Square root Diophantine neutrosophic normal interval-valued sets and their aggregated operators in application to multiple attribute decision making

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Abstract

We discuss innovative square root Diophantine neutrosophic normal interval-valued set (SRDioNSNIVS)-based approaches to multiple attribute decision-making (MADM) problems. Square root neutrosophic sets, interval-valued Diophantine neutrosophic sets and neutrosophic normal interval-valued (NSNIV) sets are both extensions of square root Diophantine neutrosophic sets. In this section, we will look over several aggregating operations and how those interpretations have evolved over time. The article is focused on a novel idea known as square root NSNIV weighted averaging (SRDioNSNIVWA), square root NSNIV weighted geometric (SR-DioNSNIVWG), generalized square root NSNIV weighted averaging (GSRDioNSNIVWA), and generalized square root NSNIV weighted geometric (GSRDioNSNIVWG). In order to solve MADM problems, we also begin an algorithm based on the aforementioned operators. The use of the euclidean and hamming distances is described, and examples from real-world situations are given. The main characteristics of these sets under various algebraic operations will be discussed in this communication. They are more practical and straightforward, and the ideal choice may be determined quickly. As a result, the defined models are more accurate and closely tied to Φ. In order to show the reliability and usefulness of the models under examination, we also compare a few of the proposed and current models. The study's results are also fascinating and intriguing.

Keywords: SRDioNSNIVWA; SRDioNSNIVWG; GSRDioNSNIVWA; GSRDioNSNIVWG

1 Introduction

Nearly all actual problems involve some degree of ambiguity. To address the uncertainties, several theories have been put out, including the fuzzy set (FS) by Zadeh,[38] the intuitionistic fuzzy set (IFS) by Atanassov,[5] the Pythagorean fuzzy set (PFS) by Yager,[35] and the neutrosophic set (NSS) by Smarandache.[32] A FS where each component of the universal has a level of belongingness that ranges from 0 to 1, with the grades corresponding to these levels being referred to as the membership value of each element in the set. Applications of FSs, such regression prediction for fuzzy time series[34] and fuzzy c-numbers, require clustering techniques.[37] Applications that require imprecise data, including natural language processing, artificial intelligence, handwriting and speech recognition, etc., are perfect candidates for this gradation approach. Later, Atanassov[5] adds the idea of an IFS logic, which is characterized by the requirement that the total of its membership degree (MD) and non-membership degree (NMD) value is ≤ 1. We might have trouble decisions-making (DM) when the MD

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and NMD sum is ≥ 1. In order to generalize IFS, Yager[35] created the new concept of PFS logic, which is characterized by the requirement that the square total of its MD and NMD is ≤ 1. Akram et al.[1–3] discussed the many applications based on the PFS. We extend Rahman et al.[28] discussion of an interval-valued Pythagorean fuzzy set (IVPFS) logic for geometric aggregation operators to a group DM framework. Pythagorean fuzzy aggregation operator with interval values suggested by Peng et al.[26] A few methods for MADGDM based on the interval-valued Pythagorean fuzzy Einstein aggregation operator were proposed by Rahman et al.[29] Yang et al. developed the idea of IVPFS with normal aggregation operations for MADM.[36] The square root fuzzy set (SRFS) and its weighted aggregated operators were studied in the context of DM by Shami et al.

Smarandache recently developed a novel theory, the neuosophic set (NSS).[32] The understanding of neutral mind is referred to as "neutosophy" and this neutrality is the main distinction between FS and IFS. Each statement is given a truth degree (TD), an indeterminacy degree (ID), and a false degree (FD). Each component of the cosmos has a level of TD, ID, FD that falls between $[0, 1]$ in the NSS set. Philosophically, it has been shown that an NSS generalizes a classical set, an FS, an IFS, and so on. By Smarandache et al.,[16] the Pythagorean NSIV set (PNSIVS) was first introduced. The single-valued NSS is applied for medical diagnostics and context analysis.[30] Ejegwa[6] extended distance measures for IFSs, including hamming distance (HD), euclidean distance (ED), normalized euclidean distance (NED), as well as their resemblances to PFSs, and applied them to MCDM and MADM problems. Palanikumar et al.[17] addressed the reasoning behind MADM for Pythagorean NSNIV aggregation operators. As a generalization of the PNSIVS, we see that the majority of the distance functions for PNSNIVSs are presented. Palanikumar et al. discussed various ideal structure of subbisemiring theory and its applications.[18–25]

Peng and Dai[27] addressed the idea of neutrosophic MADM under the MABAC and TOPSIS, whereas Zhang and Xu[39] advocated generalizing PFS based on TOPSIS to incorporate MCDM. Hwang et al.[7] discussed a number of practical MADM uses. According to Jana et al. looked at a brand-new generalization of the bipolar fuzzy soft set logical.[8] Jana discussed the extended bipolar fuzzy MABAC based MAGDM approach.[15] Jana et al. presented a novel approach for robust single valued neutrosophic soft aggregating operators under MCDM[10] with bipolar fuzzy soft.[12] Jana et al. introduced Pythagorean fuzzy dombi aggregation operations.[11] Ullah et al.[33] dealt with distance measuring for complex PFS with practical pattern recognition applications. According to Jana et al.[14] engaged with the new aggregating operators based on the trapezoidal neutrosophic MADM logic. Utilizing a novel approach method for neutrosophic dombi power aggregating operators, Jana et al. cooperated on MCDM.[9] In recent years, Jana et al. presented the MCDM approach under single valued trigonometric number (SVTrN) dombi aggregation mappings.[13] The definition of SRDioNSNIVS data is to be expanded in this work. Aggregation operators are used to obtain SRDioNSNIVS data. By way of illustration, we will develop a ranking based on these operators and use it with DM problems.

1. A novel ED and HD measure is introduced for SRDioNSNIVSs.
2. Use of the newly introduced definition for MADM, SRDioNSNIVN aggregation operators, and a practical illustration.
3. Based on SRDioNSNIVWA, SRDioNSNIVWG, GSRDioNSNIVWA, and GSRDioNSNIVWG, determine positive and negative ideal values.
4. Making a decision based on Φ to arrive at a result.

The paper is divided into the seven sections listed below. Section 1 denotes the introduction. A brief explanation of the linked ideas is given in the section 2. Section 3 discusses MADM based on square root NSNIV number (SRDioNSNIVN) and its procedures. Section 4 uses the SRDioNSNIVNs separation distance to communicate with MADM. Section 5 discusses MADM for SRDioNSNIVN based on a few aggregation operations. Section 6 discusses MADM using SRDioNSNIV data, an algorithm with a numerical example, analysis, and discussion. The conclusion is found in section 7.

2 Preliminaries

In this section, we will quickly go over some of the fundamental terms, we will need for our future studies.

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Definition 2.1. [35] Let Ω be the universe. The PFS Γ in Ω is $\Gamma = \Big\{\eta, \big\langle \Psi_\Gamma^{\mathcal{T}}(\eta), \Psi_\Gamma^{\mathcal{F}}(\eta)\big\rangle \big| \eta \in \Omega\Big\}$, where $\Psi_\Gamma^{\mathcal{T}} : \Omega \to [0,1]$ and $\Psi_\Gamma^{\mathcal{F}} : \Omega \to [0,1]$ are denotes the MD and NMD of $\eta \in \Omega$ to Γ, respectively and $0 \leq (\Psi_\Gamma^{\mathcal{T}}(\eta))^2 + (\Psi_\Gamma^{\mathcal{F}}(\eta))^2 \leq 1$. For the sake of convenience, $\Gamma = \big\langle \Psi_\Gamma^{\mathcal{T}}, \Psi_\Gamma^{\mathcal{F}} \big\rangle$ is represent a Pythagorean fuzzy number (PFN).

Definition 2.2. [4] The square root fuzzy set (SRFS) Γ in Ω is $\Gamma = \Big\{\eta, \big\langle \Psi_\Gamma^{\mathcal{T}}(\eta), \Psi_\Gamma^{\mathcal{F}}(\eta)\big\rangle \big| \eta \in \Omega\Big\}$, where $\Psi_\Gamma^{\mathcal{T}} : \Omega \to [0,1]$ and $\Psi_\Gamma^{\mathcal{F}} : \Omega \to [0,1]$ are denotes the MD and NMD of $\eta \in \Omega$ to Γ, respectively and $0 \leq (\Psi_\Gamma^{\mathcal{T}}(\eta))^2 + \sqrt{\Psi_\Gamma^{\mathcal{F}}(\eta)} \leq 1$. For the sake of convenience, $\Gamma = \big\langle \Psi_\Gamma^{\mathcal{T}}, \Psi_\Gamma^{\mathcal{F}} \big\rangle$ is represent a square root fuzzy number (SRFN).

Definition 2.3. [26] The Pythagorean interval-valued fuzzy set (PIVFS) Γ in Ω is $\widetilde{\Gamma} = \Big\{\eta, \Big\langle \widetilde{\Psi_\Gamma^{\mathcal{T}}}(\eta), \widetilde{\Psi_\Gamma^{\mathcal{F}}}(\eta)\Big\rangle \Big| \eta \in \Omega\Big\}$, where $\widetilde{\Psi_\Gamma^{\mathcal{T}}} : \Omega \to Int([0,1])$ and $\widetilde{\Psi_\Gamma^{F}} : \Omega \to Int([0,1])$ are denotes the MD and NMD of $\eta \in \Omega$ to Γ, respectively, and $0 \leq (\Psi_\Gamma^{\mathcal{T}\mathcal{U}}(\eta))^2 + (\Psi_\Gamma^{\mathcal{F}\mathcal{U}}(\eta))^2 \leq 1$. For the sake of convenience, $\widetilde{\Gamma} = \Big\langle \Big[\Psi_\Gamma^{\mathcal{T}\mathcal{L}}, \Psi_\Gamma^{\mathcal{T}\mathcal{U}}\Big], \Big[\Psi_\Gamma^{\mathcal{F}\mathcal{L}}, \Psi_\Gamma^{\mathcal{F}\mathcal{U}}\Big] \Big\rangle$ is represent a Pythagorean interval-valued fuzzy number (PIVFN).

Definition 2.4. [32] The NSS Γ in Ω is $\Gamma = \Big\{\eta, \big\langle \Psi_\Gamma^{\mathcal{T}}(\eta), \Psi_\Gamma^{\mathcal{I}}(\eta), \Psi_\Gamma^{\mathcal{F}}(\eta)\big\rangle \big| \eta \in \Omega\Big\}$, $\Psi_\Gamma^{\mathcal{T}} : \Omega \to [0,1]$, $\Psi_\Gamma^{\mathcal{I}} : \Omega \to [0,1]$ and $\Psi_\Gamma^{\mathcal{F}} : \Omega \to [0,1]$ are denotes the TD, ID and FD of $\eta \in \Omega$ to Γ, respectively and $0 \leq \Psi_\Gamma^{\mathcal{T}}(\eta) + \Psi_\Gamma^{\mathcal{I}}(\eta) + \Psi_\Gamma^{\mathcal{F}}(\eta) \leq 3$. For the sake of convenience, $\Gamma = \big\langle \Psi_\Gamma^{\mathcal{T}}, \Psi_\Gamma^{\mathcal{I}}, \Psi_\Gamma^{\mathcal{F}} \big\rangle$ is represent a neutrosophic number (NSN).

Definition 2.5. [16] The Pythagorean neutrosophic set (PNSS) Γ in Ω is $\Gamma = \Big\{\eta, \big\langle \Psi_\Gamma^{\mathcal{T}}(\eta), \Psi_\Gamma^{\mathcal{I}}(\eta), \Psi_\Gamma^{\mathcal{F}}(\eta)\big\rangle \big| \eta \in \Omega\Big\}$, $\Psi_\Gamma^{\mathcal{T}} : \Omega \to [0,1]$, $\Psi_\Gamma^{\mathcal{I}} : \Omega \to [0,1]$ and $\Psi_\Gamma^{\mathcal{F}} : \Omega \to [0,1]$ are denotes the TD, ID and FD of $\eta \in \Omega$ to Γ, respectively and $0 \leq (\Psi_\Gamma^{\mathcal{T}}(\eta))^2 + (\Psi_\Gamma^{\mathcal{I}}(\eta))^2 + (\Psi_\Gamma^{\mathcal{F}}(\eta))^2 \leq 2$. For the sake of convenience, $\Gamma = \big\langle \Psi_\Gamma^{\mathcal{T}}, \Psi_\Gamma^{\mathcal{I}}, \Psi_\Gamma^{\mathcal{F}} \big\rangle$ is represent a Pythagorean neutrosophic number (PNSN).

Definition 2.6. [26] Let $\widetilde{\Gamma} = \Big\langle [\Psi^{\mathcal{T}\mathcal{L}}, \Psi^{\mathcal{T}\mathcal{U}}], [\Psi^{\mathcal{F}\mathcal{L}}, \Psi^{\mathcal{F}\mathcal{U}}] \Big\rangle$, $\widetilde{\Gamma}_1 = \Big\langle [\Psi_1^{\mathcal{T}\mathcal{L}}, \Psi_1^{\mathcal{T}\mathcal{U}}], [\Psi_1^{\mathcal{F}\mathcal{L}}, \Psi_1^{\mathcal{F}\mathcal{U}}] \Big\rangle$ and $\widetilde{\Gamma}_2 = \Big\langle [\Psi_2^{\mathcal{T}\mathcal{L}}, \Psi_2^{\mathcal{T}\mathcal{U}}], [\Psi_2^{\mathcal{F}\mathcal{L}}, \Psi_2^{\mathcal{F}\mathcal{U}}] \Big\rangle$ be the PIVFNs, and $\Phi > 0$. Then,

1. $\widetilde{\Gamma}_1 \oplus \Gamma_2 = \left[\begin{array}{c}\Big[\sqrt{(\Psi_1^{\mathcal{T}\mathcal{L}})^2 + (\Psi_2^{\mathcal{T}\mathcal{L}})^2 - (\Psi_1^{\mathcal{T}\mathcal{L}})^2 \cdot (\Psi_2^{\mathcal{T}\mathcal{L}})^2}, \sqrt{(\Psi_1^{\mathcal{T}\mathcal{U}})^2 + (\Psi_2^{\mathcal{T}\mathcal{U}})^2 - (\Psi_1^{\mathcal{T}\mathcal{U}})^2 \cdot (\Psi_2^{\mathcal{T}\mathcal{U}})^2}\Big], \\ \Big[\Psi_1^{\mathcal{F}\mathcal{L}} \cdot \Psi_2^{\mathcal{F}\mathcal{L}}, \Psi_1^{\mathcal{F}\mathcal{U}} \cdot \Psi_2^{\mathcal{F}\mathcal{U}}\Big]\end{array}\right]$,
2. $\widetilde{\Gamma}_1 \otimes \widetilde{\Gamma}_2 = \left[\begin{array}{c}\Big[\Psi_1^{\mathcal{T}\mathcal{L}} \cdot \Psi_2^{\mathcal{T}\mathcal{L}}, \Psi_1^{\mathcal{T}\mathcal{U}} \cdot \Psi_2^{\mathcal{T}\mathcal{U}}\Big], \\ \Big[\sqrt{(\Psi_1^{\mathcal{F}\mathcal{L}})^2 + (\Psi_2^{\mathcal{F}\mathcal{L}})^2 - (\Psi_1^{\mathcal{F}\mathcal{L}})^2 \cdot (\Psi_2^{\mathcal{F}\mathcal{L}})^2}, \sqrt{(\Psi_1^{\mathcal{F}\mathcal{U}})^2 + (\Psi_2^{\mathcal{F}\mathcal{U}})^2 - (\Psi_1^{\mathcal{F}\mathcal{U}})^2 \cdot (\Psi_2^{\mathcal{F}\mathcal{U}})^2}\Big]\end{array}\right]$,
3. $\Phi \cdot \widetilde{\Gamma} = \Big[\Big[\sqrt{1 - \big(1 - (\Psi^{\mathcal{T}\mathcal{L}})^2\big)^{\Phi}}, \sqrt{1 - \big(1 - (\Psi^{\mathcal{T}\mathcal{U}})^2\big)^{\Phi}}\ \Big], \Big[(\Psi^{\mathcal{F}\mathcal{L}})^{\Phi}, (\Psi^{\mathcal{F}\mathcal{U}})^{\Phi}\Big]\Big]$,
4. $\widetilde{\Gamma}^{\Phi} = \Big[\Big[(\Psi^{\mathcal{T}\mathcal{L}})^{\Phi}, (\Psi^{\mathcal{T}\mathcal{U}})^{\Phi}\Big], \Big[\sqrt{1 - \big(1 - (\Psi^{\mathcal{F}\mathcal{L}})^2\big)^{\Phi}}, \sqrt{1 - \big(1 - (\Psi^{\mathcal{F}\mathcal{U}})^2\big)^{\Phi}}\ \Big]\Big]$.

Definition 2.7. [26] For any PIVFN $\widetilde{\Gamma} = \Big\langle [\Psi^{\mathcal{T}\mathcal{L}}, \Psi^{\mathcal{T}\mathcal{U}}], [\Psi^{\mathcal{F}\mathcal{L}}, \Psi^{\mathcal{F}\mathcal{U}}] \Big\rangle$ and score function $S(\widetilde{\Gamma})$ is defined as $S(\widetilde{\Gamma}) = \frac{1}{2}\Big((\Psi^{\mathcal{T}\mathcal{L}})^2 + (\Psi^{\mathcal{T}\mathcal{U}})^2 - (\Psi^{\mathcal{F}\mathcal{L}})^2 - (\Psi^{\mathcal{F}\mathcal{U}})^2\Big)$, $S(\widetilde{\Gamma}) \in [-1,1]$, and the accuracy function $H(\widetilde{\Gamma})$ is defined as $H(\widetilde{\Gamma}) = \frac{1}{2}\Big((\Psi^{\mathcal{T}\mathcal{L}})^2 + (\Psi^{\mathcal{T}\mathcal{U}})^2 + (\Psi^{\mathcal{F}\mathcal{L}})^2 + (\Psi^{\mathcal{F}\mathcal{U}})^2\Big)$, $H(\widetilde{\Gamma}) \in [0,1]$.

Definition 2.8. For any SRIVFN $\widetilde{\Gamma} = \Big\langle [\Psi^{\mathcal{T}\mathcal{L}}, \Psi^{\mathcal{T}\mathcal{U}}], [\Psi^{\mathcal{F}\mathcal{L}}, \Psi^{\mathcal{F}\mathcal{U}}] \Big\rangle$ and score function $S(\widetilde{\Gamma})$ is defined as $S(\widetilde{\Gamma}) = \frac{1}{2}\Big((\Psi^{\mathcal{T}\mathcal{L}})^2 + (\Psi^{\mathcal{T}\mathcal{U}})^2 - \sqrt{\Psi^{\mathcal{F}\mathcal{L}}} - \sqrt{\Psi^{\mathcal{F}\mathcal{U}}}\Big)$, $S(\widetilde{\Gamma}) \in [-1,1]$, and the accuracy function $H(\widetilde{\Gamma})$ is defined as

$$H(\widetilde{\Gamma}) = \frac{1}{2}\Big((\Psi^{\mathcal{T}\mathcal{L}})^2 + (\Psi^{\mathcal{T}\mathcal{U}})^2 + \sqrt{\Psi^{\mathcal{F}\mathcal{L}}} + \sqrt{\Psi^{\mathcal{F}\mathcal{U}}}\Big),\ H(\widetilde{\Gamma}) \in [0,1].$$

Definition 2.9. [37] The fuzzy number $M(\eta) = e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}, (\mu > 0)$ is called a normal fuzzy number (NFN) if $M = (\lambda, \mu)$ and the NFN set (NFNS) is denoted by $\widetilde{N}$, where R is a real number.

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Definition 2.10. [34] Let $\widetilde{N}$ be the NFNS, $M_1 = (\lambda_1, \mu_1) \in \widetilde{N}$and $M_2 = (\lambda_2, \mu_2) \in \widetilde{N}$, $(\mu_1, \mu_2 > 0)$. Then distance between M_1 and M_2 is $\mathbb{D}(M_1, M_2) = \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{2}(\mu_1 - \mu_2)^2}$.

3 Basic operations for SRDioNSNIVN approach

We defined the SRDioNSNIVN and its operations in relation to the concepts of square root NSIVN (SRDioNSIVN) and NFN.

Definition 3.1. Let Ω be the universe. The SRDioNSIV set $\widetilde{\Gamma}$ in Ω is

$$\widetilde{\Gamma} = \Big\{\eta, \Big\langle \Big(\widetilde{\Psi^{\mathcal{T}}_{\Gamma}}(\eta), \widetilde{\Psi^{\mathcal{I}}_{\Gamma}}(\eta), \widetilde{\Psi^{\mathcal{F}}_{\Gamma}}(\eta)\Big), (\beta_1, \beta_2, \beta_3) \Big\rangle \Big| \eta \in \Omega\Big\},$$

where $\widetilde{\Psi^{T}_{\Gamma}} : \Omega \to Int([0, 1])$, $\widetilde{\Psi^{I}_{\Gamma}} : \Omega \to Int([0, 1])$ and $\widetilde{\Psi^{F}_{\Gamma}} : \Omega \to INT([0, 1])$ are denotes the TD, ID and FD of $\eta \in \Omega$ to $\widetilde{\Gamma}$, respectively and $0 \leq (\widetilde{\Psi^{\mathcal{T}}_{\Gamma}}(\eta))^2 + \sqrt{\widetilde{\Psi^{\mathcal{I}}_{\Gamma}}(\eta)} + \sqrt{\widetilde{\Psi^{\mathcal{F}}_{\Gamma}}(\eta)} \leq 2$, implies $0 \leq (\beta_1\Psi^{\mathcal{TU}}_{\Gamma}(\eta))^2 + \sqrt{\beta_2\Psi^{\mathcal{IU}}_{\Gamma}(\eta)} + \sqrt{\Psi^{\mathcal{FU}}_{\Gamma}(\eta)} \leq 2$, where $\beta_1, \beta_2, \beta_3 \in [0, 1]$ and $\beta_1 + \beta_2 + \beta_3 \leq 1$. For the sake of convenience, $\widetilde{\Gamma} = \Big\langle \Big[\beta_1\Psi^{\mathcal{TL}}_{\Gamma}, \beta_1\Psi^{\mathcal{TU}}_{\Gamma}\Big], \Big[\beta_2\Psi^{\mathcal{IL}}_{\Gamma}, \beta_2\Psi^{\mathcal{IU}}_{\Gamma}\Big], \Big[\Psi^{\mathcal{FL}}_{\Gamma}, \Psi^{\mathcal{FU}}_{\Gamma}\Big]\Big\rangle$ is called a square root Diophantine neutrosophic interval-valued number(SRDioNSIVN).

Definition 3.2. For $\widetilde{\Gamma} = \Big\langle \Big[\beta_1\Psi^{\mathcal{TL}}_{\Gamma}, \beta_1\Psi^{\mathcal{TU}}_{\Gamma}\Big], \Big[\beta_2\Psi^{\mathcal{IL}}_{\Gamma}, \beta_2\Psi^{\mathcal{IU}}_{\Gamma}\Big], \Big[\Psi^{\mathcal{FL}}_{\Gamma}, \Psi^{\mathcal{FU}}_{\Gamma}\Big]\Big\rangle$, the score function

$$S(\widetilde{\Gamma}) = \frac{\lambda}{2}\left(\frac{(\beta_1\Psi^{\mathcal{TL}})^2 + (\beta_1\Psi^{\mathcal{TU}})^2}{2} - \frac{\sqrt{\beta_2\Psi^{\mathcal{IL}}} + \sqrt{\beta_2\Psi^{\mathcal{IU}}}}{2} + 1 - \frac{\sqrt{\beta_3\Psi^{\mathcal{FL}}} + \sqrt{\beta_3\Psi^{\mathcal{FU}}}}{2}\right),$$

where $S(\widetilde{\Gamma}) \in [-1, 1]$.

Definition 3.3. Let $\widetilde{\Gamma} = \Big\langle(\lambda, \mu); [\beta_1\Psi^{\mathcal{TL}}, \beta_1\Psi^{\mathcal{TU}}], [\beta_2\Psi^{\mathcal{IL}}, \beta_2\Psi^{\mathcal{IU}}], [\beta_3\Psi^{\mathcal{FL}}, \beta_3\Psi^{\mathcal{FU}}]\Big\rangle$ is a square root neutrosophic normal interval-valued number (SRDioNSNIVN). The FD, ID and FD are defined as $\big[\beta_1\Psi^{\mathcal{TL}}, \beta_1\Psi^{\mathcal{TU}}\big] = \Big[\beta_1\Psi^{\mathcal{TL}}e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}, \beta_1\Psi^{\mathcal{TU}}e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}\Big]$, $\big[\beta_2\Psi^{\mathcal{IL}}, \beta_2\Psi^{\mathcal{IU}}\big] = \Big[\beta_2\Psi^{\mathcal{IL}}e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}, \beta_2\Psi^{\mathcal{IU}}e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}\Big]$ and $\big[\beta_3\Psi^{\mathcal{FL}}, \beta_3\Psi^{\mathcal{FU}}\big] = \Big[1-\big(1-\beta_3\Psi^{\mathcal{FL}}\big)e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}, 1-\big(1-\beta_3\Psi^{\mathcal{FU}}\big)e^{-\left(\frac{\eta-\lambda}{\mu}\right)^2}\Big]$, $x \in X$ respectively, where X is a non-empty set and $\big[\beta_1\Psi^{\mathcal{TL}}, \beta_1\Psi^{\mathcal{TU}}\big]$, $\big[\beta_2\Psi^{\mathcal{IL}}, \beta_2\Psi^{\mathcal{IU}}\big]$, $\big[\beta_3\Psi^{\mathcal{FL}}, \beta_3\Psi^{\mathcal{FU}}\big] \in ([0, 1])$ and $0 \leq \big(\beta_1\Psi^{\mathcal{TU}}(\eta)\big)^2 + \sqrt{\beta_2\Psi^{\mathcal{IU}}(\eta)} + \sqrt{\beta_3\Psi^{\mathcal{FU}}(\eta)} \leq 2$, where $(\lambda, \mu) \in \widetilde{N}$.

Definition 3.4. Let $\widetilde{\Gamma} = \Big\langle(\lambda, \mu); [\beta\Psi^{\mathcal{TL}}, \Psi^{\mathcal{TU}}], [\beta\Psi^{\mathcal{IL}}, \beta\Psi^{\mathcal{IU}}], [\beta\Psi^{\mathcal{FL}}, \beta\Psi^{\mathcal{FU}}]\Big\rangle$, $\widetilde{\Gamma}_1 = \Big\langle(\lambda_1, \mu_1); [\beta_1\Psi^{\mathcal{TL}}_1, \beta_1\Psi^{\mathcal{TU}}_1]$, $[\beta_2\Psi^{\mathcal{IL}}_1, \beta_2\Psi^{\mathcal{IU}}_1], [\beta_3\Psi^{\mathcal{FL}}_1, \beta_3\Psi^{\mathcal{FU}}_1]\Big\rangle$ and
$\widetilde{\Gamma}_2 = \Big\langle(\lambda_2, \mu_2); [\beta_1\Psi^{\mathcal{TL}}_2, \beta_1\Psi^{\mathcal{TU}}_2], [\beta_2\Psi^{\mathcal{IL}}_2, \beta_2\Psi^{\mathcal{IU}}_2], [\beta_3\Psi^{\mathcal{FL}}_2, \beta_3\Psi^{\mathcal{FU}}_2]\Big\rangle$ be the any three SRDioNSNIVNs, and $\Phi > 0$. Then,

$$1.\ \widetilde{\Gamma}_1 \oplus \widetilde{\Gamma}_2 = \left[\begin{array}{c} (\lambda_1 + \lambda_2, \mu_1 + \mu_2; \\ \left[\begin{array}{c} \left(\sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TL}}_1} + \sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TL}}_2} - \sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TL}}_1}\cdot\sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TL}}_2}\right)^{2\Phi}, \\ \left(\sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TU}}_1} + \sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TU}}_2} - \sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TU}}_1}\cdot\sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TU}}_2}\right)^{2\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[\Phi]{\beta_2\Psi^{\mathcal{IL}}_1} + \sqrt[\Phi]{\beta_2\Psi^{\mathcal{IL}}_2} - \sqrt[\Phi]{\beta_2\Psi^{\mathcal{IL}}_1}\cdot\sqrt[\Phi]{\beta_2\Psi^{\mathcal{IL}}_2}\right)^{\Phi}, \\ \left(\sqrt[\Phi]{\beta_2\Psi^{\mathcal{IU}}_1} + \sqrt[\Phi]{\beta_2\Psi^{\mathcal{IU}}_2} - \sqrt[\Phi]{\beta_2\Psi^{\mathcal{IU}}_1}\cdot\sqrt[\Phi]{\beta_2\Psi^{\mathcal{IU}}_2}\right)^{\Phi} \end{array}\right], \\ \left[\beta_3\Psi^{\mathcal{FL}}_1\cdot\beta_3\Psi^{\mathcal{FL}}_2, \beta_3\Psi^{\mathcal{FU}}_1\cdot\beta_3\Psi^{\mathcal{FU}}_2\right] \end{array}\right],$$

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2. $\widetilde{\Gamma}_1 \otimes \widetilde{\Gamma}_2 = \left[\begin{array}{c} (\lambda_1 \cdot \lambda_2, \mu_1 \cdot \mu_2; \left[\beta_1\Psi_1^{\mathcal{TL}} \cdot \beta_1\Psi_2^{\mathcal{TL}}, \beta_1\Psi_1^{\mathcal{TU}} \cdot \beta_1\Psi_2^{\mathcal{TU}}\right], \\ \left[\begin{array}{c} \left(\sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IL}}} + \sqrt[\Phi]{\beta_2\Psi_2^{\mathcal{IL}}} - \sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IL}}} \cdot \sqrt[\Phi]{\beta_2\Psi_2^{\mathcal{IL}}}\right)^{\Phi}, \\ \left(\sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IU}}} + \sqrt[\Phi]{\beta_2\Psi_2^{\mathcal{IU}}} - \sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IU}}} \cdot \sqrt[\Phi]{\beta_2\Psi_2^{\mathcal{IU}}}\right)^{\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[2\Phi]{\beta_3\Psi_1^{\mathcal{FL}}} + \sqrt[2\Phi]{\beta_3\Psi_2^{\mathcal{FL}}} - \sqrt[2\Phi]{\beta_3\Psi_1^{\mathcal{FL}}} \cdot \sqrt[2\Phi]{\beta_3\Psi_2^{\mathcal{FL}}}\right)^{2\Phi}, \\ \left(\sqrt[2\Phi]{\beta_3\Psi_1^{\mathcal{FU}}} + \sqrt[2\Phi]{\beta_3\Psi_2^{\mathcal{FU}}} - \sqrt[2\Phi]{\beta_3\Psi_1^{\mathcal{FU}}} \cdot \sqrt[2\Phi]{\beta_3\Psi_2^{\mathcal{FU}}}\right)^{2\Phi} \end{array}\right] \end{array}\right],$

3. $\Phi \cdot \widetilde{\Gamma} = \left[\begin{array}{c} (\Phi \cdot \lambda, \Phi \cdot \mu); \\ \left[\left(1 - \left(1 - \sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TL}}}\right)^{\Phi}\right)^{2\Phi}, \left(1 - \left(1 - \sqrt[2\Phi]{\beta_1\Psi^{\mathcal{TU}}}\right)^{\Phi}\right)^{2\Phi}\right], \\ \left[\sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IL}}}, \sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IU}}}\right], \left[(\beta_3\Psi^{\mathcal{FL}})^{\Phi}, (\beta_3\Psi^{\mathcal{FU}})^{\Phi}\right] \end{array}\right],$

4. $\widetilde{\Gamma}^{\Phi} = \left[\begin{array}{c} (\lambda^{\Phi}, \mu^{\Phi}); \left[(\beta_1\Psi^{\mathcal{TL}})^{\Phi}, (\beta_1\Psi^{\mathcal{TU}})^{\Phi}\right], \left[\sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IL}}}, \sqrt[\Phi]{\beta_2\Psi_1^{\mathcal{IU}}}\right], \\ \left[\left(1 - \left(1 - \sqrt[2\Phi]{\beta_3\Psi^{\mathcal{FL}}}\right)^{\Phi}\right)^{2\Phi}, \left(1 - \left(1 - \sqrt[2\Phi]{\beta_3\Psi^{\mathcal{FU}}}\right)^{\Phi}\right)^{2\Phi}\right] \end{array}\right].$

4 Various distance measure for SRDioNSNIVN approach

We discuss some of its mathematical features and introduce the ED and HD measures for SRDioNSNIVNs.

Definition 4.1. For SRDioNSNIVNs $\widetilde{\Gamma}_1 = \left\langle (\lambda_1, \mu_1; [\beta_1\Psi_1^{\mathcal{TL}}, \beta_1\Psi_1^{\mathcal{TU}}], [\beta_2\Psi_1^{\mathcal{IL}}, \beta_2\Psi_1^{\mathcal{IU}}], [\beta_3\Psi_1^{\mathcal{FL}}, \beta_3\Psi_1^{\mathcal{FU}}] \right\rangle$ and $\widetilde{\Gamma}_2 = \left\langle (\lambda_2, \mu_2; [\beta_1\Psi_2^{\mathcal{TL}}, \beta_1\Psi_2^{\mathcal{TU}}], [\beta_2\Psi_2^{\mathcal{IL}}, \beta_2\Psi_2^{\mathcal{IU}}], [\beta_3\Psi_2^{\mathcal{FL}}, \beta_3\Psi_2^{\mathcal{FU}}] \right\rangle$. Then

$$\mathbb{D}_E\left(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2\right) = \frac{1}{2}\sqrt{\begin{array}{c} \left[\begin{array}{c} \frac{1+(\beta_1\Psi_1^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FL}}} + 1 + (\beta_1\Psi_1^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4}\lambda_1 \\ -\frac{1+(\beta_1\Psi_2^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FL}}} + 1 + (\beta_1\Psi_2^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\lambda_2 \end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{c} \frac{1+(\beta_1\Psi_1^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FL}}} + 1 + (\beta_1\Psi_1^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4}\mu_1 \\ -\frac{1+(\beta_1\Psi_2^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FL}}} + 1 + (\beta_1\Psi_2^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\mu_2 \end{array}\right]^2 \end{array}}$$

where $\mathbb{D}_E\left(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2\right)$ is denote the euclidean distance between $\widetilde{\Gamma}_1$ and $\widetilde{\Gamma}_2$.
Also, the hamming distance $\mathbb{D}_H\left(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2\right)$=

$$\frac{1}{2}\left[\begin{array}{c} \left|\begin{array}{c} \frac{1 + (\beta_1\Psi_1^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FL}}} + 1 + (\beta_1\Psi_1^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4}\lambda_1 \\ -\frac{1 + (\beta_1\Psi_2^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FL}}} + 1 + (\beta_1\Psi_2^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\lambda_2 \end{array}\right| \\ +\frac{1}{2}\left|\begin{array}{c} \frac{1 + (\beta_1\Psi_1^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FL}}} + 1 + (\beta_1\Psi_1^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4}\mu_1 \\ -\frac{1 + (\beta_1\Psi_2^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FL}}} + 1 + (\beta_1\Psi_2^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\mu_2 \end{array}\right| \end{array}\right]$$

where $\mathbb{D}_H\left(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2\right)$ is denote the hamming distance between $\widetilde{\Gamma}_1$ and $\widetilde{\Gamma}_2$.

Theorem 4.2. *If any three SRDioNSNIVNs* $\widetilde{\Gamma}_1 = \left\langle (\lambda_1, \mu_1; [\beta_1\Psi_1^{\mathcal{TL}}, \beta_1\Psi_1^{\mathcal{TU}}], [\beta_2\Psi_1^{\mathcal{IL}}, \beta_2\Psi_1^{\mathcal{IU}}], [\beta_3\Psi_1^{\mathcal{FL}}, \beta_3\Psi_1^{\mathcal{FU}}] \right\rangle, \widetilde{\Gamma}_2 = \left\langle (\lambda_2, \mu_2; [\beta_1\Psi_2^{\mathcal{TL}}, \beta_1\Psi_2^{\mathcal{TU}}], [\beta_2\Psi_2^{\mathcal{IL}}, \beta_2\Psi_2^{\mathcal{IU}}], [\beta_3\Psi_2^{\mathcal{FL}}, \beta_3\Psi_2^{\mathcal{FU}}] \right\rangle$, $\widetilde{\Gamma_3} = \left\langle (\lambda_3, \mu_3; [\beta_1\Psi_3^{\mathcal{TL}}, \beta_1\Psi_3^{\mathcal{TU}}], [\beta_2\Psi_3^{\mathcal{IL}}, \beta_2\Psi_3^{\mathcal{IU}}], [\beta_3\Psi_3^{\mathcal{FL}}, \beta_3\Psi_3^{\mathcal{FU}}] \right\rangle$*, then* $\mathbb{D}_E(\Gamma_1, \Gamma_2$ *satisfies the following properties are holds.*

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1. $\mathbb{D}_E(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2$ *is zero, whenever* $\widetilde{\Gamma}_1 = \widetilde{\Gamma}_2$ *and vice versa.*

2. $\mathbb{D}_E(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2$ *and* $\mathbb{D}_E(\widetilde{\Gamma}_2, \widetilde{\Gamma}_1$ *are co-occur.*

3. $\mathbb{D}_E(\widetilde{\Gamma}_1, \widetilde{\Gamma_3}) \leq \mathbb{D}_E(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2 + \mathbb{D}_E(\widetilde{\Gamma}_2, \widetilde{\Gamma_3})$.

Proof. Now, $\Big(\mathbb{D}_E(\Gamma_1, \Gamma_2 + \mathbb{D}_E(\Gamma_2, \Gamma_3)\Big)^2 =$

$$\left[\begin{array}{l} \frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\beta_1\Psi_1^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_1^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_1^{\mathcal{FL}}}+1+(\beta_1\Psi_1^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_1^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4}\lambda_1\\ -\frac{1+(\beta_1\Psi_2^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FL}}}+1+(\beta_1\Psi_2^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\lambda_2\end{array}\right]^2\\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\beta_1\Psi_1^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_1^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_1^{\mathcal{FL}}}+1+(\beta_1\Psi_1^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_1^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4}\mu_1\\ -\frac{1+(\beta_1\Psi_2^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FL}}}+1+(\beta_1\Psi_2^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\mu_2\end{array}\right]^2\end{array}} \\ +\frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\beta_1\Psi_2^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FL}}}+1+(\beta_1\Psi_2^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\lambda_2\\ -\frac{1+(\beta_1\Psi_3^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_3^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_3^{\mathcal{FL}}}+1+(\beta_1\Psi_3^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_3^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_3^{\mathcal{FU}}}}{4}\lambda_3\end{array}\right]^2\\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\beta_1\Psi_2^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FL}}}+1+(\beta_1\Psi_2^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_2^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4}\mu_2\\ -\frac{1+(\beta_1\Psi_3^{\mathcal{TL}})^2-\sqrt{\beta_2\Psi_3^{\mathcal{IL}}}-\sqrt{\beta_3\Psi_3^{\mathcal{FL}}}+1+(\beta_1\Psi_3^{\mathcal{TU}})^2-\sqrt{\beta_2\Psi_3^{\mathcal{IU}}}-\sqrt{\beta_3\Psi_3^{\mathcal{FU}}}}{4}\mu_3\end{array}\right]^2\end{array}}\end{array}\right]^2$$

implies

$$\begin{aligned}&\frac{1}{4}\Big((\Lambda_1\lambda_1 - \Lambda_2\lambda_2^2) + \frac{1}{2}(\Lambda_1\mu_1 - \Lambda_2\mu_2^2)\Big) + \frac{1}{4}\Big((\Lambda_2\lambda_2 - \Lambda_3\lambda_3^2 + \frac{1}{2}(\Lambda_2\mu_2 - \Lambda_3\mu_3^2)\Big)\\ &+\frac{1}{2}\left(\sqrt{(\Lambda_1\lambda_1 - \Lambda_2\lambda_2^2) + \frac{1}{2}(\Lambda_1\mu_1 - \Lambda_2\mu_2^2)} \times \sqrt{(\Lambda_2\lambda_2 - \Lambda_3\lambda_3^2) + \frac{1}{2}(\Lambda_2\mu_2 - \Lambda_3\mu_3^2)}\right),\end{aligned}$$

Since,

$$\Lambda_1 = \frac{1 + (\beta_1\Psi_1^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FL}}} + 1 + (\beta_1\Psi_1^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_1^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_1^{\mathcal{FU}}}}{4},$$

$$\Lambda_2 = \frac{1 + (\beta_1\Psi_2^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FL}}} + 1 + (\beta_1\Psi_2^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_2^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_2^{\mathcal{FU}}}}{4},$$

$$\Lambda_3 = \frac{1 + (\beta_1\Psi_3^{\mathcal{TL}})^2 - \sqrt{\beta_2\Psi_3^{\mathcal{IL}}} - \sqrt{\beta_3\Psi_3^{\mathcal{FL}}} + 1 + (\beta_1\Psi_3^{\mathcal{TU}})^2 - \sqrt{\beta_2\Psi_3^{\mathcal{IU}}} - \sqrt{\beta_3\Psi_3^{\mathcal{FU}}}}{4}.$$

Hence, $\Big(\mathbb{D}_E(\Gamma_1, \Gamma_2 + \mathbb{D}_E(\Gamma_2, \Gamma_3)\Big)^2$

$$\begin{aligned}&\geq \frac{1}{4}\Big((\Lambda_1\lambda_1 - \Lambda_2\lambda_2^2) + \frac{1}{2}(\Lambda_1\mu_1 - \Lambda_2\mu_2^2)\Big) + \frac{1}{4}\Big((\Lambda_2\lambda_2 - \Lambda_3\lambda_3^2) + \frac{1}{2}(\Lambda_2\mu_2 - \Lambda_3\mu_3^2)\Big)\\ &\quad +\frac{1}{2}\Big((\Lambda_1\lambda_1 - \Lambda_2\lambda_2) \times (\Lambda_2\lambda_2 - \Lambda_3\lambda_3) + \frac{1}{2}\Big((\Lambda_1\mu_1 - \Lambda_2\mu_2) \times (\Lambda_2\mu_2 - \Lambda_3\mu_3)\Big)\\ &= \frac{1}{4}(\Lambda_1\lambda_1 - \Lambda_2\lambda_2 + \Lambda_2\lambda_2 - \Lambda_3\lambda_3^2) + \frac{1}{8}(\Lambda_1\mu_1 - \Lambda_2\mu_2 + \Lambda_2\mu_2 - \Lambda_3\mu_3^2)\\ &= \frac{1}{4}(\Lambda_1\lambda_1 - \Lambda_3\lambda_3^2) + \frac{1}{8}(\Lambda_1\mu_1 - \Lambda_3\mu_3^2)\\ &= \frac{1}{4}\Big[(\Lambda_1\lambda_1 - \Lambda_3\lambda_3^2) + \frac{1}{2}(\Lambda_1\mu_1 - \Lambda_3\mu_3^2)\Big]\\ &= \mathbb{D}_E(\Gamma_1, \Gamma_3^2).\end{aligned}$$

If any three SRDioNSNIVNs $\widetilde{\Gamma}_1 = \Big\langle(\lambda_1, \mu_1; [\beta_1\Psi_1^{\mathcal{TL}}, \beta_1\Psi_1^{\mathcal{TU}}], [\beta_2\Psi_1^{\mathcal{IL}}, \beta_2\Psi_1^{\mathcal{IU}}],$ $[\beta_3\Psi_1^{\mathcal{FL}}, \beta_3\Psi_1^{\mathcal{FU}}]\Big\rangle, \widetilde{\Gamma}_2 = \Big\langle(\lambda_2, \mu_2; [\beta_1\Psi_2^{\mathcal{TL}}, \beta_1\Psi_2^{\mathcal{TU}}], [\beta_2\Psi_2^{\mathcal{IL}}, \beta_2\Psi_2^{\mathcal{IU}}], [\beta_3\Psi_2^{\mathcal{FL}}, \beta_3\Psi_2^{\mathcal{FU}}]\Big\rangle,$ $\widetilde{\Gamma_3} = \Big\langle(\lambda_3, \mu_3; [\beta_1\Psi_3^{\mathcal{TL}}, \beta_1\Psi_3^{\mathcal{TU}}], [\beta_2\Psi_3^{\mathcal{IL}}, \beta_2\Psi_3^{\mathcal{IU}}], [\beta_3\Psi_3^{\mathcal{FL}}, \beta_3\Psi_3^{\mathcal{FU}}]\Big\rangle$, then $\mathbb{D}_H(\Gamma_1, \Gamma_2$ satisfies the following properties are holds.

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1. $\mathbb{D}_H(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2$ is zero, whenever $\widetilde{\Gamma}_1 = \widetilde{\Gamma}_2$ and vice versa.
2. $\mathbb{D}_H(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2$ and $\mathbb{D}_H(\widetilde{\Gamma}_2, \widetilde{\Gamma}_1$ are co-occur.
3. $\mathbb{D}_H(\widetilde{\Gamma}_1, \widetilde{\Gamma_3}) \leq \mathbb{D}_H(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2 + \mathbb{D}_H(\widetilde{\Gamma}_2, \widetilde{\Gamma_3})$.

5 Various aggregation operators for SRDioNSNIVN

The new operators for SRDioNSNIVWA, SRDioNSNIVWG, GSRDioNSNIVWA, and GSRDioNSNIVWG are introduced in this section.

5.1 SRDioNSNIV weighted averaging(SRDioNSNIVWA) operator

Definition 5.1. Let $\widetilde{\Gamma}_i = \Big\langle (\lambda_i, \mu_i); [\beta_1 \Psi_i^{\mathcal{TL}}, \beta_1 \Psi_i^{\mathcal{TU}}], [\beta_2 \Psi_i^{\mathcal{IL}}, \beta_2 \Psi_i^{\mathcal{IU}}], [\beta_3 \Psi_i^{\mathcal{FL}}, \beta_3 \Psi_i^{\mathcal{FU}}] \Big\rangle$ be the family of SRDioNSNIVNs, $W = (\zeta_1, \zeta_2, ..., \zeta_n)$ be the weight of $\widetilde{\Gamma}_i$, $\zeta_i \geq 0$ and $\bigwedge_{i=1}^{n} \zeta_i = 1$. Then SRDioNSNIVWA $(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n) = \bigwedge_{i=1}^{n} \zeta_i \widetilde{\Gamma}_i \ (i = 1, 2, ..., n)$.

Theorem 5.2. *Let $\widetilde{\Gamma}_i = \Big\langle (\lambda_i, \mu_i); [\beta_1 \Psi_i^{\mathcal{TL}}, \beta_1 \Psi_i^{\mathcal{TU}}], [\beta_2 \Psi_i^{\mathcal{IL}}, \beta_2 \Psi_i^{\mathcal{IU}}], [\beta_3 \Psi_i^{\mathcal{FL}}, \beta_3 \Psi_i^{\mathcal{FU}}] \Big\rangle$ be the family of SRDioNSNIVNs. Then $SRDioNSNIVWA(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n)$=*

$$\begin{bmatrix} \Big(\bigwedge_{i=1}^{n} \zeta_i \lambda_i, \bigwedge_{i=1}^{n} \zeta_i \mu_i\Big); \\ \left[\left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi}, \left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi}, \left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi}\right], \\ \left[\bigvee_{i=1}^{n} (\beta_3 \Psi_i^{\mathcal{FL}})^{\zeta_i}, \bigvee_{i=1}^{n} (\beta_3 \Psi_i^{\mathcal{FU}})^{\zeta_i}\right] \end{bmatrix}.$$

Proof. The mathematical induction method served as the basis for the proof.
Put $n = 2$, SRDioNSNIVWA$(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2) = \zeta_1 \widetilde{\Gamma}_1 \oplus \zeta_2 \widetilde{\Gamma}_2$, where

$$\zeta_1 \widetilde{\Gamma}_1 = \begin{bmatrix} \Big(\zeta_1 \lambda_1, \zeta_1 \mu_1\Big); \\ \left[\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_1^{\mathcal{TL}})}\right)^{\zeta_1}\right)^{2\Phi}, \left(1 - \left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_1^{\mathcal{TU}})}\right)^{\zeta_1}\right)^{2\Phi}\right], \\ \left[\left(1 - \left(1 - \sqrt[\Phi]{(\beta_2 \Psi_1^{\mathcal{IL}})}\right)^{\zeta_1}\right)^{\Phi}, \left(1 - \left(1 - \sqrt[\Phi]{(\beta_2 \Psi_1^{\mathcal{IU}})}\right)^{\zeta_1}\right)^{\Phi}\right], \\ \left[(\beta_3 \Psi_1^{\mathcal{FL}})^{\zeta_1}, (\beta_3 \Psi_1^{\mathcal{FU}})^{\zeta_1}\right] \end{bmatrix}$$

$$\zeta_2 \widetilde{\Gamma}_2 = \begin{bmatrix} \Big(\zeta_2 \lambda_2, \zeta_2 \mu_2\Big); \\ \left[\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_2^{\mathcal{TL}})}\right)^{\zeta_2}\right)^{2\Phi}, \left(1 - \left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_2^{\mathcal{TU}})}\right)^{\zeta_2}\right)^{2\Phi}\right], \\ \left[\left(1 - \left(1 - \sqrt[\Phi]{(\beta_2 \Psi_2^{\mathcal{IL}})}\right)^{\zeta_2}\right)^{\Phi}, \left(1 - \left(1 - \sqrt[\Phi]{(\beta_2 \Psi_2^{\mathcal{IU}})}\right)^{\zeta_2}\right)^{\Phi}\right], \\ \left[(\beta_3 \Psi_2^{\mathcal{FL}})^{\zeta_2}, (\beta_3 \Psi_2^{\mathcal{FU}})^{\zeta_2}\right] \end{bmatrix}.$$

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Now,

$$\zeta_1\widetilde{\Gamma}_1 \oplus \zeta_2\widetilde{\Gamma}_2 = \begin{bmatrix} \left(\zeta_1\lambda_1+\zeta_2\lambda_2, \zeta_1\mu_1+\zeta_2\mu_2\right); \\ \left[\begin{array}{c} \left(\begin{array}{c} \left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TL}})}\right)^{\zeta_1}\right)+\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TL}})}\right)^{\zeta_2}\right) \\ -\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TL}})}\right)^{\zeta_1}\right)\cdot\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TL}})}\right)^{\zeta_2}\right) \end{array}\right)^{2\Phi}, \\ \left(\begin{array}{c} \left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TU}})}\right)^{\zeta_1}\right)+\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TU}})}\right)^{\zeta_2}\right) \\ -\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TU}})}\right)^{\zeta_1}\right)\cdot\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TU}})}\right)^{\zeta_2}\right) \end{array}\right)^{2\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(\begin{array}{c} \left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IL}})}\right)^{\zeta_1}\right)+\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IL}})}\right)^{\zeta_2}\right) \\ -\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IL}})}\right)^{\zeta_1}\right)\cdot\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IL}})}\right)^{\zeta_2}\right) \end{array}\right)^{\Phi}, \\ \left(\begin{array}{c} \left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IU}})}\right)^{\zeta_1}\right)+\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IU}})}\right)^{\zeta_2}\right) \\ -\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IU}})}\right)^{\zeta_1}\right)\cdot\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IU}})}\right)^{\zeta_2}\right) \end{array}\right)^{\Phi} \end{array}\right], \\ \left[(\beta_3\Psi_1^{\mathcal{FL}})^{\zeta_1}(\beta_3\Psi_2^{\mathcal{FL}})^{\zeta_2}, (\beta_3\Psi_1^{\mathcal{FU}})^{\zeta_1}(\beta_3\Psi_2^{\mathcal{FU}})^{\zeta_2}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \left(\zeta_1\lambda_1+\zeta_2\lambda_2, \zeta_1\mu_1+\zeta_2\mu_2\right); \\ \left[\begin{array}{c} \left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TL}})}\right)^{\zeta_1}\cdot\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TL}})}\right)^{\zeta_2}\right)^{2\Phi}, \\ \left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TU}})}\right)^{\zeta_1}\cdot\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TU}})}\right)^{\zeta_2}\right)^{2\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IL}})}\right)^{\zeta_1}\cdot\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IL}})}\right)^{\zeta_2}\right)^{\Phi}, \\ \left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IU}})}\right)^{\zeta_1}\cdot\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IU}})}\right)^{\zeta_2}\right)^{\Phi} \end{array}\right], \\ \left[(\beta_3\Psi_1^{\mathcal{FL}})^{\zeta_1}\cdot(\beta_3\Psi_2^{\mathcal{FL}})^{\zeta_2}, (\beta_3\Psi_1^{\mathcal{FU}})^{\zeta_1}\cdot(\beta_3\Psi_2^{\mathcal{FU}})^{\zeta_2}\right] \end{bmatrix}$$

$SRDioNSNIVWA(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2) =$

$$\begin{bmatrix} \left(\bigwedge_{i=1}^{2}\zeta_i\lambda_i, \bigwedge_{i=1}^{2}\zeta_i\mu_i\right); \\ \left[\left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi}\right], \\ \left[\bigvee_{i=1}^{2}(\beta_3\Psi_i^{\mathcal{FL}})^{\zeta_i}, \bigvee_{i=1}^{2}(\beta_3\Psi_i^{\mathcal{FU}})^{\zeta_i}\right] \end{bmatrix}.$$

ALso, valid for $n \geq 3$, hence $SRDioNSNIVWA(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_l) =$

$$\begin{bmatrix} \left(\bigwedge_{i=1}^{l}\zeta_i\lambda_i, \bigwedge_{i=1}^{l}\zeta_i\mu_i\right); \\ \left[\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi}\right], \\ \left[\bigvee_{i=1}^{l}(\beta_3\Psi_i^{\mathcal{FL}})^{\zeta_i}, \bigvee_{i=1}^{l}(\beta_3\Psi_i^{\mathcal{FU}})^{\zeta_i}\right] \end{bmatrix}.$$

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If $n = l + 1$, then SRDioNSNIVWA $(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_l, \widetilde{\Gamma}_{l+1}) =$

$$\begin{bmatrix} \Big(\bigwedge_{i=1}^{l} \zeta_i\lambda_i + \zeta_{l+1}\lambda_{l+1}, \bigwedge_{i=1}^{l} \zeta_i\mu_i + \zeta_{l+1}\mu_{l+1}\Big); \\ \left[\begin{array}{c} \left(\begin{array}{c} \bigwedge_{i=1}^{l}\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TL}})}\Big)^{\zeta_{l+1}}\Big) \\ -\bigvee_{i=1}^{l}\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TL}})}\Big)^{\zeta_{l+1}}\Big) \end{array}\right)^{2\Phi}, \\ \left(\begin{array}{c} \bigwedge_{i=1}^{l}\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TU}})}\Big)^{\zeta_{l+1}}\Big) \\ -\bigvee_{i=1}^{l}\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TU}})}\Big)^{\zeta_{l+1}}\Big) \end{array}\right)^{2\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(\begin{array}{c} \bigwedge_{i=1}^{l}\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IL}})}\Big)^{\zeta_{l+1}}\Big) \\ -\bigvee_{i=1}^{l}\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IL}})}\Big)^{\zeta_{l+1}}\Big) \end{array}\right)^{\Phi}, \\ \left(\begin{array}{c} \bigwedge_{i=1}^{l}\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\Big)^{\zeta_i}\Big)+\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IU}})}\Big)^{\zeta_{l+1}}\Big) \\ -\bigvee_{i=1}^{l}\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\Big)^{\zeta_i}\Big)\cdot\Big(1-\Big(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IU}})}\Big)^{\zeta_{l+1}}\Big) \end{array}\right)^{\Phi} \end{array}\right], \\ \Big[\bigvee_{i=1}^{l}(\beta_3\Psi_i^{\mathcal{FL}})^{\zeta_i}\cdot(\beta_3\Psi_{l+1}^{\mathcal{FL}})^{\zeta_{l+1}}, \bigvee_{i=1}^{l}(\beta_3\Psi_i^{\mathcal{FU}})^{\zeta_i}\cdot(\beta_3\Psi_{l+1}^{\mathcal{FU}})^{\zeta_{l+1}}\Big] \end{bmatrix}$$

$$= \begin{bmatrix} \Big(\bigwedge_{i=1}^{l+1}\zeta_i\lambda_i, \bigwedge_{i=1}^{l+1}\zeta_i\mu_i\Big); \\ \left[\left(1-\bigvee_{i=1}^{l+1}\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\Big)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{l+1}\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\Big)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{l+1}\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\Big)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{l+1}\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\Big)^{\zeta_i}\right)^{\Phi}\right], \\ \Big[\bigvee_{i=1}^{l+1}(\beta_3\Psi_i^{\mathcal{FL}})^{\zeta_i}, \bigvee_{i=1}^{l+1}(\beta_3\Psi_i^{\mathcal{FU}})^{\zeta_i}\Big] \end{bmatrix}.$$

Theorem 5.3. *If all* $\widetilde{\Gamma}_i = \Big\langle(\lambda_i, \mu_i); [\beta_1\Psi_i^{\mathcal{TL}}, \beta_1\Psi_i^{\mathcal{TU}}], [\beta_2\Psi_i^{\mathcal{IL}}, \beta_2\Psi_i^{\mathcal{IU}}][\beta_3\Psi_i^{\mathcal{FL}}, \beta_3\Psi_i^{\mathcal{FU}}]\Big\rangle$ *are equal, then SRDioNSNIVWA*$(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n) = \widetilde{\Gamma}$*(idempotency property).*

Proof. Given that $(\lambda_i, \mu_i) = (\lambda, \mu)$, $[\beta_1\Psi_i^{\mathcal{TL}}, \beta_1\Psi_i^{\mathcal{TU}}] = [\beta_1\Psi^{\mathcal{TL}}, \beta_1\Psi^{\mathcal{TU}}]$, $[\beta_2\Psi_i^{\mathcal{IL}}, \beta_2\Psi_i^{\mathcal{IU}}] = [\beta_2\Psi^{\mathcal{IL}}, \beta_2\Psi^{\mathcal{IU}}]$ and $[\beta_3\Psi_i^{\mathcal{FL}}, \beta_3\Psi_i^{\mathcal{FU}}] = [\beta_3\Psi^{\mathcal{FL}}, \beta_3\Psi^{\mathcal{FU}}]$ and $\bigwedge_{i=1}^{n}\zeta_i = 1$.

Now, $SRDioNSNIVWA(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n)$

$$= \begin{bmatrix} \Big(\bigwedge_{i=1}^{n}\zeta_i\lambda_i, \bigwedge_{i=1}^{n}\zeta_i\mu_i\Big); \\ \left[\left(1-\bigvee_{i=1}^{n}\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\Big)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{n}\Big(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\Big)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{n}\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\Big)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{n}\Big(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\Big)^{\zeta_i}\right)^{\Phi}\right], \\ \Big[\bigvee_{i=1}^{n}(\beta_3\Psi_i^{\mathcal{FL}})^{\zeta_i}, \bigvee_{i=1}^{n}(\beta_3\Psi_i^{\mathcal{FU}})^{\zeta_i}\Big] \end{bmatrix}$$

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$$= \begin{bmatrix} \left(\lambda \bigwedge_{i=1}^{n} \zeta_i, \mu \bigwedge_{i=1}^{n} \zeta_i\right); \\ \left[\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\right)^{\bigwedge_{i=1}^{n}\zeta_i}\right)^{2\Phi}, \left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\right)^{\bigwedge_{i=1}^{n}\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\right)^{\bigwedge_{i=1}^{n}\zeta_i}\right)^{\Phi}, \left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\right)^{\bigwedge_{i=1}^{n}\zeta_i}\right)^{\Phi}\right], \\ \left[(\beta_3\Psi_i^{\mathcal{FL}})^{\bigwedge_{i=1}^{n}\zeta_i}, (\beta_3\Psi_i^{\mathcal{FU}})^{\bigwedge_{i=1}^{n}\zeta_i}\right] \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda,\mu); \\ \left[\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})}\right)\right)^{2\Phi}, \left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})}\right)\right)^{2\Phi}\right], \\ \left[\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\right)\right)^{\Phi}, \left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\right)\right)^{\Phi}\right], \\ \left[(\beta_3\Psi_i^{\mathcal{FL}}), (\beta_3\Psi_i^{\mathcal{FU}})\right] \end{bmatrix}$$

$$= \widetilde{\Gamma}.$$

Theorem 5.4. *Let* $\widetilde{\Gamma}_i = \left\langle(\lambda_{ij},\mu_{ij}); [\beta_1\Psi_{ij}^{\mathcal{TL}}, \beta_1\Psi_{ij}^{\mathcal{TU}}], [\beta_2\Psi_{ij}^{\mathcal{IL}}, \beta_2\Psi_{ij}^{\mathcal{IU}}][\beta_3\Psi_{ij}^{\mathcal{FL}}, \beta_3\Psi_{ij}^{\mathcal{FU}}]\right\rangle (i = 1\ to\ n); (j = 1\ to\ i_j)$ *be the SRDioNSNIVWA, where*
$\widehat{\lambda} = \inf \lambda_{ij}$, $\underbrace{\lambda} = \sup \lambda_{ij}$, $\widehat{\mu} = \sup \mu_{ij}$, $\underbrace{\mu} = \inf \mu_{ij}$,
$\widehat{\beta_1\Psi^{\mathcal{TL}}} = \inf \beta_1\Psi_{ij}^{\mathcal{TL}}$, $\underbrace{\beta_1\Psi^{\mathcal{TL}}} = \sup \beta_1\Psi_{ij}^{\mathcal{TL}}$, $\widehat{\beta_1\Psi^{\mathcal{TU}}} = \inf \beta_1\Psi_{ij}^{\mathcal{TU}}$, $\underbrace{\beta_1\Psi^{\mathcal{TU}}} = \sup \beta_1\Psi_{ij}^{\mathcal{TU}}$,
$\widehat{\beta_2\Psi^{\mathcal{IL}}} = \inf \beta_2\Psi_{ij}^{\mathcal{IL}}$, $\underbrace{\beta_2\Psi^{\mathcal{IL}}} = \sup \beta_2\Psi_{ij}^{\mathcal{IL}}$, $\widehat{\beta_2\Psi^{\mathcal{IU}}} = \inf \beta_2\Psi_{ij}^{\mathcal{IU}}$, $\underbrace{\beta_2\Psi^{\mathcal{IU}}} = \sup \beta_2\Psi_{ij}^{\mathcal{IU}}$,
$\widehat{\beta_3\Psi^{\mathcal{FL}}} = \inf \beta_3\Psi_{ij}^{\mathcal{FL}}$, $\underbrace{\beta_3\Psi^{\mathcal{FL}}} = \sup \beta_3\Psi_{ij}^{\mathcal{FL}}$, $\widehat{\beta_3\Psi^{\mathcal{FU}}} = \inf \beta_3\Psi_{ij}^{\mathcal{FU}}$, $\underbrace{\beta_3\Psi^{\mathcal{FU}}} = \sup \beta_3\Psi_{ij}^{\mathcal{FU}}$.
Then, $\left\langle(\widehat{\lambda},\widehat{\mu}); [\widehat{\beta_1\Psi^{\mathcal{TL}}}, \widehat{\beta_1\Psi^{\mathcal{TU}}}], [\widehat{\beta_2\Psi^{\mathcal{IL}}}, \widehat{\beta_2\Psi^{\mathcal{IU}}}], [\underbrace{\beta_3\Psi^{\mathcal{FL}}}, \underbrace{\beta_3\Psi^{\mathcal{FU}}}]\right\rangle$
$\leq SRDioNSNIVWA(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n)$
$\leq \left\langle(\underbrace{\lambda}, \underbrace{\mu}); [\underbrace{\beta_1\Psi^{\mathcal{TL}}}, \underbrace{\beta_1\Psi^{\mathcal{TU}}}], [\underbrace{\beta_2\Psi^{\mathcal{IL}}}, \underbrace{\beta_2\Psi^{\mathcal{IU}}}], [\widehat{\beta_3\Psi^{\mathcal{FL}}}, \widehat{\beta_3\Psi^{\mathcal{FU}}}]\right\rangle$, *where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$ *(boundedness property).*

Proof. Since, $\widehat{\beta_1\Psi^{\mathcal{TL}}} = \inf \beta_1\Psi_{ij}^{\mathcal{TL}}$, $\underbrace{\beta_1\Psi^{\mathcal{TL}}} = \sup \beta_1\Psi_{ij}^{\mathcal{TL}}$
$\widehat{\beta_1\Psi^{\mathcal{TU}}} = \inf \beta_1\Psi_{ij}^{\mathcal{TU}}$, $\underbrace{\beta_1\Psi^{\mathcal{TU}}} = \sup \beta_1\Psi_{ij}^{\mathcal{TU}}$ and $\widehat{\beta_1\Psi^{\mathcal{TL}}} \leq \beta_1\Psi_{ij}^{\mathcal{TL}} \leq \underbrace{\beta_1\Psi^{\mathcal{TL}}}$ and $\widehat{\beta_1\Psi^{\mathcal{TU}}} \leq \beta_1\Psi_{ij}^{\mathcal{TU}} \leq \underbrace{\beta_1\Psi^{\mathcal{TU}}}$. Now,

$$\begin{aligned} \widehat{\beta_1\Psi^{\mathcal{TL}}} + \widehat{\beta_1\Psi^{\mathcal{TU}}} &= \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\widehat{\beta_1\Psi^{\mathcal{TL}}})}\right)^{\zeta_i}\right)^{2\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\widehat{\beta_1\Psi^{\mathcal{TU}}})}\right)^{\zeta_i}\right)^{2\Phi} \\ &\leq \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{ij}^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{ij}^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi} \\ &\leq \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\underbrace{(\beta_1\Psi^{\mathcal{TL}})}}\right)^{\zeta_i}\right)^{2\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\underbrace{(\beta_1\Psi^{\mathcal{TU}})}}\right)^{\zeta_i}\right)^{2\Phi} \\ &= \underbrace{\beta_1\Psi^{\mathcal{TL}}} + \underbrace{\beta_1\Psi^{\mathcal{TU}}}. \end{aligned}$$

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Since, $\widehat{\beta_2\Psi^{\mathcal{IL}}} = \inf \beta_2\Psi_{ij}^{\mathcal{IL}}$, $\underbrace{\beta_2\Psi^{\mathcal{IL}}} = \sup \beta_2\Psi_{ij}^{\mathcal{IL}}$
$\widehat{\beta_2\Psi^{\mathcal{IU}}} = \inf \beta_2\Psi_{ij}^{\mathcal{IU}}$, $\underbrace{\beta_2\Psi^{\mathcal{IU}}} = \sup \beta_2\Psi_{ij}^{\mathcal{IU}}$ and $\widehat{\beta_2\Psi^{\mathcal{IL}}} \leq \beta_2\Psi_{ij}^{\mathcal{IL}} \leq \underbrace{\beta_2\Psi^{\mathcal{IL}}}$ and $\widehat{\beta_2\Psi^{\mathcal{IU}}} \leq \beta_2\Psi_{ij}^{\mathcal{IU}} \leq \underbrace{\beta_2\Psi^{\mathcal{IU}}}$. Now,

$$\begin{aligned}
\widehat{\beta_2\Psi^{\mathcal{IL}}} + \widehat{\beta_2\Psi^{\mathcal{IU}}} &= \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\widehat{\beta_2\Psi^{\mathcal{IL}}})}\right)^{\zeta_i}\right)^{\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\widehat{\beta_2\Psi^{\mathcal{IU}}})}\right)^{\zeta_i}\right)^{\Phi} \\
&\leq \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{ij}^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{ij}^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi} \\
&\leq \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\underbrace{\beta_2\Psi^{\mathcal{IL}}})}\right)^{\zeta_i}\right)^{\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\underbrace{\beta_2\Psi^{\mathcal{IU}}})}\right)^{\zeta_i}\right)^{\Phi} \\
&= \underbrace{\beta_2\Psi^{\mathcal{IL}}} + \underbrace{\beta_2\Psi^{\mathcal{IU}}}.
\end{aligned}$$

Since, $\widehat{\beta_3\Psi^{\mathcal{FL}}} = \inf \beta_3\Psi_{ij}^{\mathcal{FL}}$, $\underbrace{\beta_3\Psi^{\mathcal{FL}}} = \sup \beta_3\Psi_{ij}^{\mathcal{FL}}$
$\widehat{\beta_3\Psi^{\mathcal{FU}}} = \inf \beta_3\Psi_{ij}^{\mathcal{FU}}$, $\underbrace{\beta_3\Psi^{\mathcal{FU}}} = \sup \beta_3\Psi_{ij}^{\mathcal{FU}}$ and $\widehat{\beta_3\Psi^{\mathcal{FL}}} \leq \beta_3\Psi_{ij}^{\mathcal{FL}} \leq \underbrace{\beta_3\Psi^{\mathcal{FL}}}$ and $\widehat{\beta_3\Psi^{\mathcal{FU}}} \leq \beta_3\Psi_{ij}^{\mathcal{FU}} \leq \underbrace{\beta_3\Psi^{\mathcal{FU}}}$. Now,

$$\begin{aligned}
\widehat{\beta_3\Psi^{\mathcal{FL}}} + \widehat{\beta_3\Psi^{\mathcal{FU}}} &= \bigvee_{i=1}^{n}(\widehat{\beta_3\Psi^{\mathcal{FL}}})^{\zeta_i} + \bigvee_{i=1}^{n}(\widehat{\beta_3\Psi^{\mathcal{FU}}})^{\zeta_i} \\
&\leq \bigvee_{i=1}^{n}(\beta_3\Psi_{ij}^{\mathcal{FL}})^{\zeta_i} + \bigvee_{i=1}^{n}(\beta_3\Psi_{ij}^{\mathcal{FU}})^{\zeta_i} \\
&\leq \bigvee_{i=1}^{n}(\underbrace{\beta_3\Psi^{\mathcal{FL}}})^{\zeta_i} + \bigvee_{i=1}^{n}(\underbrace{\beta_3\Psi^{\mathcal{FU}}})^{\zeta_i} \\
&= \underbrace{\beta_3\Psi^{\mathcal{FL}}} + \underbrace{\beta_3\Psi^{\mathcal{FU}}}.
\end{aligned}$$

Since, $\widehat{\lambda} = \inf \lambda_{ij}$, $\underbrace{\lambda} = \sup \lambda_{ij}$, $\widehat{\mu} = \sup \mu_{ij}$, $\underbrace{\mu} = \inf \mu_{ij}$ and $\widehat{\lambda} \leq \lambda_{ij} \leq \underbrace{\lambda}$ and $\underbrace{\mu} \leq \mu_{ij} \leq \widehat{\mu}$.
Thus, $\bigwedge_{i=1}^{n}\zeta_i\widehat{\lambda} \leq \bigwedge_{i=1}^{n}\zeta_i\lambda_{ij} \leq \bigwedge_{i=1}^{n}\zeta_i\underbrace{\lambda}$ and $\bigwedge_{i=1}^{n}\zeta_i\underbrace{\mu} \leq \bigwedge_{i=1}^{n}\zeta_i\mu_{ij} \leq \bigwedge_{i=1}^{n}\zeta_i\widehat{\mu}$.
Hence,

$$\begin{aligned}
&\frac{\bigwedge_{i=1}^{n}\zeta_i\widehat{\lambda}}{2} \times \left[\begin{array}{c}
\dfrac{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\widehat{\beta_1\Psi^{\mathcal{TL}}})}\right)^{\zeta_i}\right)^{2\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\widehat{\beta_1\Psi^{\mathcal{TU}}})}\right)^{\zeta_i}\right)^{2\Phi}}{2} \\
-\dfrac{\sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\widehat{\beta_2\Psi^{\mathcal{IL}}})}\right)^{\zeta_i}\right)^{\Phi}} + \sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\widehat{\beta_2\Psi^{\mathcal{IU}}})}\right)^{\zeta_i}\right)^{\Phi}}}{2} \\
+1-\dfrac{\sqrt{\left(\bigvee_{i=1}^{n}(\underbrace{\beta_3\Psi^{\mathcal{FL}}})^{\zeta_i}\right)} + \sqrt{\left(\bigvee_{i=1}^{n}(\underbrace{\beta_3\Psi^{\mathcal{FU}}})^{\zeta_i}\right)}}{2}
\end{array}\right] \\
\leq\ &\frac{\bigwedge_{i=1}^{n}\zeta_i\lambda_{ij}}{2} \times \left[\begin{array}{c}
\dfrac{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{ij}^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi} + \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{ij}^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi}}{2} \\
-\dfrac{\sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{ij}^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi}} + \sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{ij}^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi}}}{2} \\
+1-\dfrac{\sqrt{\left(\bigvee_{i=1}^{n}(\beta_3\Psi_{ij}^{\mathcal{FL}})^{\zeta_i}\right)} + \sqrt{\left(\bigvee_{i=1}^{n}(\beta_3\Psi_{ij}^{\mathcal{FU}})^{\zeta_i}\right)}}{2}
\end{array}\right]
\end{aligned}$$

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$$\leq \frac{\bigwedge_{i=1}^{n} \zeta_i \underbrace{\lambda}}{2} \times \left[\begin{array}{c} \frac{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\underbrace{(\beta_1\Psi_{ij}^{\mathcal{TL}})}}\right)^{\zeta_i}\right)^{2\Phi}+\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\underbrace{(\beta_1\Psi_{ij}^{\mathcal{TU}})}}\right)^{\zeta_i}\right)^{2\Phi}}{2} \\ -\frac{\sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{\underbrace{(\beta_2\Psi_{ij}^{\mathcal{IL}})}}\right)^{\zeta_i}\right)^{\Phi}}+\sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{\underbrace{(\beta_2\Psi_{ij}^{\mathcal{IU}})}}\right)^{\zeta_i}\right)^{\Phi}}}{2} \\ +1-\frac{\sqrt{\left(\bigvee_{i=1}^{n}(\beta_3\Psi_{ij}^{\mathcal{FL}})^{\zeta_i}\right)}+\sqrt{\left(\bigvee_{i=1}^{n}(\beta_3\Psi_{ij}^{\mathcal{FU}})^{\zeta_i}\right)}}{2} \end{array} \right].$$

Therefore, $\left\langle(\widehat{\lambda},\widehat{\mu});[\widehat{\beta_1\Psi^{\mathcal{TL}}},\widehat{\beta_1\Psi^{\mathcal{TU}}}],[\widehat{\beta_2\Psi^{\mathcal{IL}}},\widehat{\beta_2\Psi^{\mathcal{IU}}}],[\underbrace{\beta_3\Psi^{\mathcal{FL}}},\underbrace{\beta_3\Psi^{\mathcal{FU}}}]\right\rangle$
$\leq SRDioNSNIVWA(\widetilde{\Gamma}_1,\widetilde{\Gamma}_2,...,\widetilde{\Gamma}_n)$
$\leq \left\langle(\underbrace{\lambda},\underbrace{\mu});[\underbrace{\beta_1\Psi^{\mathcal{TL}}},\underbrace{\beta_1\Psi^{\mathcal{TU}}}],[\underbrace{\beta_2\Psi^{\mathcal{IL}}},\underbrace{\beta_2\Psi^{\mathcal{IU}}}],[\widehat{\beta_3\Psi^{\mathcal{FL}}},\widehat{\beta_3\Psi^{\mathcal{FU}}}]\right\rangle.$

Theorem 5.5. *Let* $\widetilde{\Gamma}_i = \left\langle(\lambda_{t_{ij}},\mu_{t_{ij}});[\beta_1\Psi_{t_{ij}}^{\mathcal{TL}},\beta_1\Psi_{t_{ij}}^{\mathcal{TU}}],[\beta_2\Psi_{t_{ij}}^{\mathcal{IL}},\beta_2\Psi_{t_{ij}}^{\mathcal{IU}}],[\beta_3\Psi_{t_{ij}}^{\mathcal{FL}},\beta_3\Psi_{t_{ij}}^{\mathcal{FU}}]\right\rangle$ *and* $\widetilde{W}_i = \left\langle(\lambda_{h_{ij}},\mu_{h_{ij}});[\beta_1\Psi_{h_{ij}}^{\mathcal{TL}},\beta_1\Psi_{h_{ij}}^{\mathcal{TU}}],[\beta_2\Psi_{h_{ij}}^{\mathcal{IL}},\beta_2\Psi_{h_{ij}}^{\mathcal{IU}}],[\beta_3\Psi_{h_{ij}}^{\mathcal{FL}},\beta_3\Psi_{h_{ij}}^{\mathcal{FU}}]\right\rangle$ *be the two families of SRDioNSNIVWAs. For any i, if there is* $\lambda_{t_{ij}} \leq \mu_{h_{ij}}$, $\sqrt{\left(\beta_1\Psi_{t_{ij}}^{\mathcal{TL}}\right)}+\sqrt{\left(\beta_1\Psi_{t_{ij}}^{\mathcal{TU}}\right)} \leq \sqrt{\left(\beta_1\Psi_{h_{ij}}^{\mathcal{TL}}\right)}+\sqrt{\left(\beta_1\Psi_{h_{ij}}^{\mathcal{TU}}\right)}$ *and* $\sqrt{\left(\beta_2\Psi_{t_{ij}}^{\mathcal{IL}}\right)}+\sqrt{\left(\beta_2\Psi_{t_{ij}}^{\mathcal{IU}}\right)} \leq \sqrt{\left(\beta_2\Psi_{h_{ij}}^{\mathcal{IL}}\right)}+\sqrt{\left(\beta_2\Psi_{h_{ij}}^{\mathcal{IU}}\right)}$ *and* $\left(\beta_3\Psi_{t_{ij}}^{\mathcal{FL}}\right)+\left(\beta_3\Psi_{t_{ij}}^{\mathcal{FU}}\right) \geq \left(\beta_3\Psi_{h_{ij}}^{\mathcal{FL}}\right)+\left(\beta_3\Psi_{h_{ij}}^{\mathcal{FU}}\right)$ *or* $\widetilde{\Gamma}_i \leq \widetilde{W}_i$*, then* $SRDioNSNIVWA\left(\widetilde{\Gamma}_1,\widetilde{\Gamma}_2,...,\widetilde{\Gamma}_n\right) \leq SRDioNSNIVWA\left(\widetilde{W}_1,\widetilde{W}_2,...,\widetilde{W}_n\right)$*, where* $(i = 1\ to\ n);(j = 1\ to\ i_j)$ *(monotonicity property).*

Proof. For any i, $\lambda_{t_{ij}} \leq \mu_{h_{ij}}$. Thus, $\bigwedge_{i=1}^{n}\lambda_{t_{ij}} \leq \bigwedge_{i=1}^{n}\mu_{h_{ij}}$.
For any i, $\sqrt{\left(\beta_1\Psi_{t_{ij}}^{\mathcal{TL}}\right)}+\sqrt{\left(\beta_1\Psi_{t_{ij}}^{\mathcal{TU}}\right)} \leq \sqrt{\left(\beta_1\Psi_{h_{ij}}^{\mathcal{TL}}\right)}+\sqrt{\left(\beta_1\Psi_{h_{ij}}^{\mathcal{TU}}\right)}$.
Therefore, $1-\sqrt[2\Phi]{\left(\beta_1\Psi_{t_i}^{\mathcal{TL}}\right)}+1-\sqrt[2\Phi]{\left(\beta_1\Psi_{t_i}^{\mathcal{TU}}\right)} \geq 1-\sqrt[2\Phi]{\left(\beta_1\Psi_{h_i}^{\mathcal{TL}}\right)}+1-\sqrt[2\Phi]{\left(\beta_1\Psi_{h_i}^{\mathcal{TU}}\right)}$.
Hence,

$$\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\left(\beta_1\Psi_{t_i}^{\mathcal{TL}}\right)}\right)^{\zeta_i}+\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\left(\beta_1\Psi_{t_i}^{\mathcal{TU}}\right)}\right)^{\zeta_i} \geq \bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\left(\beta_1\Psi_{h_i}^{\mathcal{TL}}\right)}\right)^{\zeta_i}+\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{\left(\beta_1\Psi_{h_i}^{\mathcal{TU}}\right)}\right)^{\zeta_i}$$

and

$$\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{t_i}^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi}+\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{t_i}^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi} \leq \left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{h_i}^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi}+\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{h_i}^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi}.$$

For any i, $\sqrt{\left(\beta_2\Psi_{t_{ij}}^{\mathcal{IL}}\right)}+\sqrt{\left(\beta_2\Psi_{t_{ij}}^{\mathcal{IU}}\right)} \leq \sqrt{\left(\beta_2\Psi_{h_{ij}}^{\mathcal{IL}}\right)}+\sqrt{\left(\beta_2\Psi_{h_{ij}}^{\mathcal{IU}}\right)}$.
Therefore, $1-\sqrt[\Phi]{\left(\beta_2\Psi_{t_i}^{\mathcal{IL}}\right)}+1-\sqrt[\Phi]{\left(\beta_2\Psi_{t_i}^{\mathcal{IU}}\right)} \geq 1-\sqrt[\Phi]{\left(\beta_2\Psi_{h_i}^{\mathcal{IL}}\right)}+1-\sqrt[\Phi]{\left(\beta_2\Psi_{h_i}^{\mathcal{IU}}\right)}$.
Hence,

$$\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{\left(\beta_2\Psi_{t_i}^{\mathcal{IL}}\right)}\right)^{\zeta_i}+\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{\left(\beta_2\Psi_{t_i}^{\mathcal{IU}}\right)}\right)^{\zeta_i} \geq \bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{\left(\beta_2\Psi_{h_i}^{\mathcal{IL}}\right)}\right)^{\zeta_i}+\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{\left(\beta_2\Psi_{h_i}^{\mathcal{IU}}\right)2}\right)^{\zeta_i}$$

and

$$\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{t_i}^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi}+\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{t_i}^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi} \leq$$

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$$\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{h_i}^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi}+\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{h_i}^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi}.$$

For any i, $\left(\beta_3\Psi_{t_{ij}}^{\mathcal{FL}}\right)+\left(\beta_3\Psi_{t_{ij}}^{\mathcal{FU}}\right)\geq\left(\beta_3\Psi_{h_{ij}}^{\mathcal{FL}}\right)+\left(\beta_3\Psi_{h_{ij}}^{\mathcal{FU}}\right)$.

Therefore, $1-\frac{\left(\bigvee_{i=1}^{n}\beta_3\Psi_{t_{ij}}^{\mathcal{FL}}\right)+\left(\bigvee_{i=1}^{n}\beta_3\Psi_{t_{ij}}^{\mathcal{FU}}\right)}{2}\leq 1-\frac{\left(\bigvee_{i=1}^{n}\beta_3\Psi_{h_{ij}}^{\mathcal{FL}}\right)+\left(\bigvee_{i=1}^{n}\beta_3\Psi_{h_{ij}}^{\mathcal{FU}}\right)}{2}$.
Hence,

$$\leq\frac{\bigwedge_{i=1}^{n}\lambda_{hij}}{2}\times\left[\begin{array}{c}\frac{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{hi}^{\mathcal{TL}})}\right)^{\zeta_i}\right)^{2\Phi}+\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{hi}^{\mathcal{TU}})}\right)^{\zeta_i}\right)^{2\Phi}}{2}\\-\frac{\sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{hi}^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{2\Phi}}+\sqrt{\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_{hi}^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{2\Phi}}}{2}\\+1-\frac{\sqrt{\left(\bigvee_{i=1}^{n}(\beta_3\Psi_{hij}^{\mathcal{FL}})\right)}+\sqrt{\left(\bigvee_{i=1}^{n}(\beta_3\Psi_{hij}^{\mathcal{FU}})\right)}}{2}\end{array}\right].$$

Hence, $SRDioNSNIVWA\left(\widetilde{\Gamma}_1,\widetilde{\Gamma}_2,...,\widetilde{\Gamma}_n\right)\leq SRDioNSNIVWA\left(\widetilde{W}_1,\widetilde{W}_2,...,\widetilde{W}_n\right)$.

5.2 SRDioNSNIV weighted geometric(SRDioNSNIVWG) operator

Definition 5.6. Let $\widetilde{\Gamma}_i=\left\langle(\lambda_i,\mu_i);[\beta_1\Psi_i^{\mathcal{TL}},\beta_1\Psi_i^{\mathcal{TU}}],[\beta_2\Psi_i^{\mathcal{IL}},\beta_2\Psi_i^{\mathcal{IU}}],[\beta_3\Psi_i^{\mathcal{FL}},\beta_3\Psi_i^{\mathcal{FU}}]\right\rangle$ be the family of SRDioNSNIVNs. Then SRDioNSNIVWG $(\widetilde{\Gamma}_1,\widetilde{\Gamma}_2,...,\widetilde{\Gamma}_n)=\bigvee_{i=1}^{n}\widetilde{\Gamma}_i^{\zeta_i}$ $(i=1\ to\ n)$.

Theorem 5.7. *Let $\widetilde{\Gamma}_i=\left\langle(\lambda_i,\mu_i);[\beta_1\Psi_i^{\mathcal{TL}},\beta_1\Psi_i^{\mathcal{TU}}],[\beta_3\Psi_i^{\mathcal{FL}},\beta_3\Psi_i^{\mathcal{FU}}]\right\rangle$ be the family of SRDioNSNIVNs. Prove that*

$$\left[\begin{array}{c}\left(\bigvee_{i=1}^{n}\lambda_i^{\zeta_i},\bigvee_{i=1}^{n}\mu_i^{\zeta_i}\right);\left[\bigvee_{i=1}^{n}(\beta_1\Psi_i^{\mathcal{TL}})^{\zeta_i},\bigvee_{i=1}^{n}(\beta_1\Psi_i^{\mathcal{TU}})^{\zeta_i}\right],\\\left[\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})}\right)^{\zeta_i}\right)^{\Phi},\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})}\right)^{\zeta_i}\right)^{\Phi}\right],\\\left[\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FL}})}\right)^{\zeta_i}\right)^{2\Phi},\left(1-\bigvee_{i=1}^{n}\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FU}})}\right)^{\zeta_i}\right)^{2\Phi}\right]\end{array}\right].$$

Theorem 5.8. *If all $\widetilde{\Gamma}_i=\left\langle(\lambda_i,\mu_i);[\beta_1\Psi_i^{\mathcal{TL}},\beta_1\Psi_i^{\mathcal{TU}}],[\beta_2\Psi_i^{\mathcal{IL}},\beta_2\Psi_i^{\mathcal{IU}}][\beta_3\Psi_i^{\mathcal{FL}},\beta_3\Psi_i^{\mathcal{FU}}]\right\rangle(i=1,2,...,n)$ are equal, then SRDioNSNIVWG$(\widetilde{\Gamma}_1,\widetilde{\Gamma}_2,...,\widetilde{\Gamma}_n)=\widetilde{\Gamma}$.*

Using the SRDioNSNIVWG operator, the boundedness and monotonicity properties are met.

5.3 Generalized SRDioNSNIVWA (GSRDioNSNIVWA) operator

Definition 5.9. Let $\widetilde{\Gamma}_i=\left\langle(\lambda_i,\mu_i);[\beta_1\Psi_i^{\mathcal{TL}},\beta_1\Psi_i^{\mathcal{TU}}],[\beta_2\Psi_i^{\mathcal{IL}},\beta_2\Psi_i^{\mathcal{IU}}],[\beta_3\Psi_i^{\mathcal{FL}},\beta_3\Psi_i^{\mathcal{FU}}]\right\rangle$ be the family of SRDioNSNIVN. Then GSRDioNSNIVWA $(\widetilde{\Gamma}_1,\widetilde{\Gamma}_2,...,\widetilde{\Gamma}_n)=\left(\bigwedge_{i=1}^{n}\zeta_i\widetilde{\Gamma}_i^{\Phi}\right)^{1/\Phi}$.

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Theorem 5.10. *Let* $\widetilde{\Gamma}_i = \left\langle (\lambda_i, \mu_i); [\beta_1 \Psi_i^{\mathcal{TL}}, \beta_1 \Psi_i^{\mathcal{TU}}], [\beta_2 \Psi_i^{\mathcal{IL}}, \beta_2 \Psi_i^{\mathcal{IU}}], [\beta_3 \Psi_i^{\mathcal{FL}}, \beta_3 \Psi_i^{\mathcal{FU}}] \right\rangle$ *be the family of SRDioNSNIVNs. Then GSRDioNSNIVWA* $(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n) =$

$$\left[\begin{array}{c}
\left(\left(\bigwedge_{i=1}^{n} \zeta_i \lambda_i^{\Phi}\right)^{1/\Phi}, \left(\bigwedge_{i=1}^{n} \zeta_i \mu_i^{\Phi}\right)^{1/\Phi}\right); \\
\left[\left(\left(1 - \bigvee_{i=1}^{n}\left(1 - \left(\sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TL}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{2\Phi}\right)^{\Phi}, \left(\left(1 - \bigvee_{i=1}^{n}\left(1 - \left(\sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TU}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{2\Phi}\right)^{\Phi}\right], \\
\left[\left(\left(1 - \bigvee_{i=1}^{n}\left(1 - \left(\sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IL}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{\Phi}\right)^{\Phi}, \left(\left(1 - \bigvee_{i=1}^{n}\left(1 - \left(\sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IU}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{\Phi}\right)^{\Phi}\right], \\
\left[\begin{array}{c}
\left(1 - \left(1 - \sqrt[2\Phi]{\bigvee_{i=1}^{n}\left(\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_3 \Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}}\right)^{\Phi}\right)^{2\Phi}, \\
\left(1 - \left(1 - \sqrt[2\Phi]{\bigvee_{i=1}^{n}\left(\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_3 \Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}}\right)^{\Phi}\right)^{2\Phi}
\end{array}\right]
\end{array}\right].$$

Proof. It must be demonstrated that,

$$\bigwedge_{i=1}^{n} \zeta_i \widetilde{\Gamma}_i^{\Phi} = \left[\begin{array}{c}
\left(\left(\bigwedge_{i=1}^{n} \zeta_i \lambda_i^{\Phi}\right), \left(\bigwedge_{i=1}^{n} \zeta_i \mu_i^{\Phi}\right)\right); \\
\left[\left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}, \left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}\right], \\
\left[\left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}, \left(1 - \bigvee_{i=1}^{n}\left(1 - \sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}\right], \\
\left[\bigvee_{i=1}^{n}\left(\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_3 \Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}, \bigvee_{i=1}^{n}\left(\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_3 \Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}\right]
\end{array}\right]$$

. Put

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$n = 2, \zeta_1\Gamma_1 \oplus \zeta_2\Gamma_2 =$

$$\left[\begin{array}{c} \left(\zeta_1\lambda_1^{\Phi} + \zeta_2\lambda_2^{\Phi}, \zeta_1\mu_1^{\Phi} + \zeta_2\mu_2^{\Phi}\right); \\ \left[\begin{array}{c} \left(\begin{array}{c} \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} + \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_2}\right)^{2\Phi}} \\ -\sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} \cdot \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_2}\right)^{2\Phi}} \end{array}\right)^{2\Phi}, \\ \left(\begin{array}{c} \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} + \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_2}\right)^{2\Phi}} \\ -\sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_1^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} \cdot \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_2^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_2}\right)^{2\Phi}} \end{array}\right)^{2\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(\begin{array}{c} \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} + \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_2}\right)^{\Phi}} \\ -\sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} \cdot \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_2}\right)^{\Phi}} \end{array}\right)^{\Phi}, \\ \left(\begin{array}{c} \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} + \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_2}\right)^{\Phi}} \\ -\sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_1^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} \cdot \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_2^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_2}\right)^{\Phi}} \end{array}\right)^{\Phi} \end{array}\right], \\ \left[\begin{array}{c} \left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_1^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_1} \cdot \left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_2^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_1}, \\ \left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_1^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_1} \cdot \left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_2^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_1} \end{array}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\left(\bigwedge_{i=1}^{2}\zeta_i\lambda_i^{\Phi}\right), \left(\bigwedge_{i=1}^{2}\zeta_i\mu_i^{\Phi}\right)\right); \\ \left[\left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{2}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}\right], \\ \left[\bigvee_{i=1}^{2}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}, \bigvee_{i=1}^{2}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}\right] \end{array}\right].$$

In general,

$$\left[\begin{array}{c} \left(\left(\bigwedge_{i=1}^{l}\zeta_i\lambda_i^{\Phi}\right), \left(\bigwedge_{i=1}^{l}\zeta_i\mu_i^{\Phi}\right)\right); \\ \left[\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}\right], \\ \left[\bigvee_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}, \bigvee_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}\right] \end{array}\right].$$

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If $n = l+1$, then $\bigwedge_{i=1}^{l} \zeta_i \widetilde{\Gamma}_i^{\Phi} + \zeta_{l+1}\widetilde{\Gamma}_{l+1}^{\Phi} = \bigwedge_{i=1}^{l+1} \zeta_i \widetilde{\Gamma}_i^{\Phi}$.

Now, $\bigwedge_{i=1}^{l} \zeta_i \widetilde{\Gamma}_i^{\Phi} + \zeta_{l+1}\widetilde{\Gamma}_{l+1}^{\Phi} = \bigwedge_{i=1}^{l+1} \zeta_i \widetilde{\Gamma}_i^{\Phi} = \zeta_1\widetilde{\Gamma}_1^{\Phi} \oplus \zeta_2\widetilde{\Gamma}_2^{\Phi} \oplus ... \oplus \zeta_l\widetilde{\Gamma}_l^{\Phi} \oplus \zeta_{l+1}\widetilde{\Gamma}_{l+1}^{\Phi}$

$$= \begin{bmatrix} \left(\zeta_i\lambda_i^{\Phi} + \zeta_{l+1}\lambda_{l+1}^{\Phi}, \zeta_i\mu_i^{\Phi} + \zeta_{l+1}\mu_{l+1}^{\Phi}\right); \\ \begin{bmatrix} \begin{pmatrix} \sqrt[2\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}} + \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} \\ - \sqrt[2\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}} \cdot \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} \end{pmatrix}^{2\Phi}, \\ \begin{pmatrix} \sqrt[2\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}} + \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} \\ - \sqrt[2\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}} \cdot \sqrt[2\Phi]{\left(1-\left(1-\sqrt[2\Phi]{(\beta_1\Psi_{l+1}^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_1}\right)^{2\Phi}} \end{pmatrix}^{2\Phi} \end{bmatrix}, \\ \begin{bmatrix} \begin{pmatrix} \sqrt[\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}} + \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} \\ - \sqrt[\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}} \cdot \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} \end{pmatrix}^{\Phi}, \\ \begin{pmatrix} \sqrt[\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}} + \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} \\ - \sqrt[\Phi]{\left(1-\bigvee_{i=1}^{l}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}} \cdot \sqrt[\Phi]{\left(1-\left(1-\sqrt[\Phi]{(\beta_2\Psi_{l+1}^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_1}\right)^{\Phi}} \end{pmatrix}^{\Phi} \end{bmatrix}, \\ \begin{bmatrix} \bigvee_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i} \cdot \left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_{l+1}^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_1}, \\ \bigvee_{i=1}^{l}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i} \cdot \left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_{l+1}^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_1} \end{bmatrix} \end{bmatrix}$$

Hence,

$$\bigwedge_{i=1}^{l+1} \zeta_i\widetilde{\Gamma}_i^{\Phi} = \begin{bmatrix} \left(\left(\bigwedge_{i=1}^{l+1}\zeta_i\lambda_i^{\Phi}\right), \left(\bigwedge_{i=1}^{l+1}\zeta_i\mu_i^{\Phi}\right)\right); \\ \left[\left(1-\bigvee_{i=1}^{l+1}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}, \left(1-\bigvee_{i=1}^{l+1}\left(1-\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})^{\Phi}}\right)^{\zeta_i}\right)^{2\Phi}\right], \\ \left[\left(1-\bigvee_{i=1}^{l+1}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}, \left(1-\bigvee_{i=1}^{l+1}\left(1-\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})^{\Phi}}\right)^{\zeta_i}\right)^{\Phi}\right], \\ \left[\bigvee_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}, \bigvee_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}\right] \end{bmatrix}.$$

Also, $\left(\bigvee_{i=1}^{l+1}\zeta_i\widetilde{\Gamma}_i^{\Phi}\right)^{1/\Phi} =$

$$\begin{bmatrix} \left(\left(\bigvee_{i=1}^{l+1}\zeta_i\lambda_i^{\Phi}\right)^{1/\Phi}, \left(\bigvee_{i=1}^{l+1}\zeta_i\mu_i^{\Phi}\right)^{1/\Phi}\right); \\ \left[\left(\left(1-\bigvee_{i=1}^{l+1}\left(1-\left(\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TL}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{2\Phi}\right)^{\Phi}, \left(\left(1-\bigvee_{i=1}^{l+1}\left(1-\left(\sqrt[2\Phi]{(\beta_1\Psi_i^{\mathcal{TU}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{2\Phi}\right)^{\Phi}\right], \\ \left[\left(\left(1-\bigvee_{i=1}^{l+1}\left(1-\left(\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IL}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{\Phi}\right)^{\Phi}, \left(\left(1-\bigvee_{i=1}^{l+1}\left(1-\left(\sqrt[\Phi]{(\beta_2\Psi_i^{\mathcal{IU}})^{\Phi}}\right)\right)^{\zeta_i}\right)^{\Phi}\right)^{\Phi}\right], \\ \begin{bmatrix} \left(1-\left(1-\sqrt[2\Phi]{\bigvee_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FL}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}}\right)^{\Phi}\right)^{2\Phi}, \\ \left(1-\left(1-\sqrt[2\Phi]{\bigvee_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Phi]{(\beta_3\Psi_i^{\mathcal{FU}})}\right)^{\Phi}\right)^{2\Phi}\right)^{\zeta_i}}\right)^{\Phi}\right)^{2\Phi} \end{bmatrix} \end{bmatrix}.$$

The above formula valid for any l.

The GSRDioNSNIVWA operator is switched to the SRDioNSNIVWA operator if $\Phi = 1$.

Theorem 5.11. *If all* $\widetilde{\Gamma}_i = \left\langle (\lambda_i, \mu_i); [\beta_1\Psi_i^{\mathcal{TL}}, \beta_1\Psi_i^{\mathcal{TU}}], [\beta_2\Psi_i^{\mathcal{IL}}, \beta_2\Psi_i^{\mathcal{IU}}][\beta_3\Psi_i^{\mathcal{FL}}, \beta_3\Psi_i^{\mathcal{FU}}] \right\rangle (i = 1\ to\ n)$ *are equal, then GSRDioNSNIVWA*$(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n) = \widetilde{\Gamma}$.

Using the GSRDioNSNIVWA operator, the boundedness and monotonicity properties are met.

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5.4 Generalized SRDioNSNIVWG (GSRDioNSNIVWG) operator

Definition 5.12. Let $\widetilde{\Gamma}_i = \left\langle (\lambda_i, \mu_i); [\beta_1 \Psi_i^{\mathcal{TL}}, \beta_1 \Psi_i^{\mathcal{TU}}], [\beta_2 \Psi_i^{\mathcal{IL}}, \beta_2 \Psi_i^{\mathcal{IU}}], [\beta_3 \Psi_i^{\mathcal{FL}}, \beta_3 \Psi_i^{\mathcal{FU}}] \right\rangle$ be the family of SRDioNSNIVNs. Then GSRDioNSNIVWG $(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n) = \frac{1}{\Phi}\left(\bigvee_{i=1}^{n} (\Phi\widetilde{\Gamma}_i)^{\zeta_i} \right)$ $(i = 1, 2, ..., n)$.

Theorem 5.13. *Let $\widetilde{\Gamma}_i = \left\langle (\lambda_i, \mu_i); [\beta_1 \Psi_i^{\mathcal{TL}}, \beta_1 \Psi_i^{\mathcal{TU}}], [\beta_2 \Psi_i^{\mathcal{IL}}, \beta_2 \Psi_i^{\mathcal{IU}}], [\beta_3 \Psi_i^{\mathcal{FL}}, \beta_3 \Psi_i^{\mathcal{FU}}] \right\rangle$ be the family of SRDioNSNIVNs. Then GSRDioNSNIVWG$(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n)$=*

$$\left[\begin{array}{c} \left(\frac{1}{\Phi} \bigvee_{i=1}^{n} (\Phi\lambda_i)^{\zeta_i}, \frac{1}{\Phi} \bigvee_{i=1}^{n} (\Phi\mu_i)^{\zeta_i} \right); \\ \left[\begin{array}{c} \left(1 - \left(1 - \sqrt[2\Phi]{\bigvee_{i=1}^{n} \left(\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TL}})} \right)^{\Phi} \right)^{2\Phi} \right)^{\zeta_i}} \right)^{\Phi} \right)^{2\Phi}, \\ \left(1 - \left(1 - \sqrt[2\Phi]{\bigvee_{i=1}^{n} \left(\left(1 - \left(1 - \sqrt[2\Phi]{(\beta_1 \Psi_i^{\mathcal{TU}})} \right)^{\Phi} \right)^{2\Phi} \right)^{\zeta_i}} \right)^{\Phi} \right)^{2\Phi} \end{array} \right], \\ \left[\left(\left(1 - \bigvee_{i=1}^{n} \left(1 - \left(\sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IL}})^{\Phi}} \right) \right)^{\zeta_i} \right)^{\Phi} \right)^{\Phi}, \left(\left(1 - \bigvee_{i=1}^{n} \left(1 - \left(\sqrt[\Phi]{(\beta_2 \Psi_i^{\mathcal{IU}})^{\Phi}} \right) \right)^{\zeta_i} \right)^{\Phi} \right)^{\Phi} \right], \\ \left[\left(\left(1 - \bigvee_{i=1}^{n} \left(1 - \left(\sqrt[2\Phi]{(\beta_3 \Psi_i^{\mathcal{FL}})^{\Phi}} \right) \right)^{\zeta_i} \right)^{2\Phi} \right)^{\Phi}, \left(\left(1 - \bigvee_{i=1}^{n} \left(1 - \left(\sqrt[2\Phi]{(\beta_3 \Psi_i^{\mathcal{FU}})^{\Phi}} \right) \right)^{\zeta_i} \right)^{2\Phi} \right)^{\Phi} \right] \end{array} \right].$$

The GSRDioNSNIVWG operator becomes the SRDioNSNIVWG operator if $\Phi = 1$.

Using the GSRDioNSNIVWG operator, the boundedness and monotonicity properties are met.

Theorem 5.14. *If all $\widetilde{\Gamma}_i = \left\langle (\lambda_i, \mu_i); [\beta_1 \Psi_i^{\mathcal{TL}}, \beta_1 \Psi_i^{\mathcal{TU}}], [\beta_2 \Psi_i^{\mathcal{IL}}, \beta_2 \Psi_i^{\mathcal{IU}}][\beta_3 \Psi_i^{\mathcal{FL}}, \beta_3 \Psi_i^{\mathcal{FU}}] \right\rangle (i = 1 \; to \; n)$ are equal, then GSRDioNSNIVWG$(\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n) = \widetilde{\Gamma}$.*

6 MADM using SRDioNSNIV data

Let $\widetilde{\Gamma} = \{\widetilde{\Gamma}_1, \widetilde{\Gamma}_2, ..., \widetilde{\Gamma}_n\}$ be the n-alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the m-attributes, $w = \{\zeta_1, \zeta_2, ..., \zeta_m\}$ be the weights of attributes,
$\widetilde{\widetilde{\Gamma}}_{ij} = \left\langle (\lambda_{ij}, \mu_{ij}); [\beta_1 \Psi_{ij}^{\mathcal{TL}}, \beta_1 \Psi_{ij}^{\mathcal{TU}}], [\beta_2 \Psi_{ij}^{\mathcal{IL}}, \beta_2 \Psi_{ij}^{\mathcal{IU}}], [\beta_3 \Psi_{ij}^{\mathcal{FL}}, \beta_3 \Psi_{ij}^{\mathcal{FU}}] \right\rangle$ is denoted by SRDioNSNIVN of $\widetilde{\Gamma}_i$ in C_j. Here,
$\left[\beta_1 \Psi_{ij}^{\mathcal{TL}}, \beta_1 \Psi_{ij}^{\mathcal{TU}}\right], \left[\beta_2 \Psi_{ij}^{\mathcal{IL}}, \beta_2 \Psi_{ij}^{\mathcal{IU}}\right], \left[\Psi_{ij}^{\mathcal{FL}}, \Psi_{ij}^{\mathcal{FU}}\right] \in [0, 1]$ and $0 \leq (\beta_1 \Psi_{ij}^{\mathcal{TU}}(\eta))^2 + \sqrt{(\beta_2 \Psi_{ij}^{\mathcal{IU}}(\eta))} + \sqrt{(\Psi_{ij}^{\mathcal{FU}}(\eta))} \leq 2$. Here, the n-alternative sets and m-attribute sets result in the $n \times m$ decision matrix, which is indicated by the equation $\mathbb{D} = (\widetilde{\Gamma}_{ij})_{n \times m}$. In a MADM problem, one tries to select the best choice from a set of constrained options using a number of attributes with preferred weights. In this scenario, each alternative is described in connection to each attribute using the euclidean and hamming distance ideas, and SRDioNSNIVSs are utilized to draw a conclusion. The representation is created by adding the positive and negative ideal values of each attribute in relation to each attributes. After applying the following algorithm, a decision is made.

6.1 Algoritheorem for SRDioNSNIV

Step-1: SRDioNSNIV choice values should be entered.
Step-2: To decide on the normalization decision values. The matrix of choices $\mathbb{D} = (\widetilde{\Gamma}_{ij})_{n \times m}$ is normalized into $\widehat{\mathbb{D}} = (\widetilde{\Gamma}_{ij})_{n \times m}$; put

$$\widetilde{\Gamma}_{ij} = \left\langle (\widehat{\lambda_{ij}}, \widehat{\mu_{ij}}); [\widehat{\beta_1 \Psi_{ij}^{\mathcal{TL}}}, \widehat{\beta_1 \Psi_{ij}^{\mathcal{TU}}}], [\widehat{\beta_2 \Psi_{ij}^{\mathcal{IL}}}, \widehat{\beta_2 \Psi_{ij}^{\mathcal{IU}}}], [\widehat{\beta_3 \Psi_{ij}^{\mathcal{FL}}}, \widehat{\beta_3 \Psi_{ij}^{\mathcal{FU}}}] \right\rangle$$

and

$$\widehat{\lambda_{ij}} = \frac{\lambda_{ij}}{\sup_i(\lambda_{ij})}, \widehat{\mu_{ij}} = \frac{\mu_{ij}}{\sup_i(\mu_{ij})} \cdot \frac{\mu_{ij}}{\lambda_{ij}}, \; \widehat{\beta_1 \Psi_{ij}^{\mathcal{TL}}} = \beta_1 \Psi_{ij}^{\mathcal{TL}}, \widehat{\beta_1 \Psi_{ij}^{\mathcal{TU}}} = \beta_1 \Psi_{ij}^{\mathcal{TU}}.$$

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Step-3: To determine the aggregate values. Using SRDioNSNIV information aggregation operators as a base, attribute C_j in $\widetilde{\Gamma}_i$, $\widetilde{\Gamma}_{ij} = \left\langle (\widehat{\lambda_{ij}}, \widehat{\mu_{ij}}); [\widehat{\beta_1 \Psi_{ij}^{\mathcal{TL}}}, \widehat{\beta_1 \Psi_{ij}^{\mathcal{TU}}}], [\widehat{\beta_2 \Psi_{ij}^{\mathcal{IL}}}, \widehat{\beta_2 \Psi_{ij}^{\mathcal{IU}}}], [\widehat{\beta_3 \Psi_{ij}^{\mathcal{FL}}}, \widehat{\beta_3 \Psi_{ij}^{\mathcal{FU}}}] \right\rangle$ is aggregated into $\widetilde{\Gamma}_i = \left\langle (\widehat{\lambda_i}, \widehat{\mu_i}); [\widehat{\beta_1 \Psi_i^{\mathcal{TL}}}, \widehat{\beta_1 \Psi_i^{\mathcal{TU}}}], [\widehat{\beta_2 \Psi_i^{\mathcal{IL}}}, \widehat{\beta_2 \Psi_i^{\mathcal{IU}}}], [\widehat{\beta_3 \Psi_i^{\mathcal{FL}}}, \widehat{\beta_3 \Psi_i^{\mathcal{FU}}}] \right\rangle$.

Step-4: Calculate the ideal values, both positive and negative, for each alternative as follows:

$$\widetilde{\Gamma}^P = \left\langle \Big(\sup_{1 \le i \le n} (\widehat{\lambda_{ij}}), \inf_{1 \le i \le n} (\widehat{\mu_{ij}}) \Big); [1, 1], [1, 1], [0, 0] \right\rangle,$$

$$\widetilde{\Gamma}^N = \left\langle \Big(\inf_{1 \le i \le n} (\widehat{\lambda_{ij}}), \sup_{1 \le i \le n} (\widehat{\mu_{ij}}) \Big); [0, 0], [0, 0], [1, 1] \right\rangle.$$

Step-5: Find the ED between each option using the following two ideal values:

$$\mathbb{D}_i^P = \mathbb{D}_E \Big(\widetilde{\Gamma}_i, \widetilde{\Gamma}^P \Big); \ \mathbb{D}_i^N = \mathbb{D}_E \Big(\widetilde{\Gamma}_i, \widetilde{\Gamma}^N \Big).$$

Step-6: The values for relative closeness are calculated as follows:

$$\mathbb{D}_i^\star = \frac{\mathbb{D}_i^N}{\mathbb{D}_i^P + \mathbb{D}_i^N}.$$

Step-7: The output that produces the best value is $\sup \mathbb{D}_i^\star$. Therefore, decision is making the best option for the given problem.

6.2 Selection process based on robotic engineering

We encounter issues with decision-making every day in fields including education, the economics, management, politics, and technology. The following five points should be taken into account as you start the robotic engineering selection process before making your ultimate choice. Out of a large number of options, we want to choose the best one based on professional evaluations against the criteria. Robotics is a branch of applied engineering that has been compared to a fusion of computer science and machine tool technology. It covers artificial intelligence, computer programming, microelectronics, production theory, and machine design. I have now randomly selected five different kinds of Robotic nurses, Pharmarobotics, Robotic-assisted biopsy, Antibacterial nanomaterials, Ai diagnostics and ai epidemiology. Four types of criteria for choosing a robotics system by robot controller features (C_1), affordable off line programming software (C_2), safety codes (C_3), experience and reputation of the robot manufacturer (C_4) and their weights are $w = \{0.4, 0.3, 0.2, 0.1\}$. Out of a large number of options, we want to choose the best one using professional evaluations against the criteria. The decision-making informations are as follows:

$$C_1 = \begin{bmatrix} (0.85, 0.6), [0.45, 0.5], [0.25, 0.3], [0.55, 0.6], [0.3, 0.35], [0.45, 0.65], [0.1, 0.15] \\ (0.85, 0.8), [0.25, 0.4], [0.3, 0.35, [0.2, 0.3], [0.1, 0.15], [0.5, 0.6], [0.15, 0.2] \\ (0.9, 0.75), [0.5, 0.55], [0.35, 0.4], [0.45, 0.6], [0.2, 0.25], [0.35, 0.5], [0.25, 0.3] \\ (0.7, 0.65), [0.35, 0.4], [0.25, 0.3], [0.3, 0.4], [0.2, 0.3], [0.55, 0.6], [0.35, 0.4] \\ (0.75, 0.65), [0.4, 0.45], [0.4, 0.45], [0.45, 0.5], [0.35, 0.4], [0.5, 0.65], [0.1, 0.15] \end{bmatrix}$$

$$C_2 = \begin{bmatrix} (0.65, 0.6), [0.5, 0.6], [0.2, 0.25], [0.5, 0.65], [0.25, 0.3], [0.65, 0.75], [0.1, 0.15] \\ (0.85, 0.7), [0.55, 0.65], [0.25, 0.3], [0.6, 0.85], [0.1, 0.15], [0.3, 0.4], [0.2, 0.25] \\ (0.8, 0.75), [0.2, 0.3], [0.3, 0.35], [0.65, 0.95], [0.15, 0.2], [0.4, 0.5], [0.2, 0.3] \\ (0.75, 0.7), [0.49, 0.5], [0.2, 0.25], [0.5, 0.73], [0.15, 0.2], [0.55, 0.7], [0.3, 0.35] \\ (0.65, 0.6), [0.25, 0.3], [0.35, 0.4], [0.35, 0.5], [0.3, 0.35], [0.75, 0.8], [0.1, 0.15] \end{bmatrix}$$

$$C_3 = \begin{bmatrix} (0.75, 0.6), [0.55, 0.6], [0.2, 0.25], [0.55, 0.75], [0.25, 0.3], [0.45, 0.5], [0.15, 0.2] \\ (0.6, 0.55), [0.45, 0.5], [0.25, 0.3], [0.75, 0.8], [0.1, 0.2], [0.65, 0.75], [0.2, 0.3] \\ (0.75, 0.7), [0.65, 0.7], [0.2, 0.25], [0.7, 0.85], [0.25, 0.3], [0.35, 0.45], [0.1, 0.25] \\ (0.85, 0.75), [0.45, 0.5], [0.2, 0.25], [0.6, 0.65], [0.35, 0.4], [0.75, 0.8], [0.1, 0.2] \\ (0.7, 0.65), [0.4, 0.65], [0.25, 0.3], [0.6, 0.75], [0.4, 0.45], [0.65, 0.7], [0.2, 0.25] \end{bmatrix}$$

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$$C_4 = \begin{bmatrix} (0.9, 0.65), [0.5, 0.65], [0.2, 0.25], [0.6, 0.75], [0.15, 0.2], [0.5, 0.6], [0.15, 0.2] \\ (0.85, 0.7), [0.55, 0.7], [0.1, 0.3], [0.3, 0.45], [0.2, 0.3], [0.4, 0.55], [0.3, 0.35] \\ (0.75, 0.6), [0.65, 0.75], [0.2, 0.35], [0.6, 0.7], [0.2, 0.25], [0.3, 0.4], [0.35, 0.4] \\ (0.85, 0.65), [0.55, 0.65], [0.3, 0.4], [0.45, 0.5], [0.15, 0.2], [0.4, 0.55], [0.25, 0.3] \\ (0.65, 0.55), [0.65, 0.7], [0.45, 0.5], [0.7, 0.75], [0.15, 0.25], [0.5, 0.7], [0.1, 0.15] \end{bmatrix}$$

You may find a normalised decision matrix here:

$$C_1 = \begin{bmatrix} (0.9444, 0.5294), [0.45, 0.5], [0.25, 0.3], [0.55, 0.6], [0.3, 0.35], [0.45, 0.65], [0.1, 0.15] \\ (0.9444, 0.9412), [0.25, 0.4], [0.3, 0.35], [0.2, 0.3], [0.1, 0.15], [0.5, 0.6], [0.15, 0.2] \\ (1, 0.7813), [0.5, 0.55], [0.35, 0.4], [0.45, 0.6], [0.2, 0.25], [0.35, 0.5], [0.25, 0.3] \\ (0.7778, 0.7545), [0.35, 0.4], [0.25, 0.3], [0.3, 0.4], [0.2, 0.3], [0.55, 0.6], [0.35, 0.4] \\ (0.8333, 0.7042), [0.4, 0.45], [0.4, 0.45], [0.45, 0.5], [0.35, 0.4], [0.5, 0.65], [0.1, 0.15] \end{bmatrix}$$

$$C_2 = \begin{bmatrix} (0.7647, 0.7385), [0.5, 0.6], [0.2, 0.25], [0.5, 0.65], [0.25, 0.3], [0.65, 0.75], [0.1, 0.15] \\ (1, 0.7686), [0.55, 0.65], [0.25, 0.3], [0.6, 0.85], [0.1, 0.15], [0.3, 0.4], [0.2, 0.25] \\ (0.9412, 0.9375), [0.2, 0.3], [0.3, 0.35], [0.65, 0.95], [0.15, 0.2], [0.4, 0.5], [0.2, 0.3] \\ (0.8824, 0.8711), [0.49, 0.5], [0.2, 0.25], [0.5, 0.73], [0.15, 0.2], [0.55, 0.7], [0.3, 0.35] \\ (0.7647, 0.7385), [0.25, 0.3], [0.35, 0.4], [0.35, 0.5], [0.3, 0.35], [0.75, 0.8], [0.1, 0.15] \end{bmatrix}$$

$$C_3 = \begin{bmatrix} (0.8824, 0.64), [0.55, 0.6], [0.2, 0.25], [0.55, 0.75], [0.25, 0.3], [0.45, 0.5], [0.15, 0.2] \\ (0.7059, 0.6722), [0.45, 0.5], [0.25, 0.3], [0.75, 0.8], [0.1, 0.2], [0.65, 0.75], [0.2, 0.3] \\ (0.8824, 0.8711), [0.65, 0.7], [0.2, 0.25], [0.7, 0.85], [0.25, 0.3], [0.35, 0.45], [0.1, 0.25] \\ (1, 0.8824), [0.45, 0.5], [0.2, 0.25], [0.6, 0.65], [0.35, 0.4], [0.75, 0.8], [0.1, 0.2] \\ (0.8235, 0.8048), [0.4, 0.65], [0.25, 0.3], [0.6, 0.75], [0.4, 0.45], [0.65, 0.7], [0.2, 0.25] \end{bmatrix}$$

$$C_4 = \begin{bmatrix} (1, 0.6706), [0.5, 0.65], [0.2, 0.25], [0.6, 0.75], [0.15, 0.2], [0.5, 0.6], [0.15, 0.2] \\ (0.9444, 0.8235), [0.55, 0.7], [0.1, 0.3], [0.3, 0.45], [0.2, 0.3], [0.4, 0.55], [0.3, 0.35] \\ (0.8333, 0.6857), [0.65, 0.75], [0.2, 0.35], [0.6, 0.7], [0.2, 0.25], [0.3, 0.4], [0.35, 0.4] \\ (0.9444, 0.7101), [0.55, 0.65], [0.3, 0.4], [0.45, 0.5], [0.15, 0.2], [0.4, 0.55], [0.25, 0.3] \\ (0.7222, 0.6648), [0.65, 0.7], [0.45, 0.5], [0.7, 0.75], [0.15, 0.25], [0.5, 0.7], [0.1, 0.15] \end{bmatrix}$$

The following aggregate data for each alternative using the SRDioNSNIVWG operator $SRDioNSNIVWG\ operator\ (\Phi = 1)$
$\widetilde{\Gamma}_1 = \langle(0.8837, 0.6284), [0.107, 0.1512], [0.1403, 0.2028], [0.0573, 0.1044]\rangle$
$\widetilde{\Gamma}_2 = \langle(0.9134, 0.8239), [0.098, 0.1652], [0.0473, 0.103], [0.0821, 0.1351]\rangle$
$\widetilde{\Gamma}_3 = \langle(0.9422, 0.8365), [0.1245, 0.1794], [0.1129, 0.1864], [0.0722, 0.1425]\rangle$
$\widetilde{\Gamma}_4 = \langle(0.8703, 0.8106), [0.0984, 0.1357], [0.0972, 0.1557], [0.1426, 0.2145]\rangle$
$\widetilde{\Gamma}_5 = \langle(0.7997, 0.7306), [0.1375, 0.1901], [0.1545, 0.2213], [0.0684, 0.1175]\rangle$
Determine the optimum values, both positive and negative, of the following alternatives:
$\widetilde{\Gamma}^P = \langle(0.9422, 0.6284), 1, 1, 0\rangle$ and $\widetilde{\Gamma}^N = \langle(0.7997, 0.8365), 0, 0, 1\rangle$
The following table shows the ED between each alternatives positive and negative ideal values:
$\mathbb{D}_1^P = 0.1805, \mathbb{D}_2^P = 0.1477, \mathbb{D}_3^P = 0.1733, \mathbb{D}_4^P = 0.1986, \mathbb{D}_5^P = 0.1914$
$\mathbb{D}_1^N = 0.08, \mathbb{D}_2^N = 0.1144, \mathbb{D}_3^N = 0.0881, \mathbb{D}_4^N = 0.0627, \mathbb{D}_5^N = 0.0699.$
The values for relative closeness are calculated as follows: $\mathbb{D}_1^\star = 0.307$, $\mathbb{D}_2^\star = 0.4364$, $\mathbb{D}_3^\star = 0.3371$, $\mathbb{D}_4^\star = 0.2398$, $\mathbb{D}_5^\star = 0.2676$.
Ranking of alternatives are as follows: $\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$.
Therefore, the optimal one is Pharmarobotics.

6.3 Comparison for proposed models and existing models

Using the aforementioned facts as a basis for analysis and discussion, we propose the SRDioNSNIVWA, SRDioNSNIVWG, GSRDioNSNIVWA, and GSRDioNSNIVWG approaches, which are based on ED and HD, respectively. The different distances are as follows:

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$\Phi = 1$	SRDioNSNIVWA	SRDioNSNIVWG	GSRDioNSNIVWA	GSRDioNSNIVWG
TOPSIS-Euclidean distance (proposed)	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$
TOPSIS-Hamming distance (proposed)	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_5 \geq \widetilde{\Gamma}_4$
Euclidean distance[17]	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_4 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_1$ $\widetilde{\Gamma}_4 \geq \widetilde{\Gamma}_5$
Hamming distance[17]	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$	$\widetilde{\Gamma}_2 \geq \widetilde{\Gamma}_3 \geq \widetilde{\Gamma}_4$ $\widetilde{\Gamma}_1 \geq \widetilde{\Gamma}_5$

7 Conclusion:

For SRDioNSNIVSs, we have presented the ED and HD measures in this study. These distance measures are advantageous due to their mathematical simplicity. With the help of suitable numerical examples, the superiority of the ED and HD measures is demonstrated. It is established that both the ED and HD metrics are applicable. With regard to SRDioNSNIVWA, SRDioNSNIVWG, GSRDioNSNIVWA, and GSRDioNSNIVWG, we have suggested aggregation operation rules. Additionally, we covered some of these operators features and provided some examples. In uncertain and inconsistent circumstances, the implementation of the SRDioNSNIV MADM technique can assist people in selecting the best option from a range of available options. To MADM problems depending on Φ, we have used the SRDioNSNIVWA, SRDioNSNIVWG, GSRDioNSNIVWA, and GSRDioNSNIVWG operators. With the SRDioNSNIVWA, SRDioNSNIVWG, GSRDioNSNIVWA, and GSRDioNSNIVWG operators based for Φ, the distinct ranking of alternatives can be discovered. The study presented above concludes by showing that the ranking of alternatives is most significantly impacted by the generalized values of Φ. The decision-makers may choose to select the values for Φ based on the real situation for the best conceivable ranking before proceeding with the necessary judgments. Therefore, based on the values of Φ, the decision-maker may choose how to arrive at the result. ED and HD metrics offer a number of practical uses in the study of data. The author is convinced that the discussions in this paper will be beneficial to future academics interested in this field of study because it is still in its early stages.

Conflicts of Interest: The authors declare no conflict of interest.

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