On Quasi $Ng^{\#}-$ Open and Quasi $Ng^{\#}-$ Closed Mappings in Neutrosophic Topological Spaces

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Abstract

The aim of this article is to investigate a new kind of Neutrosophic open and closed mappings called quasi $Ng^{\#}-$ Open and $Ng^{\#}-$ closed mappings and study their properties in Neutrosophic topological spaces with necessary examples.
Keywords: $\mathcal{N}g^{\#}-$ closed set, $\mathcal{N}g^{\#}-$ Open sets, $\mathcal{N}g^{\#}-$ open mapping, quasi $\mathcal{N}g^{\#}-$ open mapping, quasi $\mathcal{N}g^{\#}-$ closed mapping.

1 Introduction

Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache.[4] Neutrosophic set helps to study this indeterminacy in uncertain situations in real life problems.The concept of Neutrosophy has been developed into Neutrosophic topological spaces by Salama et.al.[11] in 2014. Neutrosophic topological spaces are very natural generalization of topological spaces which allows more general functions to be members of Neutrosophic topology. Many researchers have studied various topological spaces in accordance with Neutrosophy. Recently Pious Missier et.al.,[7–9] introduced the concept of $\mathcal{N}g^{\#}-$ closed sets, $\mathcal{N}g^{\#}-$ continuous functions, $\mathcal{N}g^{\#}-$ closed and $\mathcal{N}g^{\#}-$ open mappings, in Neutrosophic Topological Spaces. Further, Neutrosophic topology has been applied to the quasi open and closed mappings.

2 Preliminaries

Definition 2.1. [4]
A Neutrosophic set $\mathcal{NS}$ $\mathcal{A}_{\mathcal{N}}$ is an object having the form $\mathcal{A}_{\mathcal{N}} = \{\langle x, \mu_{\mathcal{A}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x), \gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$ where $\mu_{\mathcal{A}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in \mathcal{X}_{\mathcal{N}}$ to the set $\mathcal{A}_{\mathcal{N}}$. A Neutrosophic set $\mathcal{A}_{\mathcal{N}} = \{\langle x, \mu_{\mathcal{A}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x), \gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$ can be identified as an ordered triple $\langle \mu_{\mathcal{A}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x), \gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle$ in $]-0, 1+[$ on $\mathcal{X}_{\mathcal{N}}$.

Definition 2.2. [11] For any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}} = \{\langle x, \mu_{\mathcal{A}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x), \gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$ and $\mathcal{B}_{\mathcal{N}} = \{\langle x, \mu_{\mathcal{B}_{\mathcal{N}}}(x), \sigma_{\mathcal{B}_{\mathcal{N}}}(x), \gamma_{\mathcal{B}_{\mathcal{N}}}(x)\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$ we have

1. $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{B}_{\mathcal{N}} \iff \mu_{\mathcal{A}_{\mathcal{N}}}(x) \le \mu_{\mathcal{B}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x) \le \sigma_{\mathcal{B}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x) \ge \gamma_{\mathcal{B}_{\mathcal{N}}}(x)$
2. $\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}} = \langle x, \mu_{\mathcal{A}_{\mathcal{N}}}(x) \wedge \mu_{\mathcal{B}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x) \wedge \sigma_{\mathcal{B}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x) \vee \gamma_{\mathcal{B}_{\mathcal{N}}}(x)\rangle$

3. $\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}} = \langle x, \mu_{\mathcal{A}_{\mathcal{N}}}(x) \vee \mu_{\mathcal{B}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x) \vee \sigma_{\mathcal{B}_{\mathcal{N}}}(x)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(x) \wedge \gamma_{\mathcal{B}_{\mathcal{N}}}(x)\rangle$

Definition 2.3. [11] Let $\mathcal{A}_{\mathcal{N}} = \langle \mu_{\mathcal{A}_{\mathcal{N}}}(x), \sigma_{\mathcal{A}_{\mathcal{N}}}(x), \gamma_{\mathcal{A}_{\mathcal{N}}}(x)\rangle$ be a $\mathcal{N}S$ on $\mathcal{X}_{\mathcal{N}}$, then the complement $\mathcal{A}_{\mathcal{N}}{}^c$ defined as

- $\mathcal{A}_{\mathcal{N}}{}^c = \{\langle x, \gamma_{\mathcal{A}_{\mathcal{N}}}(x), 1 - \sigma_{\mathcal{A}_{\mathcal{N}}}(x), \mu_{\mathcal{A}_{\mathcal{N}}}(x)\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$

Note that for any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}}$ and $\mathcal{B}_{\mathcal{N}}$,

- $(\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}})^c = \mathcal{A}_{\mathcal{N}}{}^c \cap \mathcal{B}_{\mathcal{N}}{}^c$
- $(\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}})^c = \mathcal{A}_{\mathcal{N}}{}^c \cup \mathcal{B}_{\mathcal{N}}{}^c$.

Definition 2.4. [11] A Neutrosophic topology $(\mathcal{NT})$ on a non-empty set $\mathcal{X}_{\mathcal{N}}$ is a family τ_N of Neutrosophic subsets in $\mathcal{X}_{\mathcal{N}}$ satisfies the following axioms:

1. $\mathbf{0}_{\mathcal{N}}, \mathbf{1}_{\mathcal{N}} \in \tau_N$
2. $R_{N_1} \cap R_{N_2} \in \tau_N$ for any $R_{N_1}, R_{N_2} \in \tau_N$
3. $\bigcup R_{N_i} \in \tau_N \quad \forall \quad R_{N_i} : i \in I \subseteq \tau_N$

Here the empty set $\mathbf{0}_{\mathcal{N}}$ and the whole set $\mathbf{1}_{\mathcal{N}}$ may be defined as follows:

1. $\mathbf{0}_{\mathcal{N}} = \{\langle x, 0, 0, 1\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$
2. $\mathbf{1}_{\mathcal{N}} = \{\langle x, 1, 1, 0\rangle : x \in \mathcal{X}_{\mathcal{N}}\}$

Definition 2.5. [11] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}S$ in $\mathcal{NTS}$ $\mathcal{X}_{\mathcal{N}}$. Then

1. $\mathcal{N}int(\mathcal{A}_{\mathcal{N}}) = \bigcup\{\mathcal{G}_{\mathcal{N}} : \mathcal{G}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{G}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) = \bigcap\{\mathcal{K}_{\mathcal{N}} : \mathcal{K}_{\mathcal{N}}$ is a $\mathcal{NCS}$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{K}_{\mathcal{N}}\}$ is called Neutrosophic closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.6. [5] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a neutrosophic $\mathcal{N}\alpha gCS$ if $\mathcal{N}\alpha cl(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{U}_{\mathcal{N}}$, whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{U}_{\mathcal{N}}$ and $\mathcal{U}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}_{\mathcal{N}}$. The complement of $\mathcal{N}\alpha gCS$ is $\mathcal{N}\alpha gOS$.

Definition 2.7. [7]

A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a Neutrosophic $g^{\#}-$ closed $(\mathcal{N}g^{\#}CS)$ if $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{Q}_{\mathcal{N}}$ whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{Q}_{\mathcal{N}}$ and $\mathcal{Q}_{\mathcal{N}}$ is $\mathcal{N}\alpha gOS$ in $\mathcal{X}_{\mathcal{N}}$. The complement of $\mathcal{N}g^{\#}CS$ is $\mathcal{N}g^{\#}OS$.

Definition 2.8. [10] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}S$ in $\mathcal{NTS}$ $\mathcal{X}_{\mathcal{N}}$. Then

1. $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \bigcup\{\mathcal{G}_{\mathcal{N}} : \mathcal{G}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}OS$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{G}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic $g^{\#}-$ interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \bigcap\{\mathcal{K}_{\mathcal{N}} : \mathcal{K}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}CS$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{K}_{\mathcal{N}}\}$ is called Neutrosophic $g^{\#}-$ closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.9. [8] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be $\mathcal{N}g^{\#}-$ continuous function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every neutrosophic closed set $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.10. [8] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be Neutrosophic $g^{\#}-$ irresolute function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every $\mathcal{N}g^{\#}CS$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.11. [10] A Neutrosophic Topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a $T_{\mathcal{N}}g^{\#}-$ space if every $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.12. [12] A function $f_N : (X, \tau_N) \longrightarrow (Y, \zeta_N)$ is called

1. Neutrosophic closed mapping (NCM) if $f_N(V_N)$ is a $\mathcal{NCS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NCS}$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
2. Neutrosophic open mapping (NOM) if $f_N(V_N)$ is a $\mathcal{NOS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NOS}$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.13. [9] Let $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be two Neutrosophic topological spaces. A mapping $f_N : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is called $Ng^{\#}-$ closed mapping ($Ng^{\#}CM$ for short) if $f_N(A_N)$ is $Ng^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NCS}$ $\mathcal{A}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.14. [9] Let $(\mathcal{X}_\mathcal{N}, \tau_N)$ and $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ be two Neutrosophic topological spaces. A mapping f_N : $(\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is called $Ng^{\#}-$ open mapping ($Ng^{\#}OM$ for short) if $f_N(A_N)$ is ($Ng^{\#}OS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ for every $\mathcal{NOS}$ $\mathcal{A}_\mathcal{N}$ of $(\mathcal{X}_\mathcal{N}, \tau_N)$.

Definition 2.15. [9] Let $(\mathcal{X}_\mathcal{N}, \tau_N)$ and $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ be two Neutrosophic topological spaces. A mapping f_N : $(\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is called strongly Neutrosophic $g^{\#}-$ open mapping (strongly $Ng^{\#}OM$ for short) if $f_N(A_N)$ is $Ng^{\#}OS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ for every $Ng^{\#}OS$ A_N of $(\mathcal{X}_\mathcal{N}, \tau_N)$.

Definition 2.16. [9] Let $(\mathcal{X}_\mathcal{N}, \tau_N)$ and $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ be two Neutrosophic topological spaces. A mapping f_N : $(\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is called strongly Neutrosophic $g^{\#}-$ closed mapping (strongly $Ng^{\#}CM$ for short) if $f_N(A_N)$ is $Ng^{\#}CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ for every $Ng^{\#}CS$ A_N of $(\mathcal{X}_\mathcal{N}, \tau_N)$.

3 Quasi $\mathcal{N}g^{\#}-$ Open Mappings

Definition 3.1. A Neutrosophic mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is said to be quasi $\mathcal{N}g^{\#}-$ open mapping if $f_\mathcal{N}(\mathcal{V}_\mathcal{N})$ is a Neutrosophic open set in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ for every $\mathcal{N}g^{\#}OS$ $\mathcal{V}_\mathcal{N}$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$.

Example 3.2. Let $\mathcal{X}_\mathcal{N} = \{l, m\} = \mathcal{Y}_\mathcal{N}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.4, 0.6)\rangle, \langle m, (0.5, 0.4, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.2, 0.4, 0.7)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.2, 0.3, 0.5)\rangle, \langle m, (0.4, 0.3, 0.5)\rangle\}$,
$\mathcal{M}_{\mathcal{N}4} = \{\langle l, (0.1, 0.3, 0.6)\rangle, \langle m, (0.3, 0.2, 0.5)\rangle\}$.
Now $(\mathcal{X}_\mathcal{N}, \tau_N) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_\mathcal{N}\}$ and
$(\mathcal{Y}_\mathcal{N}, \zeta_N) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathcal{M}_{\mathcal{N}4}, \mathcal{M}_{\mathcal{N}4}^c, \mathbf{1}_\mathcal{N}\}$ are Neutrosophic topological spaces.
Then $\tau_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathbf{1}_\mathcal{N}\}$ and $\zeta_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}4}, \mathbf{1}_\mathcal{N}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_\mathcal{N}$ and $\mathcal{Y}_\mathcal{N}$ respectively. Define a mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ by $f_\mathcal{N}(l) = l - 0.1$ and $f_\mathcal{N}(m) = m - 0.1$. Here $\mathcal{NOS}(\mathcal{Y}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}4}, \mathbf{1}_\mathcal{N}\}, \mathcal{N}g^{\#}\mathcal{OS}(\mathcal{X}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}$. Now $\mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}$ both are $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ and $f_\mathcal{N}(\mathcal{M}_{\mathcal{N}1}) = \mathcal{M}_{\mathcal{N}3}, f_\mathcal{N}(\mathcal{M}_{\mathcal{N}2}) = \mathcal{M}_{\mathcal{N}4}$ both are $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Therefore $f_\mathcal{N}$ is quasi $\mathcal{N}g^{\#}-$ open mapping.

Theorem 3.3. *Every quasi $\mathcal{N}g^{\#}-$ open mapping is Neutrosophic open mapping.*

Proof. Let $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}OM$. Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NOS}$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Then $\mathcal{A}_\mathcal{N}$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $f_\mathcal{N}$ is quasi $\mathcal{N}g^{\#}OM$, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Hence $f_\mathcal{N}$ is Neutrosophic open mapping. □

Remark 3.4. The reverse implication of the above theorem need not be true as seen in the following example.

Example 3.5. Let $\mathcal{X}_\mathcal{N} = \{l, m\} = \mathcal{Y}_\mathcal{N}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.2, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.3, 0.3, 0.6)\rangle, \langle m, (0.4, 0.5, 0.6)\rangle\}$
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.4, 0.5, 0.6)\rangle, \langle m, (0.3, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_\mathcal{N}, \tau_N) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_\mathcal{N}\}$ and $(\mathcal{Y}_\mathcal{N}, \zeta_N) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_\mathcal{N}\}$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}$ and $\zeta_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_\mathcal{N}\}$ are Neutrosophic typologies on $\mathcal{X}_\mathcal{N}$ and $\mathcal{Y}_\mathcal{N}$ respectively. Define a mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ by $f_\mathcal{N}(l) = m$ and $f_\mathcal{N}(m) = l$. Here $\mathcal{NOS}(\mathcal{Y}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_\mathcal{N}\}, \mathcal{NOS}(\mathcal{X}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}, \mathcal{N}g^{\#}\mathcal{OS}(\mathcal{X}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}$. Hence $f_\mathcal{N}$ is Neutrosophic open mapping. Now $\mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}$ both are $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ but $f_\mathcal{N}(\mathcal{M}_{\mathcal{N}1})$ is not $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Therefore $f_\mathcal{N}$ is not quasi $\mathcal{N}g^{\#}-$ open mapping.

Theorem 3.6. *Every quasi $\mathcal{N}g^{\#}-$ open mapping is $\mathcal{N}g^{\#}-$ open mapping.*

Proof. Let $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}OM$. Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NOS}$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Then $\mathcal{A}_\mathcal{N}$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $f_\mathcal{N}$ is quasi $\mathcal{N}g^{\#}OM$, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Note that every $\mathcal{NOS}$ is $\mathcal{N}g^{\#}OS$. Therefore, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Hence $f_\mathcal{N}$ is $\mathcal{N}g^{\#}OM$. □

Remark 3.7. Converse of the above theorem need not be true as seen in the following example.

Example 3.8. Let $\mathcal{X}_\mathcal{N} = \{l, m\} = \mathcal{Y}_\mathcal{N}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.4, 0.3, 0.6)\rangle, \langle m, (0.5, 0.5, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.5, 0.5, 0.6)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_\mathcal{N}, \tau_N) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_\mathcal{N}\}$ and $(\mathcal{Y}_\mathcal{N}, \zeta_N) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_\mathcal{N}\}$ are

Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}$ and $\zeta_N = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_\mathcal{N}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_\mathcal{N}$ and $\mathcal{Y}_\mathcal{N}$ respectively. Define a mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ by $f_\mathcal{N}(l) = m$ and $f_\mathcal{N}(m) = l$. Here $\mathcal{NCS}(\mathcal{Y}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_\mathcal{N}\}, \mathcal{NCS}(\mathcal{X}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}, \mathcal{N}g^\#\mathcal{OS}(\mathcal{X}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_\mathcal{N}\}$, $\mathcal{N}g^\#\mathcal{CS}(\mathcal{Y}_\mathcal{N}) = \{\mathbf{0}_\mathcal{N}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_\mathcal{N}\}$. Hence $f_\mathcal{N}$ is $\mathcal{N}g^\#-$ open mapping. Now $\mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}$ both are $\mathcal{N}g^\#OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ but $f_\mathcal{N}(\mathcal{M}_{\mathcal{N}1})$ is not $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Therefore $f_\mathcal{N}$ is not quasi $\mathcal{N}g^\#-$ open mapping.

Theorem 3.9. *A Neutrosophic mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is quasi $\mathcal{N}g^\#-$ open mapping if and only if for every Neutrosophic set $\mathcal{A}_\mathcal{N}$ of $(\mathcal{X}_\mathcal{N}, \tau_N)$, $f_\mathcal{N}(\mathcal{N}g^\# int(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}int(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$.*

Proof. Let $f_\mathcal{N}$ be a quasi $\mathcal{N}g^\#OM$. Now, we have $\mathcal{N}int(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{A}_\mathcal{N}$ and $\mathcal{N}g^\# int(\mathcal{A}_\mathcal{N})$ is a $\mathcal{N}g^\#OS$. Hence, we get $f_\mathcal{N}(\mathcal{N}g^\# int(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}(\mathcal{A}_\mathcal{N})$. Since $f_\mathcal{N}(\mathcal{N}g^\# int(\mathcal{A}_\mathcal{N}))$ is $\mathcal{NOS}$, $f_\mathcal{N}(\mathcal{N}g^\# int(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}int(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$.
Conversely, assume that $\mathcal{A}_\mathcal{N}$ is a $\mathcal{N}g^\#OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Then $f_\mathcal{N}(\mathcal{A}_\mathcal{N}) = f_\mathcal{N}(\mathcal{N}g^\# int(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}int(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ but $\mathcal{N}int(f_\mathcal{N}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}(\mathcal{A}_\mathcal{N})$. Consequently, $f_\mathcal{N}(\mathcal{A}_\mathcal{N}) = \mathcal{N}int(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$. That is $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is Neutrosophic open set in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Therefore, $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping. □

Lemma 3.10. *If a Neutrosophic function $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is quasi $\mathcal{N}g^\#-$ open mapping then $\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}^{-1}(\mathcal{N}int(\mathcal{A}_\mathcal{N}))$ for every Neutrosophic set $\mathcal{A}_\mathcal{N}$ of $(\mathcal{Y}_\mathcal{N}, \zeta_N)$.*

Proof. Let $\mathcal{A}_\mathcal{N}$ be a Neutrosophic set in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Then $\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$ is a $\mathcal{N}g^\#OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ and $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping, then $f_\mathcal{N}(\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))) \subseteq \mathcal{N}int(f_\mathcal{N}(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))) \subseteq \mathcal{N}int(\mathcal{A}_\mathcal{N})$. Thus $\mathcal{N}g^\# int(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}^{-1}(\mathcal{N}int(\mathcal{A}_\mathcal{N}))$. □

Theorem 3.11. *A Neutrosophic mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is quasi $\mathcal{N}g^\#-$ open mapping if and only if for each Neutrosophic set $\mathcal{B}_\mathcal{N}$ of $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ and for each $\mathcal{N}g^\#CS$ $\mathcal{C}_\mathcal{N}$ of $(\mathcal{X}_\mathcal{N}, \tau_N)$ containing $f_\mathcal{N}^{-1}(\mathcal{B}_\mathcal{N})$ there is a Neutrosophic closed set $\mathcal{A}_\mathcal{N}$ of $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ such that $\mathcal{B}_\mathcal{N} \subseteq \mathcal{A}_\mathcal{N}$ and $f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{B}_\mathcal{N}$.*

Proof. Assume that $f_\mathcal{N}$ is a quasi $\mathcal{N}g^\#-$ open mapping. Let $\mathcal{B}_\mathcal{N}$ be a Neutrosophic set in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ and $\mathcal{C}_\mathcal{N}$ is a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ such that $f_\mathcal{N}^{-1}(\mathcal{B}_\mathcal{N}) \subseteq \mathcal{C}_\mathcal{N}$. Then $\mathcal{A}_\mathcal{N} = (f_\mathcal{N}^{-1}(\mathcal{C}_\mathcal{N}{}^c))^c$ is $\mathcal{NCS}$ of $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ such that $f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{C}_\mathcal{N}$.

Conversely, Assume that $\mathcal{G}_\mathcal{N}$ is a $\mathcal{N}g^\#OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Then$f_\mathcal{N}^{-1}(f_\mathcal{N}(\mathcal{G}_\mathcal{N})^c) \subseteq \mathcal{G}_\mathcal{N}{}^c$ and $\mathcal{G}_\mathcal{N}{}^c$ is $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. By hypothesis, there is a Neutrosophic closed set $\mathcal{A}_\mathcal{N}$ of $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ such that $(f_\mathcal{N}(\mathcal{G}_\mathcal{N}))^c \subseteq \mathcal{A}_\mathcal{N}$ and $f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{G}_\mathcal{N}{}^c$. Therefore, $\mathcal{G}_\mathcal{N} \subseteq (f_\mathcal{N}(\mathcal{A}_\mathcal{N}))^c$. Hence $\mathcal{A}_\mathcal{N}{}^c \subseteq f_\mathcal{N}(\mathcal{G}_\mathcal{N}) \subseteq f_\mathcal{N}(((f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))^c) \subseteq \mathcal{A}_\mathcal{N}{}^c$ which implies that $f_\mathcal{N}(\mathcal{G}_\mathcal{N}) = \mathcal{A}_\mathcal{N}{}^c$. Since $\mathcal{A}_\mathcal{N}{}^c$ is a $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$, $f_\mathcal{N}(\mathcal{G}_\mathcal{N})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Hence $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping. □

Theorem 3.12. *A Neutrosophic mapping $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ is quasi $\mathcal{N}g^\#-$ open mapping if and only if $f_\mathcal{N}^{-1}(\mathcal{N}cl(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{N}g^\# cl(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$ for every neutrosophic set $\mathcal{A}_\mathcal{N}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$.*

Proof. Suppose that $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping. For any neutrosophic set $\mathcal{A}_\mathcal{N}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$, $f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{N}g^\# cl(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$. Therefore, by Theorem 3.11, there exists a $\mathcal{NCSB}_\mathcal{N}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ such that $\mathcal{A}_\mathcal{N} \subseteq \mathcal{B}_\mathcal{N}$ and $f_\mathcal{N}^{-1}(\mathcal{B}_\mathcal{N}) \subseteq \mathcal{N}g^\# cl(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$. Therefore, we obtain $f_\mathcal{N}^{-1}(\mathcal{N}cl(\mathcal{A}_\mathcal{N})) \subseteq f_\mathcal{N}^{-1}(\mathcal{B}_\mathcal{N}) \subseteq \mathcal{N}g^\# cl(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N}))$.

Conversely, let $\mathcal{A}_\mathcal{N}$ be a Neutrosophic set in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$ and $\mathcal{B}_\mathcal{N}$ be a $\mathcal{N}g^\#CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$ containing $f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N})$. Put $\mathcal{N}cl(\mathcal{A}_\mathcal{N}) = \mathcal{W}_\mathcal{N}$, then we have $\mathcal{A}_\mathcal{N} \subseteq \mathcal{W}_\mathcal{N}$ and $\mathcal{W}_\mathcal{N}$ is $\mathcal{NCS}$ and $f_\mathcal{N}^{-1}(\mathcal{W}_\mathcal{N}) \subseteq \mathcal{N}g^\# cl(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N})) \subseteq \mathcal{B}_\mathcal{N}$. Then, by Theorem 3.11, $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping. □

Theorem 3.13. *Let $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ and $g_\mathcal{N} : (\mathcal{Y}, \zeta_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ be two Neutrosophic mappings and let $(g_\mathcal{N} \circ f_\mathcal{N}) : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ is quasi $\mathcal{N}g^\#-$ open mapping. If $g_\mathcal{N}$ is Neutrosophic continuous and one to one function, then $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping.*

Proof. Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{N}g^\#OS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$, then $(g_\mathcal{N} \circ f_\mathcal{N})(\mathcal{A}_\mathcal{N})$ is $\mathcal{NOS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$, since $(g_\mathcal{N} \circ f_\mathcal{N})$ is quasi $\mathcal{N}g^\#-$ open. Since $g_\mathcal{N}$ is Neutrosophic continuous and one to one function, $f_\mathcal{N}(\mathcal{A}_\mathcal{N}) = g_\mathcal{N}^{-1}(g_\mathcal{N} \circ f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ is $\mathcal{NOS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. This shows that $f_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping. □

Theorem 3.14. *Let $f_\mathcal{N} : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Y}_\mathcal{N}, \zeta_N)$ and $g_\mathcal{N} : (\mathcal{Y}_\mathcal{N}, \zeta_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ be any two Neutrosophic mappings. Then*

1. *$(g_\mathcal{N} \circ f_\mathcal{N}) : (\mathcal{X}_\mathcal{N}, \tau_N) \longrightarrow (\mathcal{Z}_\mathcal{N}, \eta_N)$ is Neutrosophic open mapping if $g_\mathcal{N}$ is quasi $\mathcal{N}g^\#-$ open mapping and $f_\mathcal{N}$ is $\mathcal{N}g^\#-$ open mapping.*

2. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is strongly $\mathcal{N}g^{\#}-$ open mapping if $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping and $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping.*

3. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is quasi $\mathcal{N}g^{\#}-$ open mapping if $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping and $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ open mapping.*

Proof. :
(1) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NOS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{NOS}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is Neutrosophic open mapping.
(2) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NOS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ open mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ open mapping.
(3) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ open mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}OS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{NOS}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ open mapping. □

4 Quasi $\mathcal{N}g^{\#}-$ closed Mappings

Definition 4.1. A Neutrosophic mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be quasi $\mathcal{N}g^{\#}-$ closed mapping if $f_{\mathcal{N}}(\mathcal{V}_{\mathcal{N}})$ is a Neutrosophic closed set in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{N}g^{\#}CS\ \mathcal{V}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Example 4.2. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.4, 0.6)\rangle, \langle m, (0.5, 0.4, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.2, 0.4, 0.7)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\} = (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = l$ and $f_{\mathcal{N}}(m) = m$. Here $= \mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\} = \mathcal{NCS}(\mathcal{Y}_{\mathcal{N}})$. Therefore $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping.

Theorem 4.3. *Every quasi $\mathcal{N}g^{\#}-$ closed mapping is Neutrosophic closed mapping.*

Proof. Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}CM$. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}CM,\ f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is Neutrosophic closed map. □

Remark 4.4. Converse of the above theorem need not be true as shown in the following example.

Example 4.5. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.2, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$,
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.3, 0.3, 0.6)\rangle, \langle m, (0.4, 0.5, 0.6)\rangle\}$,
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.4, 0.5, 0.6)\rangle, \langle m, (0.3, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{NT}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = m$ and $f_{\mathcal{N}}(m) = l$. Here $\mathcal{NCS}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}, \mathcal{NCS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}\mathcal{CS}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$. Hence $f_{\mathcal{N}}$ is Neutrosophic closed mapping. Now $\mathcal{M}_{\mathcal{N}1}^c$ and $\mathcal{M}_{\mathcal{N}2}^c$ both are $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but $f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}1}^c)$ is not $\mathcal{NCS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore, $f_{\mathcal{N}}$ is not quasi $\mathcal{N}g^{\#}-$ closed mapping.

Theorem 4.6. *Every quasi $\mathcal{N}g^{\#}-$ closed mapping is $\mathcal{N}g^{\#}-$ closed mapping.*

Proof. Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}CM$. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}CM,\ f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{NCS}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Note that every $\mathcal{NCS}$ is $\mathcal{N}g^{\#}CS$. Therefore, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CM$. □

Remark 4.7. Every $\mathcal{N}g^{\#}-$ closed mapping is need not be a quasi $\mathcal{N}g^{\#}-$ closed mapping can be proved by following example.

Example 4.8. Let $\mathcal{X}_{\mathcal{N}} = \{l, m\} = \mathcal{Y}_{\mathcal{N}}$. Consider the Neutrosophic sets
$\mathcal{M}_{\mathcal{N}1} = \{\langle l, (0.3, 0.1, 0.7)\rangle, \langle m, (0.2, 0.3, 0.8)\rangle\}$
$\mathcal{M}_{\mathcal{N}2} = \{\langle l, (0.4, 0.3, 0.6)\rangle, \langle m, (0.5, 0.5, 0.6)\rangle\}$
$\mathcal{M}_{\mathcal{N}3} = \{\langle l, (0.5, 0.5, 0.6)\rangle, \langle m, (0.4, 0.3, 0.6)\rangle\}$.
Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$ and $(\mathcal{Y}_{\mathcal{N}}, \zeta_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$ are Neutrosophic topological spaces. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}, \mathbf{1}_{\mathcal{N}}\}$ and $\zeta_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}, \mathbf{1}_{\mathcal{N}}\}$ are $\mathcal{N}\mathcal{T}s$ on $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{Y}_{\mathcal{N}}$ respectively. Define a mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ by $f_{\mathcal{N}}(l) = m$ and $f_{\mathcal{N}}(m) = l$. Here $\mathcal{N}\mathcal{C}\mathcal{S}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}\mathcal{C}\mathcal{S}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}\mathcal{C}\mathcal{S}(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}\mathcal{C}\mathcal{S}(\mathcal{Y}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}3}^c, \mathbf{1}_{\mathcal{N}}\}$. Hence $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ closed mapping. Now $\mathcal{M}_{\mathcal{N}1}^c, \mathcal{M}_{\mathcal{N}2}^c$ both are $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but $f_{\mathcal{N}}(\mathcal{M}_{\mathcal{N}1}^c)$ is not $\mathcal{N}\mathcal{C}\mathcal{S}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore $f_{\mathcal{N}}$ is not quasi $\mathcal{N}g^{\#}-$ closed mapping.

Theorem 4.9. *A Neutrosophic mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}-$ closed if and only if for every neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, $\mathcal{N}cl(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))$.*

Proof. Assume that $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping and $\mathcal{A}_{\mathcal{N}}$ is any Neutrosophic set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Therefore, $f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}) \subseteq f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))$ which implies that $\mathcal{N}cl(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}cl(f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))) = f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))$. This implies $\mathcal{N}cl(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))$.

Conversely, Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}} = \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$. Therefore, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}) = f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))$. By hypothesis, $\mathcal{N}cl(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$. Hence $\mathcal{N}cl(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})) \subseteq f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$. But $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{N}cl(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$. This implies $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{N}\mathcal{C}\mathcal{S}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Therefore, $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping. □

Lemma 4.10. *A Neutrosophic mapping $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ be a quasi $\mathcal{N}g^{\#}-$ closed then for every neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$, $f_{\mathcal{N}}^{-1}(\mathcal{N}int(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be any neutrosophic set in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Then $\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ and $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed. Hence $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))) \subseteq \mathcal{N}int(f_{\mathcal{N}}(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))) \subseteq \mathcal{N}int(\mathcal{A}_{\mathcal{N}})$. Which implies $f_{\mathcal{N}}(\mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))) \subseteq \mathcal{N}int(\mathcal{A}_{\mathcal{N}})$. Therefore, $f_{\mathcal{N}}^{-1}(\mathcal{N}int(\mathcal{A}_{\mathcal{N}})) \subseteq \mathcal{N}g^{\#}int(f_{\mathcal{N}}^{-1}(\mathcal{A}_{\mathcal{N}}))$. □

Theorem 4.11. *Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ and $g_{\mathcal{N}} : (\mathcal{Y}_{\mathcal{N}}, \zeta_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ be any two Neutrosophic mappings. Then*

1. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is Neutrosophic closed mapping if $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping and $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ closed mapping.*

2. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is strongly $\mathcal{N}g^{\#}-$ closed mapping if $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ closed mapping and $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping.*

3. *$(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is quasi $\mathcal{N}g^{\#}-$ closed mapping if $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping and $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ closed mapping.*

Proof. : (1) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}\mathcal{C}\mathcal{S}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ closed mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}\mathcal{C}\mathcal{S}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is Neutrosophic closed mapping.

(2) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}\mathcal{C}\mathcal{S}$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ closed mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ closed mapping.

(3) Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ closed mapping, $f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$. Since $g_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping, $(g_{\mathcal{N}} \circ f_{\mathcal{N}})(\mathcal{A}_{\mathcal{N}}) = g_{\mathcal{N}}(f_{\mathcal{N}}(\mathcal{A}_{\mathcal{N}}))$ is $\mathcal{N}\mathcal{C}\mathcal{S}$ in $(\mathcal{Z}_{\mathcal{N}}, \eta_N)$. Therefore $g_{\mathcal{N}} \circ f_{\mathcal{N}}$ is quasi $\mathcal{N}g^{\#}-$ closed mapping. □

Theorem 4.12. *Let $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ and $g_{\mathcal{N}} : (\mathcal{Y}_{\mathcal{N}}, \zeta_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ be any two Neutrosophic mappings such that $(g_{\mathcal{N}} \circ f_{\mathcal{N}}) : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Z}_{\mathcal{N}}, \eta_N)$ is quasi $\mathcal{N}g^{\#}-$ closed mapping.*

1. *If $f_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ irresolute and onto then $g_{\mathcal{N}}$ is Neutrosophic closed mapping.*

2. *If $g_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ continuous and one to one then $f_{\mathcal{N}}$ is strongly $\mathcal{N}g^{\#}-$ closed mapping.*

Proof. : (1) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NCS}$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Then $\mathcal{A}_\mathcal{N}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. Since $f_\mathcal{N}$ is $\mathcal{N}g^{\#}-$ irresolute mapping, $f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $(g_\mathcal{N} \circ f_\mathcal{N}$ is quasi $\mathcal{N}g^{\#}-$ closed and $f_\mathcal{N})$ is onto, $(g_\mathcal{N} \circ f_\mathcal{N})(f_\mathcal{N}^{-1}(\mathcal{A}_\mathcal{N})) = g_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{NCS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. This implies that $g_\mathcal{N}$ is Neutrosophic closed mapping.

(2) Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_\mathcal{N}, \tau_N)$. Since $(g_\mathcal{N} \circ f_\mathcal{N})$ is quasi $\mathcal{N}g^{\#}-$ closed, $(g_\mathcal{N} \circ f_\mathcal{N})(\mathcal{A}_\mathcal{N}) = g_\mathcal{N}(f_\mathcal{N}(\mathcal{A}_\mathcal{N}))$ is $\mathcal{NCS}$ in $(\mathcal{Z}_\mathcal{N}, \eta_N)$. Since $g_\mathcal{N}$ is $\mathcal{N}g^{\#}-$ continuous and one to one mapping, $g_\mathcal{N}^{-1}(g_\mathcal{N} \circ f_\mathcal{N}(\mathcal{A}_\mathcal{N})) = f_\mathcal{N}(\mathcal{A}_\mathcal{N})$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{Y}_\mathcal{N}, \zeta_N)$. This implies that $f_\mathcal{N}$ is strongly $\mathcal{N}g^{\#}-$ closed mapping. □

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