On The Topological Spaces of Neutrosophic Real Intervals

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Abstract

In this paper, we present the topological space of intervals based neutrosophic real numbers $a + bI$, where we clarify how neutrosophic real intervals can be expressed according to the neutrosophic partial order relation, and we use these intervals to build a topological space. On the other hand, we use a similar argument to build a topological space over the intervals of refined neutrosophic numbers, with many illustrated and related examples on open and closed sets.

Keywords: neutrosophic number; topological space; open set; closed set; neutrosophic interval.

1.Introduction

Neutrosophic real numbers together form an algebraic ring see [1], this ring has been studied on a wide range by many researchers, where it has been used in matrix theory, linear structures, and in geometry see [2-5,9-13].

In [3] a partial order relation defined on neutrosophic real numbers as follows:
$a + bI \leq c + dI \leftrightarrow a \leq c, a + b \leq c + d$.
This partial order relation allows us to study number theory and cryptography through neutrosophic point of view see [21-26].
Also, Refined neutrosophic real numbers together form an algebraic ring [6], this ring has been studied on a wide range by many researchers, where it has been used in matrix theory, linear structures, and in geometry see [8].
A partial order relation is defined on refined neutrosophic real numbers as follows:
$a + bI_1 + tI_2 \leq c + dI_1 + kI_2 \leftrightarrow a \leq c, a + t \leq c + k, a + b + t \leq c + d + k$. This partial order relation allows us to study number theory through refined neutrosophic point of view.
In this work, we depend on the partial order relations defined on neutrosophic numbers and refined neutrosophic numbers to build topological spaces on these intervals, and to study their elementary properties such as closed and open sets.
For more details about neutrosophic topology, see [16-18, 28-30].

2.Main Concepts and Discussion

Definition 2.1.
Let $R(I) = \{a + bI \,; a.b \in R \,. I^2 = I\}$ be the ring of real neutrosophic numbers, the interval $[a + bI, c + dI]$ is defined as:
$\{x + yI \in R(I) \,; a \leq x \leq c \,, a + b \leq x + y \leq c + d\}$.
Example:
We have $3 + 2I \in [1 + 6I, 4 + 4I]$.
Definition 2.2.
The interval
$$[a + bI \,, \infty[= \{x + yI \in R(I) \,; a \leq x < \infty \,, a + b \leq x + y < \infty\}.$$
The interval
$$]-\infty, a + bI] = \{x + yI \in R(I) \,; -\infty < x \leq a \,, -\infty < x + y \leq a + b\}.$$
The interval
$$]a + bI, \infty[= \{x + yI \in R(I) \,; a < x < \infty, a + b < x + y < \infty\},$$
The interval
$$]-\infty, a + bI[= \{x + yI \in R(I) \,; -\infty < x < a \,, -\infty < x + y < a + b\}.$$

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Definition 2.3.
We define different types of sub-empty sets:

$$\begin{cases} \{\varphi_1\} = \{\varphi\} + \{\varphi\}I \\ \{\varphi_2\} = \{\varphi\} + aI \ ; a \in R \\ \{\varphi_3\} = a + \{\varphi\}I \ ; a \in R. \end{cases}$$

Definition 2.4.
We define some different types of sub-global sets:

$$\begin{cases} R_1 = R + RI = \mathbb{R}(I) \\ R_2 = R + \{\varphi\}I \\ R_3 = \{\varphi\} + RI. \end{cases}$$

Also,

$$\begin{cases} R^S = S + RI \ ; S \subseteq R \\ R_S = R + SI \ ; S \subseteq R. \end{cases}$$

Definition 2.5.
We define the following different types of spaces:

$$\begin{cases} T_1 = (]-\infty, a + bI[, \cap, \cup, \{\varphi_1\}) \\ T_2 = (]a + bI, \infty[, \cap, \cup, \{\varphi_1\}) \end{cases} \ ; a + bI \in R(I) \,.$$

Theorem 2.1.
T_1, T_2 are topological spaces.
Proof.
Let $F_1 =]-\infty, a_1 + b_1 I[\,, F_2 =]-\infty, a_2 + b_2 I[\ \in T_1$, then:
$F_1 \cap F_2 = \{x + yI \ \in R(I) \,;\ x + yI \in]-\infty, a_1 + b_1 I[,]-\infty, a_2 + b_2 I[\}$,

So that

$$\begin{cases} -\infty < x < a_1 \ , -\infty < x + y < a_1 + b_1 \\ -\infty < x < a_2 \ , -\infty < x + y < a_2 + b_2\,. \end{cases}$$

Hence

$$x \in]-\infty, a_1[\ \cap]-\infty, a_2[\,, x + y \in]-\infty, a_1 + b_1[\ \cap]a_2 + b_2[.$$

Thus

$$F_1 \cap F_2 \in T_1.$$

Let

$$F_i =]-\infty, a_i + b_i I[\ ; a_i, b_i \in R,$$

then

$$\cup F_i = \left\{x + yI \in R(I) \,;\ x + yI \ \in F_j \text{ for some } j\right\}.$$

This means that:

$$x \in \cup\,]-\infty, a_i[\,, x + y \in]-\infty, a_i + b_i[\,,$$

which implies that
$\cup F_i \in T_1$ and T_1 is topological space.
Now, let

$$L_1 =]a_1 + b_1 I, \infty[\,, L_2 =]a_2 + b_2 I, \infty[\ \dots \dots$$
$$L_i =]a_i + b_i I, \infty[\in T_2 \ ; a_i, b_i \in R.$$
$$L_1 \cap L_2 = \{x + yI \ \in R(I) \,;\ x + yI \in]a_1 + b_1 I, \infty[,]a_2 + b_2 I, \infty[\}.$$

Hence

$$\begin{cases} a_1 < x < \infty \ , a_1 + b_1 < x + y < \infty \\ a_2 < x < \infty \ , a_2 + b_2 < x + y < \infty. \end{cases}$$

Thus

$$\begin{cases} x \in]a_1, \infty[\ \cap]a_2, \infty[\\ x + y \in]a_1 + b_1, \infty[\ \cap]a_2 + b_2, \infty[\,' \end{cases}$$

and

$$L_1 \cap L_2 \in T_2,$$
$$\cup L_i = \left\{x + yI \in R(I) \,;\ x + yI \ \in L_j \text{ for some } j\right\}.$$

This means that:

$$x \in \cup\,]a_i, \infty[\,, x + y \in]a_i + b_i, \infty[\,,$$

which implies that:
$\cup L_i \in T_2$ and T_2 is topological space.

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Remark 2.1.
Open sets in T_1 are:

$$]-\infty, a+bI[\ ; a, b \in R.$$

Open sets in T_2 are:

$$]a+bI, \infty[\ ; a, b \in R.$$

Closed sets in T_1 are:

$$(]-\infty, a+bI[)^c = \{x+yI \in R(I)\ ;\ x+yI \notin\ ]-\infty, a+bI[\} = [a+bI, \infty[.$$

Closed sets in T_2 are:

$$(]a+bI, \infty[)^c = \{x+yI \in R(I)\ ;\ x+yI \notin\ ]a+bI, \infty[\} =]-\infty, a+bI].$$

Example:
$]2+3I, \infty[\ \in T_2$ is an open set, $(]2+3I, \infty[)^c =]-\infty, 2+3I]$ is the corresponding closed set.
$]-\infty, 2+3I[\ \in T_1$ is an open set, $(]-\infty, 2+3I[)^c = [2+3I, \infty[$ is the corresponding closed set.

Definition 2.6.
The ring of refined neutrosophic real numbers is:

$$R(I_1, I_2) = \{a+bI_1+cI_2\ ; I_1^2 = I_1, I_2^2 = I_2, I_1I_2 = I_2I_1 = I_1;\ a, b, c \in R\}.$$

Definition 2.7.
The interval
$[a+bI_1+cI_2, m+nI_1+kI_2] =$
$\{x+yI_1+zI_2 \in R(I_1, I_2); a \le x \le m, a+c \le x+z \le m+k, a+b+c \le x+y+z \le m+n+k\}$.
The interval
$[a+bI_1+cI_2, \infty[= \{x+yI_1+zI_2 \in R(I_1, I_2); a \le x < \infty, a+c \le x+z < \infty, a+b+c \le x+y+z < \infty\}$.
The interval
$]a+bI_1+cI_2, \infty[= \{x+yI_1+zI_2 \in R(I_1, I_2); a < x < \infty, a+c < x+z < \infty, a+b+c < x+y+z < \infty\}$.
The interval
$]-\infty, a+bI_1+cI_2] = \{x+yI_1+zI_2 \in R(I_1, I_2); -\infty < x \le a, -\infty < x+z \le a+c, -\infty < x+y+z \le a+b+c\}$.
The interval
$]-\infty, a+bI_1+cI_2[= \{x+yI_1+zI_2 \in R(I_1, I_2); -\infty < x < a, -\infty < x+z < a+c, -\infty < x+y+z < a+b+c\}$.

Definition 2.8.
We define

$$T_1 = (]-\infty, a+bI_1+cI_2[, \cup, \cap, \{\emptyset\}),$$
$$T_2 = (]a+bI_1+cI_2, \infty[, \cup, \cap, \{\emptyset\}).$$

Theorem 2.2.
T_1, T_2 are topological spaces.

Proof.
Let

$$F_1 =]a_1+b_1I_1+c_1I_2, \infty[\ , F_2 =]a_2+b_2I_1+c_2I_2, \infty[\ \in T_2,$$
$$L_1 =]-\infty, a_1+b_1I_1+c_1I_2[\ , L_2 =]-\infty, a_2+b_2I_1+c_2I_2[\ , \ldots$$
$$F_i =]a_i+b_iI_1+c_iI_2, \infty[\ \in T_2, L_i =]-\infty, a_i+b_iI_1+c_iI_2[\ \in T_1.$$

We have

$$F_1 \cap F_2 = \{x+yI_1+zI_2 \in R(I_1, I_2)\ ;\ x+yI_1+zI_2 \in F_1, F_2\}.$$

Thus,

$$\begin{cases} a_1 < x < \infty, a_2 < x < \infty \\ a_1+c_1 < x+z < \infty, a_2+c_2 < x+z < \infty \\ a_1+b_1+c_1 < x+y+z < \infty, a_2+b_2+c_2 < x+y+z < \infty. \end{cases}$$

So that

$$\begin{cases} x \in\ ]a_1, \infty[\ \cap\ ]a_2, \infty[\\ x+z \in\ ]a_1+c_1, \infty[\ \cap\ ]a_2+c_2, \infty[\\ x+y+z \in\ ]a_2+b_2+c_2, \infty[\ \cap\ ]a_1+b_1+c_1, \infty[, \end{cases}$$

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and $F_1 \cap F_2 \in T_2$.

$$L_1 \cap L_2 = \{x + yI_1 + zI_2 \in R(I_1, I_2)\ ;\ x + yI_1 + zI_2 \in L_1, L_2\},$$

$$\begin{cases} -\infty < x < a_1, -\infty < x < a_2 \\ -\infty < x + z < a_1 + c_1, -\infty < x + z < a_2 + c_2 \\ -\infty < x + y + z < a_1 + b_1 + c_1, -\infty < x + y + z < a_2 + b_2 + c_2. \end{cases}$$

Hence

$$\begin{cases} x \in]-\infty, a_1[\cap]-\infty, a_2[\\ x + z \in]-\infty, a_1 + c_1[\cap]-\infty, a_2 + c_2[\\ x + y + z \in]-\infty, a_1 + b_1 + c_1[\cap]-\infty, a_2 + b_2 + c_2[. \end{cases}$$

Therefore

$$L_1 \cap L_2 \in T_1.$$

$$\cup F_i = \{x + yI_1 + zI_2 \in R(I_1, I_2)\ ;\ x + yI_1 + zI_2 \in F_j \text{ for some } j\}.$$

Therefore

$$\begin{cases} x \in \cup\ ]a_i, \infty[\\ x + z \in \cup\ ]a_i + c_i, \infty[\\ x + y + z \in\ ]a_i + b_i + c_i, \infty[. \end{cases}$$

Hence $\cup F_i \in T_2$.

$$\cup L_i = \{x + yI_1 + zI_2 \in R(I_1, I_2)\ ;\ x + yI_1 + zI_2 \in L_j \text{ for some } j\}.$$

Therefore

$$\begin{cases} x \in \cup\ ]-\infty, a_i[\\ x + z \in \cup\ ]-\infty, a_i + c_i[\\ x + y + z \in\ ]-\infty, a_i + b_i + c_i[. \end{cases}$$

Hence $\cup L_i \in T_1$.

Remark 2.2.

Open sets in T_1 are

$$]-\infty, a + bI_1 + cI_2[\ ; a, b, c \in R.$$

Open sets in T_2 are

$$]a + bI_1 + cI_2, \infty[\ ; a, b, c \in R.$$

Closed sets in T_1 are

$$[a + bI_1 + cI_2, \infty[\ ; a, b, c \in R.$$

Closed sets in T_2 are

$$]-\infty, a + bI_1 + cI_2]\ ; a, b, c \in R.$$

Example:

$]3 + I_1 + I_2, \infty[$ is an open set in T_2.

The corresponding closed set is:

$$(]3 + I_1 + I_2, \infty[)^c =]-\infty, 3 + I_1 + I_2[\ .$$

Definition 2.9.

$R(I_1, I_2, I_3) = \{a + bI_1 + cI_2 + dI_3\ ; a, b, c, d \in R\ , I_iI_j = I_{\min(i,j)}, {I_i}^2 = I_i\}$ is called the 3-refined neutrosophic ring of real numbers.

Definition 2.10.

The intervals on $R(I_1, I_2, I_3)$ are defined as follows:

$]x_0 + x_1I_1 + x_2I_2 + x_3I_3, \infty[=$
$\{y_0 + y_1I_1 + y_2I_2 + y_3I_3 \in R(I_1, I_2, I_3); x_0 < y_0 < \infty, x_0 + x_3 < y_0 + y_3 < \infty, x_0 + x_2 + x_3 < y_0 + y_2 + y_3 < \infty, x_0 + x_1 + x_2 + x_3 < y_0 + y_1 + y_2 + y_3 < \infty\}$.

$[x_0 + x_1I_1 + x_2I_2 + x_3I_3, \infty[=$
$\{y_0 + y_1I_1 + y_2I_2 + y_3I_3 \in R(I_1, I_2, I_3); x_0 \le y_0 < \infty, x_0 + x_3 \le y_0 + y_3 < \infty, x_0 + x_2 + x_3 \le y_0 + y_2 + y_3 < \infty, x_0 + x_1 + x_2 + x_3 \le y_0 + y_1 + y_2 + y_3 < \infty\}$.

$]-\infty, x_0 + x_1I_1 + x_2I_2 + x_3I_3[=$
$\{y_0 + y_1I_1 + y_2I_2 + y_3I_3 \in R(I_1, I_2, I_3); -\infty < y_0 < x_0, -\infty < y_0 + y_3 < x_0 + x_3, -\infty < y_0 + y_2 + y_3 < x_0 + x_2 + x_3, -\infty < y_0 + y_1 + y_2 + y_3 < x_0 + x_1 + x_2 + x_3\}$.

$]-\infty, x_0 + x_1I_1 + x_2I_2 + x_3I_3] =$
$\{y_0 + y_1I_1 + y_2I_2 + y_3I_3 \in R(I_1, I_2, I_3); -\infty < y_0 \le x_0, -\infty < y_0 + y_3 \le x_0 + x_3, -\infty < y_0 + y_2 + y_3 \le x_0 + x_2 + x_3, -\infty < y_0 + y_1 + y_2 + y_3 \le x_0 + x_1 + x_2 + x_3\}$.

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Definition 2.11.
We define the following spaces in $R(I_1, I_2, I_3)$.

$$T_1 = (]-\infty, a + bI_1 + cI_2 + dI_3[, \cup, \cap, \emptyset),$$
$$T_2 = (]a + bI_1 + cI_2 + dI_3, \infty[, \cup, \cap, \emptyset).$$

Theorem 2.3.
T_1, T_2 are topological spaces.

Proof.
Let

$$F_i =]-\infty, a_i + b_iI_1 + c_iI_2 + d_iI_3[\in T_1\ ;\ a_i, b_i, c_i, d_i \in R,$$
$$L_i =]a_i + b_iI_1 + c_iI_2 + d_iI_3, \infty[\ \in T_2;\ a_i, b_i, c_i, d_i \in R.$$
$$F_i \cap F_j = \{x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in R(I_1, I_2, I_3)\ ;\ x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in F_i, F_j\},$$
$$\begin{cases} -\infty < x_0 < a_i\ , -\infty < x_0 < a_j \\ -\infty < x_0 + x_3 < a_i + d_i, -\infty < x_0 + x_3 < a_j + d_j \\ -\infty < x_0 + x_2 + x_3 < a_i + c_i + d_i, -\infty < x_0 + x_2 + x_3 < a_j + c_j + d_j \\ -\infty < x_0 + x_1 + x_2 + x_3 < a_i + b_i + c_i + d_i, -\infty < x_0 + x_1 + x_2 + x_3 < a_j + b_j + c_j + d_j. \end{cases}$$

Thus

$$\begin{cases} x_0 \in\ ]-\infty, a_i[\ \cap\ ]-\infty, a_j[\\ x_0 + x_3 \in\ ]-\infty, a_i + d_i[\ \cap\ ]-\infty, a_j + d_j[\\ x_0 + x_2 + x_3 \in\ ]-\infty, a_i + c_i + d_i[\ \cap\ ]-\infty, a_j + c_j + d_j[\\ x_0 + x_1 + x_2 + x_3 \in\ ]-\infty, a_i + b_i + c_i + d_i[\ \cap\ ]-\infty, a_j + b_j + c_j + d_j[. \end{cases}$$

Thus

$$F_i \cap F_j \in T_1.$$
$$\cup F_i = \{x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in R(I_1, I_2, I_3)\ ;\ x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in F_j \text{ for some } j\},$$

thus

$$\begin{cases} x_0 \in \cup\ ]-\infty, a_i[\\ x_0 + x_3 \in \cup\ ]-\infty, a_i + d_i[\\ x_0 + x_2 + x_3 \in \cup\ ]-\infty, a_i + c_i + d_i[\\ x_0 + x_1 + x_2 + x_3 \in \cup\ ]-\infty, a_i + b_i + c_i + d_i[. \end{cases}$$

Hence

$$\cup F_i \in T_1.$$
$$L_i \cap L_j = \{x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in R(I_1, I_2, I_3)\ ;\ x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in L_j, L_i\}.$$

Thus

$$\begin{cases} a_i < x_0 < \infty\ , a_j < x_0 < \infty \\ a_i + d_i < x_0 + x_3 < \infty, a_j + d_j < x_0 + x_3 < \infty \\ a_i + c_i + d_i < x_0 + x_2 + x_3 < \infty, a_j + c_j + d_j < x_0 + x_2 + x_3 < \infty \\ a_i + b_i + c_i + d_i < x_0 + x_1 + x_2 + x_3 < \infty, a_j + b_j + c_j + d_j < x_0 + x_1 + x_2 + x_3 < \infty. \end{cases}$$

So that

$$\begin{cases} x_0 \in\ ]a_i, \infty[\ \cap\ ]a_j, \infty[\\ x_0 + x_3 \in\ ]a_i + d_i, \infty[\ \cap\ ]a_j + d_j, \infty[\\ x_0 + x_2 + x_3 \in\ ]a_i + c_i + d_i, \infty[\ \cap\ ]a_j + c_j + d_j, \infty[\\ x_0 + x_1 + x_2 + x_3 \in\ ]a_i + b_i + c_i + d_i, \infty[\ \cap\ ]a_j + b_j + c_j + d_j, \infty[\ , \end{cases}$$

and

$$L_i \cap L_j \in T_2.$$
$$\cup L_i = \{x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in R(I_1, I_2, I_3)\ ;\ x_0 + x_1I_1 + x_2I_2 + x_3I_3 \in L_j \text{ for some } j\}$$

This implies that:

$$\begin{cases} x_0 \in \cup\ ]a_i, \infty[\\ x_0 + x_3 \in \cup\ ]a_i + d_i, \infty[\\ x_0 + x_2 + x_3 \in \cup\ ]a_i + c_i + d_i, \infty[\\ x_0 + x_1 + x_2 + x_3 \in \cup\ ]a_i + b_i + c_i + d_i, \infty[. \end{cases}$$

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Thus

$$\cup L_i \in T_2.$$

Remark 2.3.

Open sets in T_1 are:

$$]-\infty, x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3[\ ; \ x_i \in R.$$

Open sets in T_2 are:

$$]x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3, \infty[\ ; \ x_i \in R.$$

Closed sets in T_1 are:

$$[x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3, \infty[\ ; \ x_i \in R.$$

Closed sets in T_2 are:

$$]-\infty, x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3] \ ; \ x_i \in R.$$

Example:

$F_1 = \ ]-\infty, 2 + I_1 + I_2 + I_3[$ is an open set in T_1.

The corresponding closed set is:

$$F_1^c = [2 + I_1 + I_2 + I_3, \infty[.$$

$L_1 = \ ]2 + I_1 + I_2 + I_3, \infty[$ is an open set in T_2.

The corresponding closed set is:

$$L_1^c =]-\infty, 2 + I_1 + I_2 + I_3].$$

3. Conclusion

In this paper, we presented the topological space of intervals based neutrosophic real numbers $a + bI$, where we clarified how neutrosophic real intervals can be expressed according to the neutrosophic partial order relation, and we used these intervals to build a topological space. On the other hand, we used a similar argument to build a topological space over the intervals of refined neutrosophic numbers, with many illustrated and related examples on open and closed sets.

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