) into a linguistic neutrosophic semi-compact space (T_{LN}, η_{LN}) , then the space (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact.

Proof: Let A_{LN} be a linguistic neutrosophic semi-compact set in S_{LN} then $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-compact set in T_{LN} . As the space (T_{LN}, η_{LN}) is linguistic neutrosophic semi-compact, $f_{LN}(A_{LN})$ is linguistic neutrosophic semi-closed in T_{LN} . Since f_{LN} is one to one and irresolute, $(f_{LN})^{-1}(f(A_{LN}))$ is linguistic neutrosophic semi-closed in S_{LN} . Hence, (S_{LN}, τ_{LN}) is linguistic neutrosophic semi-compact.

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