An Approach to Solve the Linear Programming Problem Using Single Valued Trapezoidal Neutrosophic Number

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Abstract

While making a decision, the neutrosophic set theory includes the uncertainty part beside certainty part (i.e., Yes or No). In the present uncertain socio-economic fashion, this pattern is highly acceptable and hence, the limitations of fuzzy set and intuitionistic fuzzy set are overcome with neutrosophic set theory. The present study provides a modified structure of linear programming problem (LP-problem) and its solution approach in neutrosophic sense. A special type of neutrosophic set defined over the set of real number, viz., single valued trapezoidal neutrosophic number (SVTN-number) is adopted here as the coefficients of the objective function, right-hand side coefficients and decision variables itself of an LP-problem. In order to solve such problem, a parameter based ranking function of SVTN-number is newly constructed from the geometrical configuration of the trapezium. It plays a key role in the development of the solution algorithm. An LP-problem is normally solved under the asset of some given constraints. Besides that, there may be some hidden parameters (e.g., awareness level of nearer society for the smooth run of a clinical pharmacy, ruined structure of road to be met a profit from a bus, etc) of an LP-problem and these affect the solution badly when experts ignore them. This study makes an attempt to solve an LP-problem by giving importance to all these to attain a fair outcome. The efficiency of the proposed concept is illustrated in a real field. A real example is stated and is solved numerically under the present view.

Keywords: Neutrosophic set; Single valued trapezoidal neutrosophic (SVTN) number; Linear programming problem in neutrosophic sense; Simplex method.

1 Introduction

Due to complex diversity and vague atmosphere in the present socio-economic scenario, making a decision on several events are being complicated day in day out. It is almost impossible to draw a decision in a straight way due to incomplete and imprecise information available in the respective ground. Work pressure, diverted mind, measurement errors, limited attention, lack of knowledge, time bounding pressure, the narrow scope of placement at the end of academic, etc will force the experts to have such information. So, decision makers focus to develop the concepts of decision making and optimization in an uncertain way. This results in the exploration of fuzzy set by Zadeh [30] and intuitionistic fuzzy set by Atanassov [2]. But these logics can't manage the situations involving indeterminacy. There are many practical facts like in sports game, the role of elector in the casting of poll, making decisions in different sectors, etc wherein one may predict three kinds of outcomes. Smarandache [27] studied this kind of facts more precisely and he then introduced the notion of the neutrosophic set (NS), a generalisation of intuitionistic fuzzy set. Each object in NS is characterised by a triplet, viz., truth-membership value, indeterminacy-membership value and falsity-membership value. Each of neutrosophic triplet is quantified explicitly and is independent in nature. The indeterministic part of uncertain data plays an important role to make a proper decision which is out of scope in intuitionistic fuzzy set theory.

The ranking technique of fuzzy number, intuitionistic fuzzy number, neutrosophic number play an important role in developing different multi-attributive decision making, optimization, mathematical structures and others. Gani and Ponnalagu [13] defined a method based on intuitionistic fuzzy linear programming for investment strategy. Li [21] developed a ratio ranking technique of triangular intuitionistic fuzzy numbers and applied it to MADM problems. Yao and Wu [29] brought a ranking method of fuzzy numbers based on the decomposition principle and signed distance while Rao and Shankar [25] developed so with an area method using the circumference of the centroid. Mukherjee and Basu [22] applied a fuzzy ranking method for solving assignment problems with fuzzy costs. Roy and Das [26] solved a neutrosophic multi-objective production planning problem. Deli and Subas [11] applied a ranking method of trapezoidal neutrosophic number in MADM problems. Biswas et al. [9] have proposed an approach for multi-attribute group decision making problems under single-valued neutrosophic environment. Hussian et al. [14] have solved the neutrosophic

Doi :10.5281/zenodo.3740647
Received: January 31, 2020 Revised: March 07, 2020 Accepted: April 02, 2020

linear programming problem by transforming it into a crisp programming model. Pramanik [24] has put a new direction to solve a neutrosophic multi-objective programming by extending Zimmermann's approach. Several approaches [3-8, 12] are seen to optimise LP-problems under real neutrosophic climate. Chakraborty [10] established a score function of pentagonal neutrosophic number and applied it on a net working problem. Khalid [15-20] handled neutrosophic geometric programming in several directions. Mullai et al. [23] developed an inventory model with neutrosophic random variable.

Decision makers generally solve an LP-problem based on some constraints provided to them. Furthermore, there are a number of parameters of an LP-problem such as, but not limited to, the awareness level of nearer society for the smooth run of a clinical pharmacy, degree of redemption from Govt. tax on raw materials used in industry to meet a profit, degree of ruined economy of society for inhaling situation of the nearer market, degree of road condition in driving a bus to get a profit, etc. While solving an LP-problem, experts ignore these facts and so the outcome is not fair as a whole. To conquer this limitation, a ranking function of SVTN-number in term of a parameter graded in [0,1] is derived here from the geometrical configuration of the trapezium. This parameter describes an additional character of an LP-problem which is modified here and thus several situations (i.e., for different grades) of respective problem are managed nicely.

This study extends the concept of LP-problem from crisp sense to neutrosophic sense. Based on a ranking function newly brought here, an efficient solution algorithm is developed to solve such a problem. The efficiency of the present thought is measured in the real field. The paper is organised as follows.
Some preliminary definitions are remembered in Section 2. Section 3 provides a ranking function of SVTN-number , and an LP-problem is given from the point of neutrosophic view. In Section 4, an algorithm is developed towards solving such problem. The method has been illustrated with the help of a real life example in Section 5. Finally, the present work and its future aspect and limitation are given in Section 6.

2 Preliminaries

Some necessary definitions and results are stated below to make out the main results.

2.1 Definition [1]

A fuzzy number P is designed by a pair of bounded functions $P^L(\alpha), P^R(\alpha), \alpha \in [0,1]$ where P^L is monotone increasing, left continuous and P^R is monotone decreasing, right continuous with $P^L(\alpha) \leq P^R(\alpha)$.

A trapezoidal fuzzy number is displayed by $P = (m_0, n_0, \gamma, \delta)$ where $[m_0, n_0]$ is interval defuzzifier and $\gamma(>0), \delta(>0)$ are respectively left fuzziness, right fuzziness and $(m_0 - \gamma, n_0 + \delta)$ is the support of P. Its membership function is defined as :

$$P(x) = \begin{cases} \frac{1}{\gamma}(x - m_0 + \gamma), \; m_0 - \gamma \leq x \leq m_0, \\ 1, \qquad\qquad x \in [m_0, n_0], \\ \frac{1}{\delta}(n_0 - x + \delta), \; n_0 \leq x \leq n_0 + \delta, \\ 0, \qquad\qquad \text{elsewhere}. \end{cases}$$

In parametric form $P^L(\alpha) = m_0 - \gamma + \gamma\alpha,\ P^R(\alpha) = n_0 + \delta - \delta\alpha$.

2.2 Definition [27]

An NS P over the universe U is defined by a triplet namely truth-membership value μ_P, indeterminacy-membership value ν_P and falsity-membership value η_P where $\mu_P, \nu_P, \eta_P : U \to]^-0, 1^+[$. Thus P is displayed as : $P = \{< x, \mu_P(x), \nu_P(x), \eta_P(x) >: x \in U\}$ with $^-0 \leq \sup \mu_P(x) + \sup \nu_P(x) + \sup \eta_P(x) \leq 3^+$. Here $1^+ = 1 + \varepsilon$, where 1 is its standard part and ε is its non-standard part. Similarly $^-0 = 0 - \varepsilon$, where 0 is its standard part and ε is its non-standard part.

This concept was primarily viewed in philosophical sense. But it is difficult to use NS with value from real standard or nonstandard subset of $]^-0, 1^+[$ in real field. To overcome this, NS with value from the subset of [0,1] is considered.

2.3 Definition [28]

A single valued neutrosophic (SVN) set M over a universe U is an NS where the components of each triplet are real standard elements of $[0,1]$. Thus an SVN-set M is executed as : $M = \{< x, \mu_M(x), \nu_M(x), \eta_M(x) >: x \in U$ and $\mu_M(x), \nu_M(x), \eta_M(x) \in [0,1]\}$ such that $0 \leq \sup \mu_M(x) + \sup \nu_M(x) + \sup \eta_M(x) \leq 3$.

2.4 Definition [11]

Let $p_i, q_i, s_i, t_i \in \mathbf{R}$ (the set of all real numbers) with ordered as $p_i \leq q_i \leq s_i \leq t_i \,(i = 1, 2, 3)$ and $w_{\tilde{m}}, u_{\tilde{m}}, y_{\tilde{m}} \in [0, 1]$. Then a SVN-number $\tilde{m} = \langle([p_1, q_1, s_1, t_1]; w_{\tilde{m}}), ([p_2, q_2, s_2, t_2]; u_{\tilde{m}}), ([p_3, q_3, s_3, t_3]; y_{\tilde{m}})\rangle$ is a special SVN-set on $\mathbf{R}$ whose truth value, indeterminacy value, falsity value are respectively defined by the mappings $\mu_{\tilde{m}} : \mathbf{R} \to [0, w_{\tilde{m}}]$, $\nu_{\tilde{m}} : \mathbf{R} \to [u_{\tilde{m}}, 1]$, $\eta_{\tilde{m}} : \mathbf{R} \to [y_{\tilde{m}}, 1]$ and they are respectively given as :

$$\begin{cases} g^l_\mu(x), & p_1 \leq x \leq q_1, \\ w_{\tilde{m}}, & q_1 \leq x \leq s_1, \\ g^r_\mu(x), & s_1 \leq x \leq t_1, \\ 0, & \text{otherwise.} \end{cases} \quad \begin{cases} g^l_\nu(x), & p_2 \leq x \leq q_2, \\ u_{\tilde{m}}, & q_2 \leq x \leq s_2, \\ g^r_\nu(x), & s_2 \leq x \leq t_2, \\ 1, & \text{otherwise.} \end{cases} \quad \begin{cases} g^l_\eta(x), & p_3 \leq x \leq q_3, \\ y_{\tilde{m}}, & q_3 \leq x \leq s_3, \\ g^r_\eta(x), & s_3 \leq x \leq t_3, \\ 1, & \text{otherwise.} \end{cases}$$

The functions $g^l_\mu : [p_1, q_1] \to [0, w_{\tilde{m}}], g^r_\nu : [s_2, t_2] \to [u_{\tilde{m}}, 1], g^r_\eta : [s_3, t_3] \to [y_{\tilde{m}}, 1]$ are continuous and non-decreasing which satisfy : $g^l_\mu(p_1) = 0, g^l_\mu(q_1) = w_{\tilde{m}}, g^r_\nu(s_2) = u_{\tilde{m}}, g^r_\nu(t_2) = 1, g^r_\eta(s_3) = y_{\tilde{m}}, g^r_\eta(t_3) = 1$. The functions $g^r_\mu : [s_1, t_1] \to [0, w_{\tilde{m}}], g^l_\nu : [p_2, q_2] \to [u_{\tilde{m}}, 1], g^l_\eta : [p_3, q_3] \to [y_{\tilde{m}}, 1]$ are continuous and non-increasing which satisfy : $g^r_\mu(s_1) = w_{\tilde{m}}, g^r_\mu(t_1) = 0, g^l_\nu(p_2) = 1, g^l_\nu(q_2) = u_{\tilde{m}}, g^l_\eta(p_3) = 1, g^l_\eta(q_3) = y_{\tilde{m}}$.

If $[p_1, q_1, s_1, t_1] = [p_2, q_2, s_2, t_2] = [p_3, q_3, s_3, t_3]$ in $\tilde{m}$, it is reduced to a SVTN-number. Thus $\tilde{n} = \langle([p, q, s, t]; w_{\tilde{n}}, u_{\tilde{n}}, y_{\tilde{n}})\rangle$ is a SVTN-number.

2.5 Definition [6]

A neutrosophic set of the form $\tilde{m} = \langle([p_1, q_1, \delta_1, \xi_1]; w_{\tilde{m}}),\ ([p_2, q_2, \delta_2, \xi_2]; u_{\tilde{m}}), ([p_3, q_3, \delta_3, \xi_3]; y_{\tilde{m}})\rangle$ and defined on $\mathbf{R}$ is called a SVN-number. $\delta_i(> 0)$ are the left spreads, $\xi_i(> 0)$ are the right spreads and $[p_i, q_i]$ are the modal intervals for degree of truth, indeterminacy, falsity-membership for $i = 1, 2, 3$ respectively in $\tilde{m}$ and $w_{\tilde{m}}, u_{\tilde{m}}, y_{\tilde{m}} \in [0, 1]$. The three neutrosophic components are designed as :

$$T_{\tilde{m}}(x) = \begin{cases} \frac{1}{\delta_1} w_{\tilde{m}}(x - p_1 + \delta_1), & p_1 - \delta_1 \leq x \leq p_1, \\ w_{\tilde{m}}, & x \in [p_1, q_1], \\ \frac{1}{\xi_1} w_{\tilde{m}}(q_1 - x + \xi_1), & q_1 \leq x \leq q_1 + \xi_1, \\ 0, & \text{elsewhere.} \end{cases}$$

$$I_{\tilde{m}}(x) = \begin{cases} \frac{1}{\delta_2}(p_2 - x + u_{\tilde{m}}(x - m_2 + \delta_2)), & p_2 - \delta_2 \leq x \leq p_2, \\ u_{\tilde{m}}, & x \in [p_2, q_2], \\ \frac{1}{\xi_2}(x - q_2 + u_{\tilde{m}}(q_2 - x + \xi_2)), & q_2 \leq x \leq q_2 + \xi_2, \\ 1, & \text{elsewhere.} \end{cases}$$

$$F_{\tilde{m}}(x) = \begin{cases} \frac{1}{\delta_3}(p_3 - x + y_{\tilde{m}}(x - p_3 + \delta_3)), & p_3 - \delta_3 \leq x \leq p_3, \\ y_{\tilde{m}}, & x \in [p_3, q_3], \\ \frac{1}{\xi_3}(x - q_3 + y_{\tilde{m}}(q_3 - x + \xi_3)), & q_3 \leq x \leq q_3 + \xi_3, \\ 1, & \text{elsewhere.} \end{cases}$$

Here $\tilde{m}$ consists of three pairs $(T^l_{\tilde{m}}, T^u_{\tilde{m}}), (I^l_{\tilde{m}}, I^u_{\tilde{m}}), (F^l_{\tilde{m}}, F^u_{\tilde{m}})$ of bounded and continuous functions so that

(i) $T^l_{\tilde{m}}, I^u_{\tilde{m}}, F^u_{\tilde{m}}$ are monotone non-decreasing and $T^u_{\tilde{m}}, I^l_{\tilde{m}}, F^l_{\tilde{m}}$ are monotone non-increasing.

(ii) $T^l_{\tilde{m}}(r) \leq T^u_{\tilde{a}}(r), I^l_{\tilde{m}}(r) \geq I^u_{\tilde{m}}(r), F^l_{\tilde{m}}(r) \geq F^u_{\tilde{a}}(m), r \in [0, 1]$.

A SVN-number $\tilde{m}$ is transformed into a SVTN-number when three modal intervals in $\tilde{m}$ are all equal. Thus $\tilde{q} = \langle([m_0, n_0, \delta_1, \xi_1]; w_{\tilde{q}}),\ ([m_0, n_0, \delta_2, \xi_2]; u_{\tilde{q}}),\ ([m_0, n_0, \delta_3, \xi_3]; y_{\tilde{q}})\rangle$ is a SVTN-number.

The truth, indeterminacy and falsity-membership values of a SVTN-number differ with respect to their corresponding height only by Definition 2.4. But to compare the various SVTN-numbers in a more flexible way, both supports (i.e. the bases of trapeziums) and heights of neutrosophic components are allowed to differ in Definition 2.5. So, it is the more generalisation of Definition 2.4.

3 Ranking technique of SVTN-number

The score value of SVTN-number is evaluated here from geometrical view and its properties are studied. Then a linear ranking function is defined with this score value.

3.1 Definition

The maximum heights of three neutrosophic components of a SVTN-number (proposed in Definition 2.5) are all taken as one (i.e., 1) to have a linear ranking function. Thus three components of a SVTN-number $\tilde{p} = \langle [p, q, \delta_1, \xi_1], [p, q, \delta_2, \xi_2], [p, q, \delta_3, \xi_3] \rangle$ are designed respectively as follows.

$$\mu_{\tilde{p}}(y) = \begin{cases} \frac{1}{\delta_1}(y - p + \delta_1), & p - \delta_1 \leq y \leq p, \\ 1, & x \in [p, q], \\ \frac{1}{\xi_1}(q - y + \xi_1), & q \leq y \leq q + \xi_1, \\ 0, & \text{elsewhere} . \end{cases}$$

$$\nu_{\tilde{p}}(y) = \begin{cases} \frac{1}{\delta_2}(p - y), & p - \delta_2 \leq y \leq p, \\ 0, & y \in [p, q], \\ \frac{1}{\xi_2}(y - q), & q \leq y \leq q + \xi_2, \\ 1, & \text{elsewhere.} \end{cases}$$

$$\eta_{\tilde{p}}(y) = \begin{cases} \frac{1}{\delta_3}(p - y), & p - \delta_3 \leq y \leq p, \\ 0, & x \in [p, q], \\ \frac{1}{\xi_3}(y - q), & q \leq y \leq q + \xi_3, \\ 1, & \text{elsewhere.} \end{cases}$$

Consider two SVTN-numbers $\tilde{a} = \langle [a, b, \omega_1, \lambda_1], [a, b, \omega_2, \lambda_2], [a, b, \omega_3, \lambda_3] \rangle$ and $\tilde{c} = \langle [c, d, \xi_1, \kappa_1], [c, d, \xi_2, \kappa_2], [c, d, \xi_3, \kappa_3] \rangle$. Then,

(i) Addition :
$\tilde{a} + \tilde{c} = \langle [a + c, b + d, \omega_1 + \xi_1, \lambda_1 + \kappa_1], [a + c, b + d, \omega_2 + \xi_2, \lambda_2 + \kappa_2], [a + c, b + d, \omega_3 + \xi_3, \lambda_3 + \kappa_3] \rangle$.

(ii) Scalar multiplication : For any real number x,
$x\tilde{a} = \langle [xa, xb, x\omega_1, x\lambda_1], [xa, xb, x\omega_2, x\lambda_2], [xa, xb, x\omega_3, x\lambda_3] \rangle$ if $x > 0$.
$x\tilde{a} = \langle [xb, xa, -x\lambda_1, -x\omega_1], [xb, xa, -x\lambda_1, -x\omega_2], [xb, xa, -x\lambda_1, -x\omega_3] \rangle$ if $x < 0$.

(iii) If $a = b = \omega_i = \lambda_i = 0$ for all i in $\tilde{a}$, then it is called a zero SVTN-number and is denoted by
$\tilde{0} = \langle [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0] \rangle$.

3.1.1 Product of two SVTN-numbers

Let $\tilde{a} = \langle [a, b, \omega_1, \lambda_1], [a, b, \omega_2, \lambda_2], [a, b, \omega_3, \lambda_3] \rangle$ and $\tilde{c} = \langle [c, d, \kappa_1, \zeta_1], [c, d, \kappa_2, \zeta_2], [c, d, \kappa_3, \zeta_3] \rangle$ be two $svtn$-numbers. Their product $\tilde{a} \cdot \tilde{c}$ is defined as :

$$\begin{aligned} \tilde{a} \cdot \tilde{c} &= \langle [ac,\, bd,\, a\kappa_1 + c\omega_1 - \omega_1\kappa_1,\, b\zeta_1 + d\lambda_1 + \lambda_1\zeta_1], \\ &\quad [ac,\, bd,\, a\kappa_2 + c\omega_2 - \omega_2\kappa_2,\, b\zeta_2 + d\lambda_2 + \lambda_2\zeta_2], \\ &\quad [ac,\, bd,\, a\kappa_3 + c\omega_3 - \omega_3\kappa_3,\, b\zeta_3 + d\lambda_3 + \lambda_3\zeta_3] \rangle \\ &\quad \text{when} \quad b + \lambda_i > 0 \quad \text{and} \quad d + \zeta_i > 0,\ \forall i = 1, 2, 3. \end{aligned} \tag{1}$$

$$\begin{aligned} \tilde{a} \cdot \tilde{c} &= \langle [ad,\, bc,\, -a\zeta_1 + d\omega_1 + \omega_1\zeta_1,\, -b\kappa_1 + c\lambda_1 - \lambda_1\kappa_1], \\ &\quad [ad,\, bc,\, -a\zeta_2 + d\omega_2 + \omega_2\zeta_2,\, -b\kappa_2 + c\lambda_2 - \lambda_2\kappa_2], \\ &\quad [ad,\, bc,\, -a\zeta_3 + d\omega_3 + \omega_3\zeta_3,\, -b\kappa_3 + c\lambda_3 - \lambda_3\kappa_3] \rangle \\ &\quad \text{when} \quad b + \lambda_i < 0, \quad \text{but} \quad d + \zeta_i > 0,\ \forall i = 1, 2, 3. \end{aligned} \tag{2}$$

$$\begin{aligned} \tilde{a} \cdot \tilde{c} &= \langle [bd,\, ac,\, -b\zeta_1 - d\lambda_1 - \lambda_1\zeta_1,\, -a\kappa_1 - c\omega_1 + \omega_1\kappa_1], \\ &\quad [bd,\, ac,\, -b\zeta_2 - d\lambda_2 - \lambda_2\zeta_2,\, -a\kappa_2 - c\omega_2 + \omega_2\kappa_2], \\ &\quad [bd,\, ac,\, -b\zeta_3 - d\lambda_3 - \lambda_3\zeta_3,\, -a\kappa_3 - c\omega_3 + \omega_3\kappa_3] \rangle \\ &\quad \text{when} \quad b + \lambda_i < 0 \quad \text{and} \quad d + \zeta_i < 0,\ \forall i = 1, 2, 3. \end{aligned} \tag{3}$$

Suppose $b + \lambda_3 < 0, d + \zeta_3 > 0$ only but $b + \lambda_k > 0, d + \zeta_k > 0$ or $b + \lambda_k < 0, d + \zeta_k < 0$ for $k = 1, 2$ (i.e., others keep same sign) then the product is also defined from above as the components in neutrosophic triplet are independent in nature. More precisely, the product of falsity components in $\tilde{a} \cdot \tilde{c}$ follows the 2nd rule whereas the product of truth and indeterminacy components in $\tilde{a} \cdot \tilde{c}$ follow either 1st rule or 3rd rule.

3.1.2 Geometrical representation of SVTN-number

The SVTN-number in different looks and their comparison are now presented geometrically by means of Definition 2.4 (Figure 1), Definition 2.5 (Figure 2), Definition 3.1 (Figure 3) respectively. In the present study,

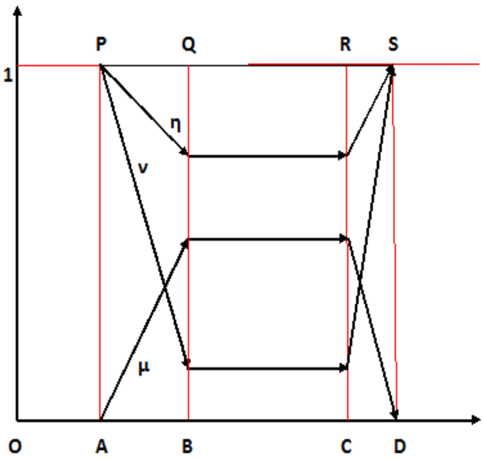

Figure 1 : SVTN-number

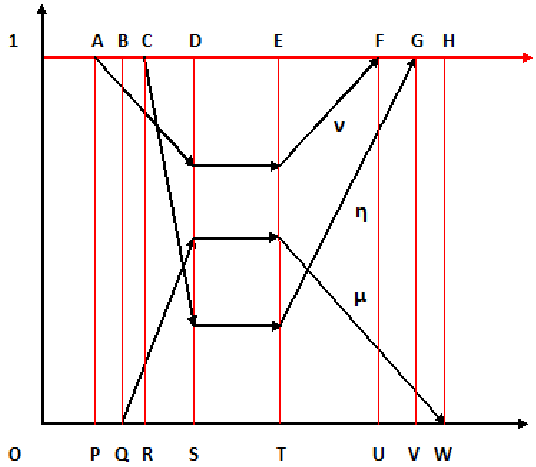

Figure 2 : SVTN-number

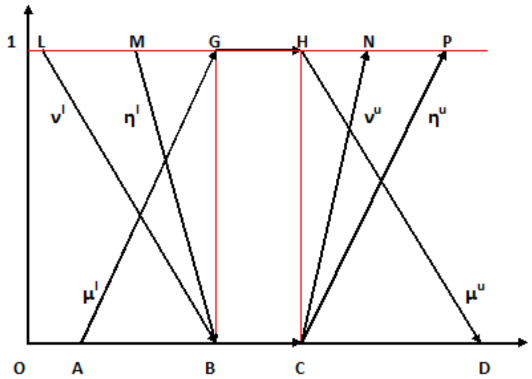

Figure 3 : SVTN-number

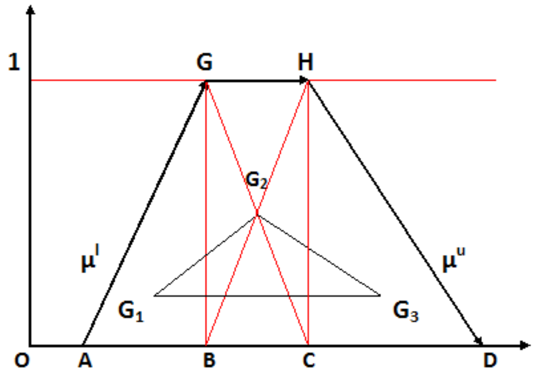

Figure 4 : Truth value

the components of neutrosophic triplet differ by their supports only. Figure 4 represents the truth value of SVTN-number alone by Definition 3.1.

3.2 Score of SVTN-number

Let, Figure 4 represent a SVTN-number $\tilde{p} = \langle [p, q, \delta_1, \zeta_1], [p, q, \delta_2, \zeta_2], [p, q, \delta_3, \zeta_3] \rangle$. Let the trapeziums $AGHD, LBCN$ and $MBCP$ correspond respectively the truth value, indeterminacy value and falsity value. Two perpendicular lines BG and CH are drawn. Then each trapezium is divided into two triangles and one rectangle/square. Now let us consider each trapezium separately.

The trapezium $AGHD$ (see Figure 5) consists of two triangles ABG, CHD and one rectangle $BCHG$. With respect to the co-ordinate of vertices of trapezium $AGHD$ corresponding to $\tilde{p}$, the centroid of triangles ABG and CHD are $G_1(p - \frac{\delta_1}{3}, \frac{1}{3})$ and $G_3(q + \frac{\zeta_1}{3}, \frac{1}{3})$ respectively. Also the centroid of rectangle $BCHG$ is the intersecting point G_2 of its two diagonals i.e., $G_2(\frac{p+q}{2}, \frac{1}{2})$. For contrary, suppose G_1, G_2, G_3 are collinear. Then the area of triangle formed with these points as vertices will be zero i.e.,

$$\frac{1}{3}(\frac{p+q}{2} - q - \frac{\zeta_1}{3}) + \frac{1}{2}(q + \frac{\zeta_1}{3} - p + \frac{\delta_1}{3}) + \frac{1}{3}(p - \frac{\delta_1}{3} - \frac{p+q}{2}) = 0 \tag{4}$$
$$\Rightarrow \quad \delta_1 + \zeta_1 = 3(p-q) < 0, \text{ (using Def. 3.1)}$$

It is a contradiction to the hypothesis that $\delta_1 + \zeta_1 > 0$. Hence G_1, G_2, G_3 are non-collinear and a triangle can be formed with these points as vertices. The centroid of this triangle is $G'(\frac{9p+9q-2\delta_1+2\zeta_1}{18}, \frac{7}{18})$. The centroid points G_1, G_2, G_3 are the balancing points for triangle ABG, rectangle $BCHG$ and triangle CHD respectively. But as the centroid G' is much more balancing point for the two triangles and one rectangle as a whole, so this point is taken to construct the ranking function.

The trapezium $LBCN$ consists of two triangles LBG, HCN and one rectangle $BCHG$. With respect to the co-ordinate of vertices of trapezium $LBCN$ corresponding to $\tilde{p}$, the centroid of the triangles LBG and HCN are $G_4(p-\frac{\delta_2}{3},\frac{2}{3})$ and $G_5(q+\frac{\zeta_2}{3},\frac{2}{3})$ respectively. The centroid of the rectangle $BCHG$ is $G_2(\frac{p+q}{2},\frac{1}{2})$. Here G_4, G_2, G_5 are non-collinear for the same fact stated above. So the centroid $G''(\frac{9p+9q-2\delta_2+2\zeta_2}{18},\frac{11}{18})$ of the triangle with vertices G_4, G_2, G_5 is taken to construct the ranking function.

Finally, the trapezium $MBCP$ consists of two triangles MBG, PCH and the rectangle $BCHG$. With respect to the co-ordinate of vertices of trapezium $MBCP$ corresponding to $\tilde{p}$, the centroid of the triangles MBG and PCH are $G_6(p-\frac{\delta_3}{3},\frac{2}{3})$ and $G_7(q+\frac{\zeta_3}{3},\frac{2}{3})$ respectively. By similar argument as above, the centroid $G'''(\frac{9p+9q-2\delta_3+2\zeta_3}{18},\frac{11}{18})$ of a triangle of vertices G_6, G_2, G_7 is considered to form the ranking function.

We now define the score of $\tilde{p}$ corresponding to truth value, indeterminacy value and falsity value respectively as :

$S_\mu(\tilde{p})=\frac{7}{18}(\frac{9p+9q-2\delta_1+2\zeta_1}{18})$, $S_\nu(\tilde{p})=\frac{11}{18}(\frac{9p+9q-2\delta_2+2\zeta_2}{18})$, $S_\eta(\tilde{p})=\frac{11}{18}(\frac{9p+9q-2\delta_3+2\zeta_3}{18})$.

For an arbitrary parameter γ lying in $[0,1]$ and for any natural number n, the γ-weighted score function of $\tilde{p}$ is denoted by $S_\gamma(\tilde{p})$ and is defined as :

$$\begin{aligned} S_\gamma(\tilde{p}) &= S_\mu(\tilde{p})\gamma^n + S_\nu(\tilde{p})(1-\gamma^n) + S_\eta(\tilde{p})(1-\gamma^n) \\ &= \frac{1}{324}[7(9p+9q-2\delta_1+2\zeta_1)\gamma^n + 11(9p+9q-2\delta_2+2\zeta_2)(1-\gamma^n) \\ &\quad +11(9p+9q-2\delta_3+2\zeta_3)(1-\gamma^n)] \end{aligned} \tag{5}$$

3.2.1 Proposition

The γ-weighted score of a SVTN-number obeys the following the norms.

(i) It is linear i.e., $S_\gamma(\tilde{c}\pm\tilde{d})=S_\gamma(\tilde{c})\pm S_\gamma(\tilde{d})$ and $S_\gamma(\pi\tilde{c})=\pi S_\gamma(\tilde{c})$, π being any real number and $\tilde{c},\tilde{d}$ are two SVTN-numbers.

(ii) $S_\gamma(\tilde{d}-\tilde{d})=S_\gamma(\tilde{0})$.

(iii) $S_\gamma(\tilde{c})$ is monotone increasing or decreasing or constant according as $S_\mu(\tilde{c}) >$ $S_\nu(\tilde{c})+S_\eta(\tilde{c})$ or $S_\mu(\tilde{c}) < S_\nu(\tilde{c})+S_\eta(\tilde{c})$ or $S_\mu(\tilde{c}) = S_\nu(\tilde{c})+S_\eta(\tilde{c})$ respectively.

Proof. (i) Let $\tilde{c}=\langle[a,b,\omega_1,\lambda_1],[a,b,\omega_2,\lambda_2],[a,b,\omega_3,\lambda_3]\rangle$ and $\tilde{d}=\langle[x,y,\kappa_1,\zeta_1],[x,y,\kappa_2,\zeta_2],[x,y,\kappa_3,\zeta_3]\rangle$ be two SVTN-numbers. Then,

$-\tilde{d}=\langle[-y,-x,\zeta_1,\kappa_1],[-y,-x,\zeta_2,\kappa_2],[-y,-x,\zeta_3,\kappa_3]\rangle$

$\tilde{c}+\tilde{d}=\langle[a+x,b+y,\omega_1+\kappa_1,\lambda_1+\zeta_1],[a+x,b+y,\omega_2+\kappa_2,\lambda_2+\zeta_2],[a+x,b+y,\omega_3+\kappa_3,\lambda_3+\zeta_3]\rangle$,

$\tilde{c}-\tilde{d}=\langle[a-y,b-x,\omega_1+\zeta_1,\lambda_1+\kappa_1],[a-y,b-x,\omega_2+\zeta_2,\lambda_2+\kappa_2],[a-y,b-x,\omega_3+\zeta_3,\lambda_3+\kappa_3]\rangle$;

Now,

$S_\gamma(\tilde{c})=\frac{1}{324}[7(9a+9b-2\omega_1+2\lambda_1)\gamma^n+11(9a+9b-2\omega_2+2\lambda_2)(1-\gamma^n)+11(9a+9b-2\omega_3+2\lambda_3)(1-\gamma^n)]$,

$S_\gamma(\tilde{d})=\frac{1}{324}[7(9x+9y-2\kappa_1+2\zeta_1)\gamma^n+11(9x+9y-2\kappa_2+2\zeta_2)(1-\gamma^n)+11(9x+9y-2\kappa_3+2\zeta_3)(1-\gamma^n)]$,

$S_\gamma(\tilde{c}+\tilde{d})=\frac{1}{324}[7(9\overline{a+x}+9\overline{b+y}-2\overline{\omega_1+\kappa_1}+2\overline{\lambda_1+\zeta_1})\gamma^n+11(9\overline{a+x}+9\overline{b+y}-2\overline{\omega_2+\kappa_2}+2\overline{\lambda_2+\zeta_2})(1-\gamma^n)+11(9\overline{a+x}+9\overline{b+y}-2\overline{\omega_3+\kappa_3}+2\overline{\lambda_3+\zeta_3})(1-\gamma^n)]$,

$S_\gamma(\tilde{c}-\tilde{d})=\frac{1}{324}[7(9\overline{a-y}+9\overline{b-x}-2\overline{\omega_1+\zeta_1}+2\overline{\lambda_1+\kappa_1})\gamma^n+11(9\overline{a-y}+9\overline{b-x}-2\overline{\omega_2+\zeta_2}+2\overline{\lambda_2+\kappa_2})(1-\gamma^n)+11(9\overline{a-y}+9\overline{b-x}-2\overline{\omega_3+\zeta_3}+2\overline{\lambda_3+\kappa_3})(1-\gamma^n)]$;

Hence the result is.

(ii) Here, $-\tilde{d}=\langle[-y,-x,\zeta_1,\kappa_1],[-y,-x,\zeta_2,\kappa_2],[-y,-x,\zeta_3,\kappa_3]\rangle$.

$\tilde{d}-\tilde{d}=\langle[x-y,y-x,\kappa_1+\zeta_1,\zeta_1+\kappa_1],[x-y,y-x,\kappa_2+\zeta_2,\zeta_2+\kappa_2],[x-y,y-x,\kappa_3+\zeta_3,\zeta_3+\kappa_3]\rangle$;

$S_\gamma(\tilde{d}-\tilde{d})=\frac{1}{324}[7(9\overline{x-y}+9\overline{y-x}-2\overline{\kappa_1+\zeta_1}+2\overline{\zeta_1+\kappa_1})\gamma^n+11(9\overline{x-y}+9\overline{y-x}-2\overline{\kappa_2+\zeta_2}+2\overline{\zeta_2+\kappa_2})(1-\gamma^n)+11(9\overline{x-y}+9\overline{y-x}-2\overline{\kappa_3+\zeta_3}+2\overline{\zeta_3+\kappa_3})(1-\gamma^n)]=0=S_\gamma(\tilde{0})$;

This ends (ii).

(iii)

$$\begin{aligned} S_\gamma(\tilde{c}) &= \gamma^n S_\mu(\tilde{c})+(1-\gamma^n)S_\nu(\tilde{c})+(1-\gamma^n)S_\eta(\tilde{c}) \\ \frac{dS_\gamma(\tilde{c})}{d\gamma} &= n\gamma^{n-1}[S_\mu(\tilde{c})-(S_\nu(\tilde{c})+S_\eta(\tilde{c}))] \end{aligned}$$

$\frac{dS_\gamma(\tilde{c})}{d\gamma} >,<,=0$ when $[S_\mu(\tilde{c})-(S_\nu(\tilde{c})+S_\eta(\tilde{c}))] >,<,=0$ respectively as $\gamma\geq 0$. This meets the fact.

3.3 Definition

Let SVTN($\mathbf{R}$) be the set of all SVTN-numbers defined over $\mathbf{R}$. For $\gamma \in [0,1]$, a mapping f_γ : SVTN($\mathbf{R}$) $\longrightarrow \mathbf{R}$ is called a ranking function and it is defined as : $f_\gamma(\tilde{c}) = S_\gamma(\tilde{c})$ for $\tilde{c} \in$ SVTN($\mathbf{R}$). The order of $\tilde{x}, \tilde{w} \in$ SVTN($\mathbf{R}$) is defined as :

$S_\gamma(\tilde{x}) > S_\gamma(\tilde{w}) \Leftrightarrow \tilde{x} >_{f_\gamma} \tilde{w}$ (i.e., $\tilde{x} > \tilde{w}$ with respect to f_γ), $S_\gamma(\tilde{x}) < S_\gamma(\tilde{w}) \Leftrightarrow \tilde{x} <_{f_\gamma} \tilde{w}$,
$S_\gamma(\tilde{x}) = S_\gamma(\tilde{w}) \Leftrightarrow \tilde{x} =_{f_\gamma} \tilde{w}$.

4 Linear programming in neutrosophic sense

Here, we shall extend the concept of crisp LP-problem under the neutrosophic environment. First we recall the structure of crisp LP-problem.

$$\begin{aligned} \text{Max } z &= cx \\ \text{such that } Ax &= b,\ x = (x_1, x_2, \cdots, x_n)^t, x_i \geq 0 \end{aligned}$$

where $c = (c_1, c_2, \cdots, c_n), b = (b_1, b_2, \cdots, b_n)^t$ and $A = [p_{ij}]_{m\times n}$ with c_j, b_j, p_{ij} all real.

The concept of LP-problem is now modified by considering the coefficients of the variables in the objective function, the right hand side coefficients in the constraints and in the decision variables regarded as SVTN-numbers. Thus a LP-problem in neutrosophic sense is designed as follows :

$$\begin{aligned} \text{Max } \tilde{z} &=_{f_\gamma} \tilde{c}\tilde{x} \\ \text{such that } \quad A\tilde{x} &=_{f_\gamma} \tilde{b},\ \tilde{x} \geq_{f_\gamma} \tilde{0} \end{aligned} \tag{6}$$

where $\tilde{b} \in (\text{SVTN}(\mathbf{R}))^m, \tilde{x} \in (\text{SVTN}(\mathbf{R}))^n, A \in \mathbf{R}^{m\times n}, \tilde{c}^t \in (\text{SVTN}(\mathbf{R}))^n$ and f_γ is a ranking function.

4.1 Definition

1. $\tilde{x} \in (\text{SVTN}(\mathbf{R}))^n$ satisfying the constraints of (6) is called a feasible solution to (6).

2. If $\tilde{c}\tilde{x}^* \geq_{f_\gamma} \tilde{c}\tilde{x}$ holds for all solutions $\tilde{x}$ to (6), then $\tilde{x}^*$ is an optimal solution to (6).

3. For the modified LP-problem (6), consider $rank(A, \tilde{b}) = rank(A) = m$. The columns of A is partitioned as $[B, N]$ where $B_{m\times m}$ and N are respectively called basis and non-basis matrix. Clearly $rank(B) = m$. Then, a feasible solution $\tilde{x} = (\tilde{x}_B, \tilde{x}_N)^t$ to (6) obtained by setting $\tilde{x}_B =_{f_\gamma} B^{-1}\tilde{b}, \tilde{x}_N =_{f_\gamma} \tilde{0}$ is called a neutrosophic basic feasible solution (NBFS). The component $\tilde{x}_B$ and $\tilde{x}_N$ are respectively called basic variable and nonbasic variable.
4. $\tilde{x}$ is non-degenerate NBFS when all components of $\tilde{x}_B >_{f_\gamma} \tilde{0}$. For $\tilde{x}$ being degenerate NBFS, at least one component of $\tilde{x}_B =_{f_\gamma} \tilde{0}$.

4.1.1 Note

In the modified LP-problem (6), let $A = [p_{ij}]_{m\times n} = [p_1, p_2, \cdots, p_n]$ where each $p_k = (p_{1k}, p_{2k}, \cdots, p_{mk})^t$ is m component column vector. Taking partition on the columns of A, let $B_{m\times m}$ be the basis matrix. Suppose $w_k = (w_{1k}, w_{2k}, \cdots, w_{mk})^t$ is a set of m component scalars required to represent any column p_k of A as a linear combination of the column vectors of basis matrix B i.e., $p_k = Bw_k$.

5 Simplex method for modified LP-problem

The modified LP-problem (6) can be put as follows :

$$\begin{aligned} \text{Max } \tilde{z} &=_{f_\gamma} \tilde{c}_B\tilde{x}_B + \tilde{c}_N\tilde{x}_N \\ \text{such that } \quad B\tilde{x}_B + N\tilde{x}_N &=_{f_\gamma} \tilde{b} \\ \tilde{x}_B, \tilde{x}_N &\geq_{f_\gamma} \tilde{0} \end{aligned}$$

where $\tilde{x}_B, \tilde{x}_N, B, N$ all are signified already. We have then,

$$\begin{aligned} & \tilde{x}_B + B^{-1}N\tilde{x}_N =_{f_\gamma} B^{-1}\tilde{b} && (7)\\ \Rightarrow\ & \tilde{c}_B\tilde{x}_B + \tilde{c}_B B^{-1}N\tilde{x}_N =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} \\ \Rightarrow\ & \tilde{z} - \tilde{c}_N\tilde{x}_N + \tilde{c}_B B^{-1}N\tilde{x}_N =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} \\ \Rightarrow\ & \tilde{z} + (\tilde{c}_B B^{-1}N - \tilde{c}_N)\tilde{x}_N =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} && (8)\end{aligned}$$

Assuming $\tilde{x}_N =_{f_\gamma} \tilde{0}$, we get $\tilde{x}_B =_{f_\gamma} B^{-1}\tilde{b}$ by (7) and $\tilde{z} =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b}$ by (8). The LP-problem (6) is thus arranged in the following table (Table 1).

Table 1 : Tabular form of modified LP-problem.

	$\tilde{c}_j$	$\tilde{c}_B$	$\tilde{c}_N$	
	$\tilde{z}$	$\tilde{x}_B$	$\tilde{x}_N$	R.H.S
$\tilde{x}_B$	0	1	$B^{-1}N$	$B^{-1}\tilde{b}$
$\tilde{z}$	1	0	$\tilde{c}_B B^{-1}N - \tilde{c}_N$	$\tilde{c}_B B^{-1}\tilde{b}$

All the required information are met by Table 1 to proceed the simplex method. The cost row in Table 1 is $\tilde{\chi}_j =_{f_\gamma} (\tilde{c}_B B^{-1}p_j - \tilde{c}_j)_{p_j \notin B}$ which implies $\tilde{\chi}_j =_{f_\gamma} (\tilde{z}_j - \tilde{c}_j)$ for non-basic variables.

5.1 Theorem

A non-degenerate NBFS $(\tilde{x}_B, \tilde{x}_N) = (B^{-1}\tilde{b}, \tilde{0})$ is optimal to the modified LP-problem (6) if and only if $\tilde{z}_j - \tilde{c}_j \geq_{f_\gamma} \tilde{0},\ \forall\, j = 1, \cdots, n.$

Proof. Let $\tilde{x}^* = (\tilde{x}_B^t, \tilde{x}_N^t)^t$ be an NBFS to (1) where $\tilde{x}_B = B^{-1}\tilde{b}, \tilde{x}_N = \tilde{0}$. Let $\tilde{z}^*$ be the objective function corresponding to $\tilde{x}^*$. Then $\tilde{z}^* =_{f_\gamma} \tilde{c}_B\tilde{x}_B =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b}$. Let $\tilde{z}$ be the objective function corresponding to another feasible solution $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n]^t$ to LP-problem (6), then $B\tilde{x}_B + N\tilde{x}_N =_{f_\gamma} \tilde{b} =_{f_\gamma} A\tilde{x}$ and objective function is :

$$\tilde{z} \ =_{f_\gamma}\ \tilde{c}_B\tilde{x}_B + \tilde{c}_N\tilde{x}_N =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} - \sum_{p_j \notin B}(\tilde{c}_B B^{-1}p_j - \tilde{c}_j)\tilde{x}_j =_{f_\gamma} \tilde{z}^* - \sum_{p_j \notin B}(\tilde{z}_j - \tilde{c}_j)\tilde{x}_j$$

Clearly, the solution is optimal if and only if $\tilde{z}_j - \tilde{c}_j \geq_{f_\gamma} \tilde{0}\, \forall\, j = 1, \cdots, n.$

5.2 Theorem

For any NBFS to the modified LP-problem (6), if there is some column not in basis for which $\tilde{z}_k - \tilde{c}_k <_{f_\gamma} \tilde{0}$ and $w_{ik} \leq 0$; $i = 1, 2, \cdots, m$, then LP-problem attains an unbounded solution.

Proof. Let $\tilde{x}_B$ be a basic solution for the problem (6). Arranging the constraints,

$$\begin{aligned} & B\tilde{x}_B + N\tilde{x}_N = \tilde{b} \\ \Rightarrow\ & \tilde{x}_B + B^{-1}N\tilde{x}_N = B^{-1}\tilde{b} \\ \Rightarrow\ & \tilde{x}_B + B^{-1}\sum_j (p_j\tilde{x}_j) = B^{-1}\tilde{b},\ p_j s \text{ are the columns of } N \\ \Rightarrow\ & \tilde{x}_B + \sum_j (B^{-1}p_j\tilde{x}_j) = B^{-1}\tilde{b} \\ \Rightarrow\ & \tilde{x}_B + \sum_j (w_j\tilde{x}_j) = \tilde{w}_0,\ \text{for } p_j = Bw_j, p_j \notin B \\ \Rightarrow\ & \tilde{x}_{B_i} + \sum_j (w_{ij}\tilde{x}_j) = \tilde{w}_{i0};\ i = 1, \cdots, m;\ j = 1, \cdots, n \\ \Rightarrow\ & \tilde{x}_{B_i} = \tilde{w}_{i0} - \sum_j (w_{ij}\tilde{x}_j);\ i = 1, \cdots, m;\ j = 1, \cdots, n \end{aligned}$$

If $\tilde{x}_k$ enters into the basis, then $\tilde{x}_k >_{f_\gamma} \tilde{0}$ and $\tilde{x}_j =_{f_\gamma} \tilde{0}$ for $j \neq B_i \cup k$ (B_i being a column of B). Since $w_{ik} \leq 0\, (i = 1, \cdots, m)$, so $\tilde{w}_{i0} - w_{ik}\tilde{x}_k \geq_{f_\gamma} \tilde{0}$. Hence, basic solution remains feasible at present and for that feasible solution, objective function is :

$$\begin{aligned}\tilde{z}^* &=_{f_\gamma} \tilde{c}_B\tilde{x}_B + \tilde{c}_N\tilde{x}_N =_{f_\gamma} \sum_{i=1}^{m} \tilde{c}_{B_i}(\tilde{w}_{i0} - w_{ik}\tilde{x}_k) + \tilde{c}_k\tilde{x}_k \\ &=_{f_\gamma} \sum_{i=1}^{m} \tilde{c}_{B_i}\tilde{w}_{i0} - (\sum_{i=1}^{m} \tilde{c}_{B_i}w_{ik} - \tilde{c}_k)\tilde{x}_k \\ &=_{f_\gamma} \tilde{c}_B\tilde{w}_0 - (\tilde{c}_B y_k - \tilde{c}_k)\tilde{x}_k =_{f_\gamma} \tilde{z} - (\tilde{z}_k - \tilde{c}_k)\tilde{x}_k\end{aligned}$$

Thus $\tilde{z}^* =_{f_\gamma} \tilde{z} - (\tilde{z}_k - \tilde{c}_k)\tilde{x}_k$. It implies $\tilde{z}^* >_{f_\gamma} \tilde{z}$, as $\tilde{z}_k - \tilde{c}_k <_{f_\gamma} \tilde{0}$.
Hence the LP-problem attains an unbounded solution.

5.3 Simplex algorithm for solving modified LP-problem

While applying simplex method to solve an LP-problem studied here, it is always assumed that the initial solution is feasible. It will be optimised through some iterations. Following steps are practiced :

Step 1. For maximization problem, go to Step 1 directly. Otherwise, convert it into a maximization problem by changing the sign of all price vectors $\tilde{c}_j$.

Step 2. Introduce slack variables to convert all '$\leq$' type inequations into equations. Consider the costs of all slack variables to $\tilde{0}$.

Step 3. Calculate a NBFS to the problem of the form $\tilde{x}_B = B^{-1}\tilde{b} = \tilde{w}_0$ and $\tilde{x}_N = \tilde{0}$ and the respective objective function as $\tilde{z} =_{f_\gamma} \tilde{c}_B B^{-1}\tilde{b} =_{f_\gamma} \tilde{c}_B\tilde{w}_0$.

Step 4. Assume $\tilde{\chi}_B =_{f_\gamma} \tilde{z}_B - \tilde{c}_B =_{f_\gamma} \tilde{0}$ for each basic variable and in the present iteration, calculate $\tilde{\chi}_j =_{f_\gamma} \tilde{z}_j - \tilde{c}_j =_{f_\gamma} \tilde{c}_B B^{-1}p_j - \tilde{c}_j$ for each non-basic variable. The present solution will be optimal, if $\tilde{z}_j - \tilde{c}_j \geq_{f_\gamma} \tilde{0}$, $\forall j$.

Step 5. If $\tilde{\chi}_j =_{f_\gamma} \tilde{z}_j - \tilde{c}_j <_{f_\gamma} \tilde{0}$ for some non-basic variables then compute $\tilde{\chi}_k = \min\{\tilde{\chi}_j\}$. If $w_{ik} < 0$ for all $i = 1, \cdots, m$, then the given problem attains unbounded solution and so terminate the iteration. Otherwise determine

$$\frac{\tilde{w}_{r0}}{w_{rk}} = \min\{\frac{\tilde{w}_{i0}}{w_{ik}} : w_{ik} > 0;\ i = 1, \cdots, m\}.$$

to find out the index of the variable $\tilde{x}_{B_r}$ to be removed from the present basis.

Step 6. Modify $\tilde{w}_{i0}$ by replacing $\tilde{w}_{i0} - \frac{\tilde{w}_{r0}}{w_{rk}}w_{ik}$ for $i \neq r$ and $\tilde{w}_{r0}$ by $\frac{\tilde{w}_{r0}}{w_{rk}}$.

Step 7. Develop new basis and perform Step 4, Step 5 repeatedly until the optimality is reached.

Step 8. Find the optimal solution and the optimal value of objective function.

6 Numerical Example

A real life problem is stated here and is solved numerically by use of proposed concept. For simplicity, we define the γ-weighted score function for $n = 1$ in rest of the paper.

6.1 Example

For business purpose, Mr. X wishes to drive his two lorries (L_1, L_2) in two different routes (R_1, R_2). The route R_1 is assigned for the lorry L_1 and route R_2 for L_2. He likes to allow a maximum of Rs. $300\tilde{b}_1$ for fuel charge and at most Rs. $320\tilde{b}_2$ for the salary of staffs in a week. The consumed fuel charge is Rs. 23/hr for L_1 and Rs. 25/hr for L_2. The salary of staffs is estimated Rs. 30/hr for L_1 and Rs. 40/hr for L_2. Such type of variation of fuel charge and salary estimation are due to road condition, mileage of lorry, distance, road tax, time bounds and different business angles. This results a profit approximately Rs. $\tilde{c}_1$ /hr from L_1 and Rs. $\tilde{c}_2$ /hr from L_2. Now suggest him what time can he allow to run his lorries in two routes depending on these criteria so that the maximum profit will be met as a whole in a week.

The problem can be summarised in the following table (Table 2):

Table 2 : Summarisation of Example 6.1

Expenditure $\Downarrow$	Route : R_1	R_2	Available cost / week $\Downarrow$
Fuel charge	Rs. 23/hr	Rs. 25/hr	Rs. $300\tilde{b}_1$
Staff salary	Rs. 30/hr	Rs. 40/hr	Rs. $320\tilde{b}_2$
Profit/hr $\Rightarrow$	Rs. $\tilde{c}_1$	Rs. $\tilde{c}_2$	

Let the lorry L_1 will run for $\tilde{x}_1$ hr in route R_1 and the lorry L_2 will run for $\tilde{x}_2$ hr in route R_2 in a week. The problem will be then designed as follows :

$$\begin{aligned} \text{Max } \tilde{z} \quad &=_{f_\gamma} \quad \tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2 \\ \text{such that} \quad & \quad 23\tilde{x}_1 + 25\tilde{x}_2 \leq_{f_\gamma} 300\tilde{b}_1 \\ & \quad 30\tilde{x}_1 + 40\tilde{x}_2 \leq_{f_\gamma} 320\tilde{b}_2 \\ & \quad \tilde{x}_1, \tilde{x}_2 \geq_{f_\gamma} \tilde{0} \end{aligned}$$

It is a modified LP-problem in neutrosophic sense with a pre-assigned $\gamma = 0.4$ where $\tilde{c}_1, \tilde{c}_2, \tilde{b}_1, \tilde{b}_2$ are all SVTN-numbers given as follows :

$$\begin{aligned} \tilde{c}_1 &= \langle [4,7,1,3], [4,7,3,4], [4,7,2,1] \rangle, \\ \tilde{c}_2 &= \langle [6,8,4,10], [6,8,5,1], [6,8,3,2] \rangle, \\ \tilde{b}_1 &= \langle [10,12,3,7], [10,12,6,12], [10,12,4,15] \rangle, \\ \tilde{b}_2 &= \langle [8,22,2,18], [8,22,4,25], [8,22,7,30] \rangle. \end{aligned}$$

Rewriting the given constraints by introducing slack variables,

$$\begin{aligned} & 23\tilde{x}_1 + 25\tilde{x}_2 + \tilde{x}_3 =_{f_\gamma} 300\tilde{b}_1 \\ & 30\tilde{x}_1 + 40\tilde{x}_2 + \tilde{x}_4 =_{f_\gamma} 320\tilde{b}_2 \\ & \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq_{f_\gamma} \tilde{0} \end{aligned}$$

The first revised simplex table is given in the following table (Table 3).

Table 3 : First revised simplex table.

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$\tilde{x}_B \Downarrow$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	R.H.S
$\tilde{x}_3$	23	25	1	0	$300\tilde{b}_1$
$\tilde{x}_4$	30	40	0	1	$320\tilde{b}_2 \rightarrow$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(1)}$	$\tilde{c}_2^{(1)} \uparrow$	$\tilde{c}_3^{(1)}$	$\tilde{c}_4^{(1)}$	

where $\tilde{c}_1^{(1)} = -\tilde{c}_1$, $\tilde{c}_2^{(1)} = -\tilde{c}_2$ and $S_\gamma(\tilde{c}_3^{(1)}) = S_\gamma(\tilde{c}_4^{(1)}) = S_\gamma(\tilde{0})$.

Now $S_\gamma(\tilde{c}_1^{(1)}) = \frac{1}{324}(1457\gamma - 2178) < 0$, $S_\gamma(\tilde{c}_2^{(1)}) = \frac{1}{324}(1696\gamma - 2662) < 0$ and $\tilde{c}_1^{(1)} >_{f_\gamma} \tilde{c}_2^{(1)}$ for $\gamma = 0.4$. So $\tilde{x}_2$ enters in the basis.

Further $S_\gamma(300\tilde{b}_1/25) = \frac{12}{324}(4730 - 3288\gamma)$, $S_\gamma(320\tilde{b}_2/40) = \frac{8}{324}(6908 - 4794\gamma)$. For $\gamma = 0.4$, $(300\tilde{b}_1/25) >_{f_\gamma} (320\tilde{b}_2/40)$ and so the leaving variable is $\tilde{x}_4$. The second revised simplex table is (Table 4):

Table 4 : Second revised simplex table.

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$\tilde{x}_B \Downarrow$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	R.H.S
$\tilde{x}_3$	17/4	0	1	-5/8	$100(3\tilde{b}_1 - 2\tilde{b}_2) \rightarrow$
$\tilde{x}_2$	3/4	1	0	1/40	$8\tilde{b}_2$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(2)} \uparrow$	$\tilde{c}_2^{(2)}$	$\tilde{c}_3^{(2)}$	$\tilde{c}_4^{(2)}$	$8\tilde{b}_2\tilde{c}_2$

where $S_\gamma(\tilde{c}_2^{(2)}) = S_\gamma(\tilde{c}_3^{(2)}) = S_\gamma(\tilde{0})$ and $\tilde{c}_1^{(2)} = \frac{3}{4}\tilde{c}_2 - \tilde{c}_1$, $\tilde{c}_4^{(2)} = \frac{1}{40}\tilde{c}_2$.

Then $S_\gamma(\tilde{c}_1^{(2)}) = \frac{1}{1296}(-726 + 740\gamma)$, $S_\gamma(\tilde{c}_4^{(2)}) = \frac{1}{12960}(2662 - 1696\gamma)$. For $\gamma = 0.4$, clearly $S_\gamma(\tilde{c}_1^{(2)}) < 0$, $S_\gamma(\tilde{c}_4^{(2)}) > 0$. So $\tilde{x}_1$ enters in the basis.

Further $S_\gamma((300\tilde{b}_1 - 200\tilde{b}_2)/\frac{17}{4}) = \frac{100}{1377}(374 - 276\gamma)$, $S_\gamma(8\tilde{b}_2/\frac{3}{4}) = \frac{32}{972}(6908 - 4794\gamma)$. So the leaving variable is $\tilde{x}_3$ for $\gamma = 0.4$. The final revised table is (Table 5):

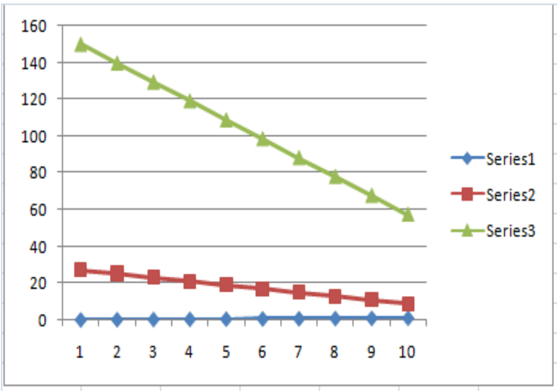

Figure 5 : Graph of run-time measurement

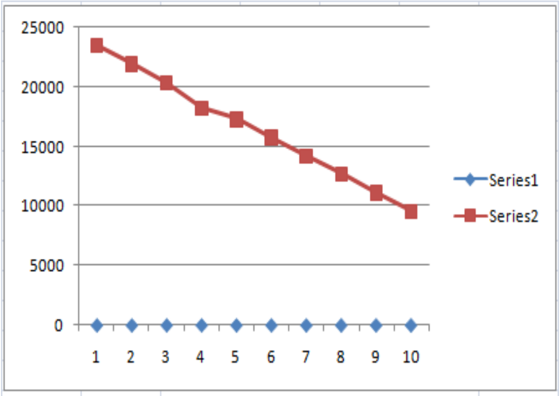

Figure 6 : Profit measurement graph.

Table 5 : Final revised simplex table.

$\tilde{c}_j \Rightarrow$	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{0}$	$\tilde{0}$	
$\tilde{x}_B \Downarrow$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	R.H.S
$\tilde{x}_1$	1	0	4/17	-5/34	$400(3\tilde{b}_1 - 2\tilde{b}_2)/17$
$\tilde{x}_2$	0	1	-3/17	23/170	$(736\tilde{b}_2 - 900\tilde{b}_1)/17$
$\tilde{z} \Rightarrow$	$\tilde{c}_1^{(3)}$	$\tilde{c}_2^{(3)}$	$\tilde{c}_3^{(3)}$	$\tilde{c}_4^{(3)}$	$\frac{400}{17}(3\tilde{b}_1 - 2\tilde{b}_2)\tilde{c}_1 + \frac{1}{17}(736\tilde{b}_2 - 900\tilde{b}_1)\tilde{c}_2$

where $S_\gamma(\tilde{c}_1^{(3)}) = S_\gamma(\tilde{c}_2^{(3)}) = S_\gamma(\tilde{0})$ and $\tilde{c}_3^{(3)} = \frac{1}{17}(4\tilde{c}_1 - 3\tilde{c}_2)$, $\tilde{c}_4^{(3)} = \frac{1}{170}(23\tilde{c}_2 - 25\tilde{c}_1)$. Then $S_\gamma(\tilde{c}_3^{(3)}) = \frac{1}{5508}(726 - 740\gamma) > 0$ and $S_\gamma(\tilde{c}_4^{(3)}) = \frac{1}{55080}(6776 - 2583\gamma) > 0$ for $\gamma = 0.4$.

Thus the optimality arises. The optimal solution is : $\tilde{x}_1 = 400(3\tilde{b}_1 - 2\tilde{b}_2)/17, \tilde{x}_2 = (736\tilde{b}_2 - 900\tilde{b}_1)/17$ and so, Max $\tilde{z} =_{f_\gamma} \frac{400}{17}(3\tilde{b}_1 - 2\tilde{b}_2)\tilde{c}_1 + \frac{1}{17}(736\tilde{b}_2 - 900\tilde{b}_1)\tilde{c}_2$.

6.1.1 Result and discussion

At optimality stage, different optimal values of Example 6.1 for different γ is displayed in the following table (Table 6).

Here $S_\gamma(\tilde{x}_1) = \frac{100}{1377}(374 - 276\gamma)$, $S_\gamma(\tilde{x}_2) = \frac{2}{1377}(103411 - 71148\gamma)$ and $S_\gamma(\tilde{z}) = \frac{1}{324}(7605114 - 5014498\gamma)$.

Table 6 : Optimal values for different γ.

γ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\tilde{x}_1$	27.16	25.16	23.15	21.15	19.14	17.14	15.13	13.13	11.13	9.12
$\tilde{x}_2$	150.2	139.86	129.53	119.2	108.86	98.53	88.19	77.86	67.53	57.19
$\tilde{z}$	23473	21925	20377	18230	17282	15734	14186	12639	11091	9543

The value for $\gamma = 1$ is excluded in Table 6, as it terminates the iteration in Table 4. There $S_\gamma(\tilde{c}_1^{(2)}) = \frac{1}{1296}(-726 + 740\gamma) > 0$ for $\gamma = 0.9811$ approximately. It is then suggested to run the lorry L_2 in route R_2 only to meet a profit. Thus, it is clear from Table 6 that the character γ plays a vital role to determine the optimal solution in modified LP-problem. With respect to different γ, it is seen that $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{z}$ are all monotone decreasing functions. This γ is here signified as the level of ruination of road. It is one of the factors determining the profit of owner from the lorry. In Figure 5, Series 2 and Series 3 represent the weekly run time of two lorries L_1 and L_2 respectively. Figure 6 deals the weekly profit (Series 2) of Mr. X. In both graphical presentation, Series 1 measures the level of ruination of road i.e., different γ.

7 Conclusion

The present study deals a modified structure of crisp LP-problem in the parlance of SVTN-number. An approach is taken to solve such problem by developing an efficient algorithm. A new ranking technique plays a key role to develop this algorithm and also to establish some well known theories. The proposed concept is illustrated by solving a real life problem. A discussion of result obtained is performed and is presented graphically.

This concept will assist the industrialists, directors of management institutes, marketing supervisors to manage the various uncertain situations and complexities. They can reach at a fair end as the present notion helps to solve a LP-problem with respect to the provided constraints and its hidden states together. Several linear, non-linear programming problem, multi criteria decision making and also many mathematical frame works may be enlighten by this attempt.

The ranking function is innovated here by taking the maximum height of each trapezium. It may be allowed within [0,1] in future.

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