



## Neutrosophic BCK-algebra and $\Omega$ -BCK-algebra

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### Abstract

In this paper, we study neutrosophic of one important types of algebra namely BCK-algebra. Some new results of a generalization of BCK-algebra ( $\Omega$ -BCK-algebra) have been introduced. Several facts about neutrosophic  $\Omega$ -BCK-algebra are presented such as neutrosophic of homomorphic image and neutrosophic of kernel homomorphism. Finally, some definitions, examples, and other properties of neutrosophic BCK-algebra and neutrosophic  $\Omega$ -BCK-algebra are given.

**Keywords:** Neutrosophic set; Neutrosophic algebra; BCK-algebra; BCI-algebra; Fuzzy set

### 1. Introduction

In 1998 Smarandache [1,2] came up with the idea of a neutrosophic set to cover the weaknesses discovered in fuzzy sets [3] and intuitionistic fuzzy sets [4]. Based on this mathematical idea, many researchers have made a lot of contributions by linking this tool with other branches of mathematics, such as complex analysis, topology, algebra, numerical analysis, and so on. In neutrosophic logic each matter is approach to the percentage of truth inside T, the percentage of indeterminacy inside I, and the percentage of falsity inside F. In fact, the neutrosophic set is the generalization of fuzzy set. We denote (NE) to neutrosophic property. The concept of fuzzy algebra was developed by Zadeh's student [5]; since then, many researchers have made significant contributions to fuzzy sets and their extension with various algebraic structures; for example, Youssef and Dib [6] studied the concept of a fuzzy group and demonstrated numerous results. Fathi and Salleh [7] generalized the idea of the fuzzy group to the intuitionistic fuzzy subgroup. Agboola et al. [8] created a neutrosophic group by adding a third membership to the definition of intuitionistic fuzzy groups [9]. Smarandache and others contributed to the neutrosophic algebraic environment [10-22]. In an environment of BCK/BCI BCK/BCI -algebras, there were several contributions discussed in [23-29]. Several authors inspired of some algebras, for example BCK-algebra, BCI-algebra and KU-algebra. An algebra  $(A, *, \circ)$  is said to be BCK-algebra if  $[(a*b)*(a*c)]*(c*b)=0$ ,  $[(a*(a*b))*b]=0$ ,  $[(a*a)=0]$ ,  $[(0*a)=0]$  and  $[(a*b)=(b*a)=0, \text{ so } a=b]$  for all  $a, b, c \in A$ . In this paper we introduced a new idea that is: neutrosophic BCK-algebra and neutrosophic  $\Omega$ -BCK-algebra as a generalization of neutrosophic BCK-algebra. In section two, we define neutrosophic BCK-algebra, neutrosophic  $\Omega$ -BCK-algebra, and some of their properties are discussed. In the next section, neutrosophic  $\Omega$ -BCK-algebra are presented with their generalization. Finally, some their properties have been constructed with some remarks and examples.

### 2. Preliminaries

This section is about some of the vital definitions, remarks and examples which are used later in this paper.

**Definition 2.1.** [3]

Let  $X$  be a nonempty set. A fuzzy set  $A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$  is characterized by a membership function  $\mu_A: X \rightarrow [0, 1]$ , where  $\mu_A(x)$  is interpreted as the degree of membership of the element  $x$  in the fuzzy subset  $A$  for any  $x \in X$ .

**Definition 2.2.** [1,2]

Let  $A$  be a universal set. The neutrosophic  $A$ , in short  $NE(A)$  is defined as

$$B = \{ \langle \xi, tH(\xi), iH(\xi), fH(\xi) \rangle : \xi \in A \} \ni tH, iH, fH : A \rightarrow [0, 1].$$

Note that there is an equivalent definition to Definition 2.2 and by the following:

Let  $X$  be a nonempty set. A neutrosophic set (NS, for short)  $A$  on  $X$  is an object of the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle \mid x \in X \}$  characterized by a membership function  $\mu_A : X \rightarrow ]-0, 1+[$  and an indeterminacy function  $\sigma_A : X \rightarrow ]-0, 1+[$  and a non-membership function  $\nu_A : X \rightarrow ]-0, 1+[$  which satisfy the condition:  $-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3+$ , for any  $x \in X$ .

**Definition 2.3.** [1,2]

Let  $S = \{ \langle x, \mu_S(x), \sigma_S(x), \gamma_S(x) \rangle : x \in X \}$  and  $T = \{ \langle x, \mu_T(x), \sigma_T(x), \gamma_T(x) \rangle : x \in X \}$  be the two neutrosophic sets in  $X$ . Then the following statements are true in  $X$ :

- (i)  $S \subseteq T \Leftrightarrow \{ \langle x, \mu_S(x) \leq \mu_T(x), \sigma_S(x) \leq \sigma_T(x), \gamma_S(x) \geq \gamma_T(x) \rangle : x \in X \}$
- (ii)  $S \cap T = \{ \langle x, \mu_S(x) \wedge \mu_T(x), \sigma_S(x) \wedge \sigma_T(x), \gamma_S(x) \vee \gamma_T(x) \rangle : x \in X \}$
- (iii)  $S \cup T = \{ \langle x, \mu_S(x) \vee \mu_T(x), \sigma_S(x) \vee \sigma_T(x), \gamma_S(x) \wedge \gamma_T(x) \rangle : x \in X \}$
- (iv)  $0N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
- (v)  $1N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$ .

**Definition 2.4.** [8]

Let  $(G, *)$  be a group with  $(GUI) = \{ a+bI \mid a \text{ and } b \text{ in } G \}$ . We say  $NE(G) = \langle GUI, * \rangle$  is neutrosophic group which it is generated by  $G, I$ .

**Remarks 2.5.**

- 1)  $I$  denote to neutrosophic element such that  $I^2 = I$ . Also,  $0 \cdot I = 0 \cdot I^{-1}$  is the inverse of  $I$  which is not exists.
- 2)  $NE(G)$  is commutative if  $ab = ba$  such that  $a$  and  $b$  in  $NE(G)$ .
- 3)  $N(G) <$  is not group in general, but it contains a group.

**Definition 2.6.** [8]

Let  $NE(G)$  be a neutrosophic group. We say  $NE(A)$  is a neutrosophic subgroup if it satisfies all conditions of neutrosophic group;  $NE(A)$  contains a subset and it is group.

**Definition 2.7.** [9]

Let  $A$  be a semigroup, the semigroup generated by  $A$  and  $I$  ( $\langle AUI \rangle$ ) denoted by  $\langle AUI \rangle$  is defined to be a neutrosophic semigroup where  $I$  is indeterminacy element and termed as neutrosophic element.

**Remark 2.8.**

All neutrosophic semigroups contain a proper subset which is a semigroup.

**Example 2.9.**

$Z$  is a semigroup with  $(.)$  and let  $NE(A) = \langle ZUI \rangle$  be a neutrosophic semigroup with  $(.)$ . Then  $Z$  subset of  $NE(A)$  is a semigroup.

**Definition 2.10.** [9]

Let  $NE(A)$  be a neutrosophic monoid under the binary operation  $*$ . Suppose  $e$  is the identity in  $N(A)$ , that is  $s * e = e * s = s$  for all  $s \in NE(A)$ .

**Remark 2.11.**

A proper subset  $B$  of  $NE(A)$  is a neutrosophic submonoid if  $B$  is a neutrosophic semigroup with  $*$  and  $e \in B$ , i.e.,  $B$  is a monoid with  $*$ .

**3. Neutrosophic BCK-algebra**

In this section, we present neutrosophic BCK-algebra with some their properties and some examples about it.

**Definition 3.1.**

An algebra  $(A, *, \circ)$  is called neutrosophic BCK-algebra if:

- 1)  $\{T((a*b) * (a*c)) * T(c*b)=0, [I(a*b)*(a*c)]*I(c*b)=0, [F(a*b)*(a*c)]*F(c*b)=0\}$ .
- 2)  $\{T[a*(a*b)*T(b)]=0, I[(a*(a*b))*I(b)]=0, F[(a*(a*b))*F(b)]=0\}$ .
- 3)  $\{T[(a*a)]=0, I(a*a) * =0, F(a*a) =0\}$ .
- 4)  $\{T[(0*a)]=0, I(0*a) =0, F(0*a)=0\}$ .
- 5)  $\{T[((a*b)=(b*a))]=0, \text{ so } a=b, I[((a*b)=(b*a))]=0, \text{ so } a=b, F[((a*b)=(b*a))]=0, \text{ so } a=b\}$  for all  $a, b, c$  in  $A$ .

**Remarks 3.2.**

Let  $(A, *, \circ)$  be a neutrosophic BCK-algebra. Then the following statements are holds:

- 1)  $\{T((a*b)*(b*c)) \leq T(c*b), (I(a*b)*(b*c)) \leq I(c*b), (F(a*b)*(b*c)) \leq F(c*b)\}$  for all  $a, b, c$  in  $A$ .
- 2)  $\{T(a*(a*(a*b))) \leq T(a*b), (I(a*(a*(a*b)))) \leq I(a*b), (F(a*(a*(a*b)))) \leq F(a*b)\}$ .
- 3)  $\{(0) \leq T(a), (0 \leq I(a)), (0 \leq F(a))\}$ .
- 4)  $\{(T(a*b)=0 \text{ iff } a \leq b), (I(a*b)=0 \text{ iff } a \leq b), (F(a*b)=0 \text{ iff } a \leq b)\}$ .
- 5)  $\{\text{If } T((a \leq b), \text{ then } T(a*c) \leq T(b*c) \text{ and } T(c*b) \leq T(c*a), (\text{If } I((a \leq b), \text{ then } I(a*c) \leq I(b*c) \text{ and } I(c*b) \leq I(c*a), (\text{If } F((a \leq b), \text{ then } F(a*c) \leq F(b*c) \text{ and } F(c*b) \leq F(c*a))\}$ .
- 6)  $\{T((a*b)*c)=T((a*c)*b), \{I((a*b)*c)=I((a*c)*b), \{F((a*b)*c)=F((a*c)*b)\}$ .
- 7)  $\{T(a*b) \leq c \text{ iff } T(a*c) \leq b, (I(a*b) \leq c \text{ iff } I(a*c) \leq b, F(a*b) \leq c \text{ iff } F(a*c) \leq b\}$ .
- 8)  $\{0*T(a*b)=T((0*a)*(0*b)), \{0*I(a*b)=I((0*a)*(0*b)), \{0*F(a*b)=F((0*a)*(0*b))\}$ .
- 9)  $\{T((a*c)*(b*c)) \leq T(a*b), I((a*c)*(b*c)) \leq I(a*b), F((a*c)*(b*c)) \leq F(a*b)\}$ .
- 10)  $\{T((a*c)*(b*c)) \leq T(a*b), I((a*c)*(b*c)) \leq I(a*b), F((a*c)*(b*c)) \leq F(a*b)\}$  for all  $a, b$  and  $c$  in  $A$ .

**Example 3.3.**

Let  $A=\{0, x, y\}$ . Then there is a neutrosophic binary operation  $*$  which define by:

$$\begin{aligned} 0*0=0, 0*x=0, 0*y=0, \\ x*0=x, x*x=0, x*y=0, \\ y*0=y, y*x=x, y*y=0. \end{aligned}$$

Hence the neutrosophic BCK-algebra is commutative.

**Definition 3.4.**

Let  $J$  be a non zero set of the neutrosophic BCK-algebra  $A$ . We say  $J$  is a neutrosophic ideal ( $NE(J)$ ) of  $A$  if :

- 1)  $NE(0) \in J$ . i.e.  $[0 \in T(0) \in J, I(0) \in J, F(0) \in J]$ .
  - 2)  $NE(a*b) \in J$ . i.e.  $[(T(a*b) \in J), (I(a*b) \in J), (F(a*b) \in J)]$ . OR  $(NE(a*b) \in NE(J))$ .
  - 3)  $[(T(b) \in J), (I(b) \in J), (F(b) \in J)]$ . OR  $\{NE(b) \in NE(J)\}$ .
- So  $[(T(a) \in J), (I(a) \in J), (F(a) \in J)]$ . OR  $\{NE(a) \in NE(J)\}$ .

**Definition 3.5.** [10]

Let  $A$  be a single valued neutrosophic set on  $X$  and  $B$  be a single valued neutrosophic set on  $Y$ , let  $f: \text{Supp } A \rightarrow \text{Supp } B$  be an ordinary mapping and  $R$  be a single valued neutrosophic relation on  $X \times Y$ . Then  $f_R$  is called a single valued neutrosophic mapping if for all  $(x, y) \in \text{Supp } A \times \text{Supp } B$  the following condition is satisfied:

$$\mu_R(x, y) = \{ \min(\mu_A(x), \mu_B(f(x))), \text{ if } y = f(x) \text{ or } 0, \text{ otherwise} \} \text{ and}$$

$$\sigma_R(x, y) = \{ \min(\sigma_A(x), \sigma_B(f(x))), \text{ if } y = f(x) \text{ or } 0, \text{ otherwise} \} \text{ and}$$

$$\nu_R(x, y) = \{ \max(\nu_A(x), \nu_B(f(x))), \text{ if } y = f(x), \text{ or } 1, \text{ otherwise} \},$$

**Definition 3.6.**

Let  $f: A_1 \rightarrow A_2$  be neutrosophic mapping where  $A_1$  and  $A_2$  are two neutrosophic BCK-algebras. Then  $f$  is called neutrosophic homomorphism if:

$$\{T_f(a*b) = T_f(a)*T_f(b), I_f(a*b) = I_f(a)*I_f(b), F_f(a*b) = F_f(a)*F_f(b)\} \text{ for all } a, b \text{ in } A_1.$$

Now we present another notion namely neutrosophic subalgebra of neutrosophic BCK-algebra  $A$

**Definition 3.7.**

Let  $\varphi \neq J$  be a neutrosophic subset of a neutrosophic BCK-algebra  $A$ . Then  $NE(J)$  is called neutrosophic subalgebra of  $A$  if:

$$\{T(a*b) \in NE(J), (I(a*b) \in NE(J), (F(a*b) \in NE(J)). \text{ OR } NE(a*b) \in NE(J) \text{ for all } a, b \in J.$$

**4. Neutrosophic  $\Omega$ -BCK-algebra**

In this section, we present a neutrosophic of a generalization of BCK-algebra namely  $\Omega$ -BCK-algebra with some properties about the topic, but before that we need to define some concepts which will be used later like  $\Omega$ -group and  $\Omega$ -semigroup.

**Definition 4.1.**

Let  $A$  be a set has an element  $0$  and let  $\varphi \neq \Omega$  be a set. If  $g: A \times \Omega \times A \rightarrow A \quad \forall a, b \text{ in } A \text{ and } \alpha \text{ in } \Omega$  such that the images denoted by  $a \alpha b$  and the following statement are hold:

$$1) [(a \alpha b) \beta (a \alpha c)] \beta (c \alpha b) = 0,$$

$$2) (a \alpha b) = (b \alpha a) = 0, \text{ so } a=b,$$

$$3) a \alpha a = 0,$$

$$4) 0 \alpha a = 0 \quad \forall a, b, c \text{ in } A \text{ and } \alpha, \beta \text{ in } \Omega.$$

Then  $A$  is called  $\Omega$ -BCK-algebra.

**Definition 4.2.**

Let  $A$  be a neutrosophic set has an element  $(0)$  and let  $\varphi \neq \Omega$  be a neutrosophic set. Suppose that  $g: A \times \Omega \times A \rightarrow A$  be a neutrosophic mapping  $\forall a, b \text{ in } A \text{ and } \alpha \text{ in } \Omega$  such that the images denoted by  $a \alpha b$  and the following statement are hold:

- 1)  $\{[T((a \alpha b) \beta (a \alpha c)) \beta (c \alpha b) = 0], [I((a \alpha b) \beta (a \alpha c)) \beta (c \alpha b) = 0], [F((a \alpha b) \beta (a \alpha c)) \beta (c \alpha b) = 0]\}$ ,
- 2)  $\{[T((a \alpha b) = (b \alpha a) = 0), \text{ so } a=b], [I((a \alpha b) = (b \alpha a) = 0), \text{ so } a=b], [F((a \alpha b) = (b \alpha a) = 0), \text{ so } a=b]\}$ ,
- 3)  $\{T(a \alpha a)=0, I(a \alpha a)=0, F(a \alpha a)=0\}$ ,
- 4)  $\{[T((0 \alpha a)=0)], [I((0 \alpha a)=0)], [F((0 \alpha a)=0)]\}, \forall a, b, c \text{ in } A \text{ and } \alpha, \beta \text{ in } \Omega$ .

Then  $A$  is called neutrosophic  $\Omega$ -BCK-algebra.

**Remark 4.3.**

Let  $A$  be a neutrosophic  $\Omega$ -BCK-algebra with  $\alpha$  in  $\Omega$ . If  $*$ :  $A \times A \rightarrow A$  such that  $NE(a*b) = NE(a \alpha b)$ , then  $(A, *, \circ)$  is a neutrosophic BCK-algebra  $(A\alpha)$  for all  $a, b$  in  $A$ .

**Example 4.4.**

Suppose that  $A = \{0, 1, 2, 3\}$  and let  $\Omega = \{\alpha, \beta\}$ . The following results satisfies neutrosophic  $\Omega$ -BCK-algebra for  $A$ :

$$\{T_A (0\alpha 0=0, 0\alpha 1=0, 0\alpha 2=0, 0\alpha 3=0, 1\alpha 0=1, 1\alpha 1=0, 1\alpha 2=2, 1\alpha 3=2, 2\alpha 0=2, 2\alpha 1=3, 2\alpha 2=0, 2\alpha 3=3, 3\alpha 0=3, 3\alpha 1=1, 3\alpha 2=1, 3\alpha 3=0), I_A (0\alpha 0=0, 0\alpha 1=0, 0\alpha 2=0, 0\alpha 3=0, 1\alpha 0=1, 1\alpha 1=0, 1\alpha 2=2, 1\alpha 3=2, 2\alpha 0=2, 2\alpha 1=3, 2\alpha 2=0, 2\alpha 3=3, 3\alpha 0=3, 3\alpha 1=1, 3\alpha 2=1, 3\alpha 3=0), F_A (0\alpha 0=0, 0\alpha 1=0, 0\alpha 2=0, 0\alpha 3=0, 1\alpha 0=1, 1\alpha 1=0, 1\alpha 2=2, 1\alpha 3=2, 2\alpha 0=2, 2\alpha 1=3, 2\alpha 2=0, 2\alpha 3=3, 3\alpha 0=3, 3\alpha 1=1, 3\alpha 2=1, 3\alpha 3=0)\}.$$

**Example 4.5.**

Suppose that  $A = \{0, a, b\}$  and let  $\Omega = \{\alpha, \beta\}$ . The following results satisfies neutrosophic  $\Omega$ -BCK-algebra for  $A$ :

$$\{T_A (0 \alpha 0=0, 0 \alpha a=0, 0 \alpha b=0, a \alpha 0=a, a \alpha a=0, a \alpha b=0, b \alpha 0=b, b \alpha a=a, b \alpha b=0), I_A (0 \alpha 0=0, 0 \alpha a=0, 0 \alpha b=0, a \alpha 0=a, a \alpha a=0, a \alpha b=0, b \alpha 0=b, b \alpha a=a, b \alpha b=0), F_A (0 \alpha 0=0, 0 \alpha a=0, 0 \alpha b=0, a \alpha 0=a, a \alpha a=0, a \alpha b=0, b \alpha 0=b, b \alpha a=a, b \alpha b=0)\}.$$

**Example 4.6.** Suppose that  $A = \{0, y_1, y_2, y_3\}$  and let  $\Omega = \{0\}$ . Then  $A$  is a neutrosophic  $\Omega$ -BCK-algebra if:

$$\{T_A (0 \circ 0=0, 0 \circ y_1=0, 0 \circ y_2=0, 0 \circ y_3, y_1 \circ 0=y_1, y_1 \circ y_1=0, y_1 \circ y_2=0, y_1 \circ y_3=0, y_2 \circ 0=y_2, y_2 \circ y_1=y_1, y_2 \circ y_2=0, y_2 \circ y_3=0, y_3 \circ 0=y_3, y_3 \circ y_1=y_3, y_3 \circ y_2=y_3, y_3 \circ y_3=0), I_A (0 \circ 0=0, 0 \circ y_1=0, 0 \circ y_2=0, 0 \circ y_3, y_1 \circ 0=y_1, y_1 \circ y_1=0, y_1 \circ y_2=0, y_1 \circ y_3=0, y_2 \circ 0=y_2, y_2 \circ y_1=y_1, y_2 \circ y_2=0, y_2 \circ y_3=0, y_3 \circ 0=y_3, y_3 \circ y_1=y_3, y_3 \circ y_2=y_3, y_3 \circ y_3=0), F_A (0 \circ 0=0, 0 \circ y_1=0, 0 \circ y_2=0, 0 \circ y_3, y_1 \circ 0=y_1, y_1 \circ y_1=0, y_1 \circ y_2=0, y_1 \circ y_3=0, y_2 \circ 0=y_2, y_2 \circ y_1=y_1, y_2 \circ y_2=0, y_2 \circ y_3=0, y_3 \circ 0=y_3, y_3 \circ y_1=y_3, y_3 \circ y_2=y_3, y_3 \circ y_3=0)\}.$$

**Definition 4.7.**

Let  $\phi \neq A_1$  be a neutrosophic subset of a neutrosophic  $\Omega$ -BCK-algebra  $A$ . If  $NE(a\alpha b)$  belong to  $NE(A)$ , then  $A_1$  is called neutrosophic subalgebra.

**Example 4.8.**

Let  $A = \{0, 1, 2, 3\}$  and let  $\Omega = \{\alpha\}$ . Then  $J = \{0, 1\}$  is a normal neutrosophic subalgebra where a neutrosophic normal subalgebra means:

Let  $\phi \neq A_1$  be a neutrosophic subset of neutrosophic  $\Omega$ -BCK-algebra  $A$ . So  $A_1$  is called neutrosophic normal subalgebra if  $NE(a \alpha a) \alpha (b \alpha b)$  belong to  $NE(A_1)$ .

**Remarks 4.9.**

i) Any neutrosophic neutrosophic  $\Omega$ -BCK-algebra  $A$  is commutative if  $NE(b\alpha(b\alpha \beta a)) = NE(a \alpha (a \beta b))$  with  $b$  belong to  $A$  and  $\alpha, \beta$  belong to  $\Omega$ .

ii) A neutrosophic  $\Omega$ -BCK-algebra  $A$  is a neutrosophic partially ordered by  $a \leq b$  if and only if  $NE(a\alpha b) = 0$  where  $\alpha$  belong to  $\Omega$  and namely neutrosophic  $\Omega$ -BCK-ordering.

iii) If  $A = \{0, a, b, c\}$  and  $\Omega = \{\alpha, \beta\}$  such that:

$\{T_A(0 \alpha 0=0, 0 \alpha a=0, 0 \alpha b=0, 0 \alpha c=0, a \alpha 0=a, a \alpha a=0, a \alpha b=a, a \alpha c=a, b \alpha 0=b, b \alpha a=b, b \alpha b=0, b \alpha c=b, c \alpha 0=c, c \alpha a=c, c \alpha b=c, c \alpha c=0, 0 \beta 0=0, 0 \beta a=0, 0 \beta b=0, 0 \beta c=0, a \beta 0=a, a \beta a=0, a \beta b=a, a \beta c=a, b \beta 0=b, b \beta a=a, b \beta b=0, b \beta c=a, c \beta 0=c, c \beta a=b, c \beta b=a, c \beta c=0), I_A(0 \alpha 0=0, 0 \alpha a=0, 0 \alpha b=0, 0 \alpha c=0, a \alpha 0=a, a \alpha a=0, a \alpha b=a, a \alpha c=a, b \alpha 0=b, b \alpha a=b, b \alpha b=0, b \alpha c=b, c \alpha 0=c, c \alpha a=c, c \alpha b=c, c \alpha c=0, 0 \beta 0=0, 0 \beta a=0, 0 \beta b=0, 0 \beta c=0, a \beta 0=a, a \beta a=0, a \beta b=a, a \beta c=a, b \beta 0=b, b \beta a=a, b \beta b=0, b \beta c=a, c \beta 0=c, c \beta a=b, c \beta b=a, c \beta c=0), F_A(0 \alpha 0=0, 0 \alpha a=0, 0 \alpha b=0, 0 \alpha c=0, a \alpha 0=a, a \alpha a=0, a \alpha b=a, a \alpha c=a, b \alpha 0=b, b \alpha a=b, b \alpha b=0, b \alpha c=b, c \alpha 0=c, c \alpha a=c, c \alpha b=c, c \alpha c=0, 0 \beta 0=0, 0 \beta a=0, 0 \beta b=0, 0 \beta c=0, a \beta 0=a, a \beta a=0, a \beta b=a, a \beta c=a, b \beta 0=b, b \beta a=a, b \beta b=0, b \beta c=a, c \beta 0=c, c \beta a=b, c \beta b=a, c \beta c=0)\}$ ,

then the neutrosophic  $\Omega$ -BCK-algebra A is commutative.

**Definition 4.10.**

Let  $A_1$  and  $A_2$  be two neutrosophic  $\Omega$ -BCK-algebras. So, the neutrosophic mapping  $g: A_1 \rightarrow A_2$  is called neutrosophic homomorphism if:

$$\{(T(a\alpha b) = T(a) \alpha T(b)), (I(a\alpha b) = T(a) \alpha I(b)), (F(a\alpha b) = T(a) \alpha F(b))\}$$

where a, b belongs to A and  $\alpha$  belong to  $\Omega$ .

**Remark 4.11.**

A neutrosophic Kernel of the homomorphism g ( $NE(Ker(g))$ ) can define it by the following:

Let  $g: A_1 \rightarrow A_2$  be a neutrosophic homomorphism where  $A_1$  and  $A_2$  are two neutrosophic  $\Omega$ -BCK-algebras. Then the  $NE(Ker(g)) = \{a \in A_1 : NE(g(a)) = 0\}$ .

On the other hand,  $NE(Img(g)) = \{NE(g(a) : a \in A_1\}$  and refer to neutrosophic  $Img(g)$ .

**Proposition 4.12.**

Let A be a  $\Omega$ -BCK-algebra. Then  $NE(a \alpha (a\beta b)) = 0$  for all a, b belongs to A and  $\alpha, \beta$  belongs to  $\Omega$ .

**Proof:** From  $NE(\Omega$ -BCK-algebra), for all a, b belongs to A and  $\alpha, \beta$  belongs to  $\Omega$

$$NE(a\beta b) \alpha (a\beta b) = 0 \text{ -----} (*)$$

Now we put  $b=0, c=b$  in (\*), so we obtain  $NE(a \alpha (a\beta b) \alpha b) = 0$ .

**Proposition 4.13.**

Let A be a neutrosophic  $\Omega$ -BCK-algebra. If A is a neutrosophic commutative  $\Omega$ -BCK-algebra and  $a \leq b$ , then  $NE(a) = NE(b \alpha (b \beta a))$  such that a, b belongs to A and  $\alpha, \beta$  belongs to  $\Omega$ .

**Proof:** Suppose that A is a neutrosophic commutative  $\Omega$ -BCK-algebra. Hence

$$NE(a \alpha (a \beta b)) = NE(b \alpha (b \beta a)).$$

Therefore

$$NE(a \alpha 0) = NE(b \alpha (b \beta a)).$$

$$NE(a) = NE(b \alpha (b \beta a))$$

Thus, ends the proof.

**Corollary 4.14.**

Let A be a neutrosophic  $\Omega$ -BCK-algebra. If  $a \leq b$ , such that  $NE(a) = NE(b \alpha (b \beta a))$ , then A is a neutrosophic commutative  $\Omega$ -BCK-algebra.

**Proof:** Assume that  $b \leq a$ . So  $NE(a) = NE(b \alpha (b \beta a))$ . But  $a \leq b$ . Hence  $NE(a \alpha b) = 0$ .

Then

$$NE(a) = NE(a \alpha 0)$$

$$= NE(a \alpha (a \alpha b))$$

Thus

$$NE(a \alpha (a \alpha b)) = NE(b \alpha (b \alpha a))$$

Then  $A$  is a neutrosophic commutative  $\Omega$ -BCK-algebra.

#### Corollary 4.15.

Every  $NE(a) \geq 0$  is a neutrosophic  $\Omega$ -BCK-algebra  $A$  for all  $a, b$  belongs to  $A$ .

**Proof:** We know that  $NE(0 \alpha a) = 0$ ,  $a$  belongs to  $\Omega$ . Therefore  $NE(a) \geq 0$ . Then  $0$  is the least an element in a neutrosophic  $\Omega$ -BCK-algebra.

#### Proposition 4.16.

Let  $g: A_1 \rightarrow A_2$  be a neutrosophic homomorphism of a neutrosophic  $\Omega$ -BCK-algebra. Then neutrosophic image of  $g$  is a neutrosophic subalgebra of a neutrosophic  $\Omega$ -BCK-algebra  $A_2$ .

**Proof:** Assume that  $g: A_1 \rightarrow A_2$  be a neutrosophic homomorphism of a neutrosophic  $\Omega$ -BCK-algebra  $A_1$  and  $A_2$ . We have  $a, b$  is belonging to  $NE(\text{Img}(g))$ . So,

$$\exists S_1, S_2 \in NE(A_1) \ni NE(g(S_1)) = NE(a) \text{ and } NE(g(S_2)) = NE(b).$$

This implies that  $NE(g(S_1) \alpha NE(g(S_2)) = NE(a \alpha b) \forall \alpha \in \Omega$ . Hence

$$NE(g(S_1 \alpha S_2)) = NE(S_1 \alpha S_2).$$

Then

$$NE(a \alpha b) \in NE(\text{Img}(g)).$$

Thus, the neutrosophic of image  $g$  is a neutrosophic subalgebra of  $A_2$ .

#### Corollary 4.17.

The neutrosophic of Kernel of  $g$  is a neutrosophic subalgebra where  $g: A_1 \rightarrow A_2$  is a neutrosophic homomorphism between two neutrosophic  $\Omega$ -BCK-algebras  $A_1$  and  $A_2$ .

**Proof:** Suppose that  $g: A_1 \rightarrow A_2$  is a neutrosophic homomorphism of neutrosophic  $\Omega$ -BCK-algebras  $A_1$  and  $A_2$ . For all  $a, b$  belongs to  $NE(\text{Ker}(g))$ , so  $NE(g(a)) = NE(g(b)) = 0$  and hence  $NE(g(a \alpha b)) = NE(g(a)) \alpha NE(g(b)) = 0 \alpha 0 = 0$ . So,  $NE(a \alpha b) \in NE(\text{Ker}(g))$ . Thus  $NE(\text{Ker}(g))$  is a neutrosophic subalgebra of  $A_1$ .

## 5. Conclusion

In this work, we have studied a new generalization about BCK-algebra named neutrosophic  $\Omega$ -BCK-algebra. some definitions, examples and other properties of neutrosophic BCK-algebra and neutrosophic  $\Omega$ -BCK-algebra are proved.

We look forward in the future to study other topics of linear programming and its applications in practical life using neutrosophic logic, accompanying programs, sensitivity analysis...etc.

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## References

- [1] Smarandache, F. Neutrosophy: Neutrosophic probability, set and logic; American Research Press: Rehoboth, IL, USA, 1998.
- [2] Smarandache, F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 2005, 24, 287-297.
- [3] Zadeh, L.A. Fuzzy sets. *Information and Control*, 1965, 8, 331–352.
- [4] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* 20(1) (1986), 87–96.
- [5] Chang, C. L. (1968). Fuzzy topological spaces. *Journal of mathematical Analysis and Applications*, 24(1), 182-190.
- [6] Youssef, N. L., & Dib, K. A. (1992). A new approach to fuzzy groupoids. *Fuzzy sets and systems*, 49(3), 381-392.
- [7] Fathi, M., & Salleh, A. R. (2009). Intuitionistic fuzzy groups. *Asian Journal of Algebra*, 2(1), 1-10.
- [8] A. A. Agboola, A. D. Akwu, and Y. T. Oyebo, "Neutrosophic groups and subgroups," *International Journal of Mathematical Combinatorics*, vol. 3, pp. 1–9, 2012.
- [9] Yuan, X. H., Li, H. X., & Lee, E. S. (2010). On the definition of the intuitionistic fuzzy subgroups. *Computers & Mathematics with Applications*, 59(9), 3117-3129.
- [10] Smarandache, F., & Ali, M. (2018). Neutrosophic triplet group. *Neural Computing and Applications*, 29(7), 595-601.
- [11] Shabir, M., Ali, M., Naz, M., & Smarandache, F. (2013). Soft neutrosophic group. *Neutrosophic Sets and Systems*, 1, 13-25.
- [12] Abobala, M., & Lattakia, S. (2019). n-refined neutrosophic groups I. *International Journal of Neutrosophic Science*, Vol. 0, (1), 27-34.
- [13] Zhang, X., Smarandache, F., & Liang, X. (2017). Neutrosophic duplet semi-group and cancellable neutrosophic triplet groups. *Symmetry*, 9(11), 275.
- [14] Mohammed Abed, M., Hassan, N., & Al-Sharq, F. (2022). On Neutrosophic Multiplication Module. *Neutrosophic Sets and Systems*, 49(1), 12.
- [15] Abuqamar, M., & Hassan, N. (2022). The Algebraic Structure of Normal Groups Associated with Q-Neutrosophic Soft Sets. *Neutrosophic Sets and Systems*, 48, 328-338.
- [16] Salama, AA & Alblowi, SA 2012, Neutrosophic set and neutrosophic topological space, *ISOR J. mathematics*, vol. (3), Issue (4), pp. 31 - 35.
- [17] Al-Sharqi, F. G., Abed, M. M., & Mhassin, A. A. (2018). On Polish Groups and their Applications. *Journal of Engineering and Applied Sciences*, 13(18), 7533-7536.
- [18] Abed, M. M., Al-Jumaili, A. F., & Al-sharqi, F. G. (2018). Some mathematical structures in a topological group. *J. Algab. Appl. Math*, 16(2), 99-117.
- [19] Jun, Y. B., Al-Masarwah, A., & Qamar, M. A. (2022). Rough Semigroups in Connection with Single Valued Neutrosophic  $(\epsilon, \epsilon)$ -Ideals. *Neutrosophic Sets & Systems*, 51.
- [20] Akinleye, S. A., Smarandache, F., & Agboola, A. A. A. (2016). On neutrosophic quadruple algebraic structures. *Neutrosophic Sets and Systems*, 12(1), 16.
- [21] Al-Obaidi, A. H., Imran, Q. H., & Broumi, S. (2022). On New Concepts of Weakly Neutrosophic Continuous Functions. *Journal of Neutrosophic and Fuzzy Systems (JNFS)* Vol, 4(01), 08-14.
- [22] Abdulkadhim, M. M., Imran, Q. H., Al-Obaidi, A. H., & Broumi, S. (2022). On Neutrosophic Crisp Generalized Alpha Generalized Closed Sets. *International Journal of Neutrosophic Science*, 19(1), 107-115.
- [23] Saeid, A.B.; Jun, Y.B. Neutrosophic subalgebras of BCK/BCI-algebras based on neutrosophic points. *Ann. Fuzzy Math. Inform.* 2017, 14, 87–97.
- [24] Jun, Y. B., Kim, S. J., & Smarandache, F. (2018). Interval neutrosophic sets with applications in BCK/BCI-algebra. *Axioms*, 7(2), 23.
- [25] Khan, M.; Anis, S.; Smarandache, F.; Jun, Y.B. Neutrosophic  $N N$  -structures and their applications in semigroups. *Ann. Fuzzy Math. Inform.* 2017, 14, 583–598.
- [26] Öztürk, M.A.; Jun, Y.B. Neutrosophic ideals in BCK/BCI-algebras based on neutrosophic points. *J. Inter. Math. Virtual Inst.* 2018, 8, 1–17.
- [27] Al-Masarwah, A., & Ahmad, A. G. (2022). A New Interpretation of Multi-Polarity Fuzziness Subalgebras of BCK/BCI-Algebras. *Fuzzy Information and Engineering*, 1-12.
- [28] Al-Masarwah, A., & Alshehri, H. (2022). Algebraic Perspective of Cubic Multi-Polar Structures on BCK/BCI-Algebras. *Mathematics*, 10(9), 1475.
- [29] Al-Masarwah, A., Ahmad, A. G., Muhiuddin, G., & Al-Kadi, D. (2021). Generalized m-polar fuzzy positive implicative ideals of BCK-algebras. *Journal of Mathematics*, 2021.