



# Complex Plithogenic Set

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## Abstract

In the last decade uncertainty analysis in multi-attributes data sets is considered as one of the major issues. Same time another issues arises due to conflict among experts for their multi-valued and its existence. To resolve these issues, the properties of Plithogenic set is somehow considered as helpful for representing the multi-valued attributes and its multiple opposites, non-opposites as well as neutrals values with its contradiction. The problem arises when these values changes at given phase of time. Hence the current paper focused on introducing Complex Plithogenic set and its graphs for knowledge discovery and representation tasks with an illustrative example.

**Keywords:** Complex fuzzy set; Graph Analytics; Knowledge representation; Plithogenic set; Plithogenic Complex Set

## 1. Introduction

The adequate representation of uncertainty and its fluctuation in given data sets is considered as one of the major issues by data science researchers [1]. To resolve this issue complex fuzzy sets [2-3] and its algebra is applied for handling the data with complex fuzzy attributes [4-5]. The complex fuzzy graph representation of concept lattice [1] and its extensive properties given a way to deal with complex data sets and its pattern [5]. One of the reasons is complex fuzzy set given a way to represents the uncertainty using amplitude and its fluctuation using phase term as shown in Figure 1. The problem arises when the given data sets contain several opposites, non-opposites and neutral sides, independently [6]. In this case the contradiction arises among two or more experts while precise representation of uncertainty in the given data sets [7]. The properties of Plithogenic set proposed by Smarandache[8-9] given a way to write those types of multi-attribute data sets with contradiction[10-11]. Recently, several researchers paid attention towards data analysis using Plithogenic set [11-15] and its graphical visualization [16-19]. The problem arises when the uncertainty or contradiction in Plithogenic attributes changes based on given phase of time as observed in neutrosophic attributes [20-22]. This types of data and its representation required extensive studies as discussed in [23]. To deal with this issues, the current paper focused on introducing complex Plithogenic set and its algebra.

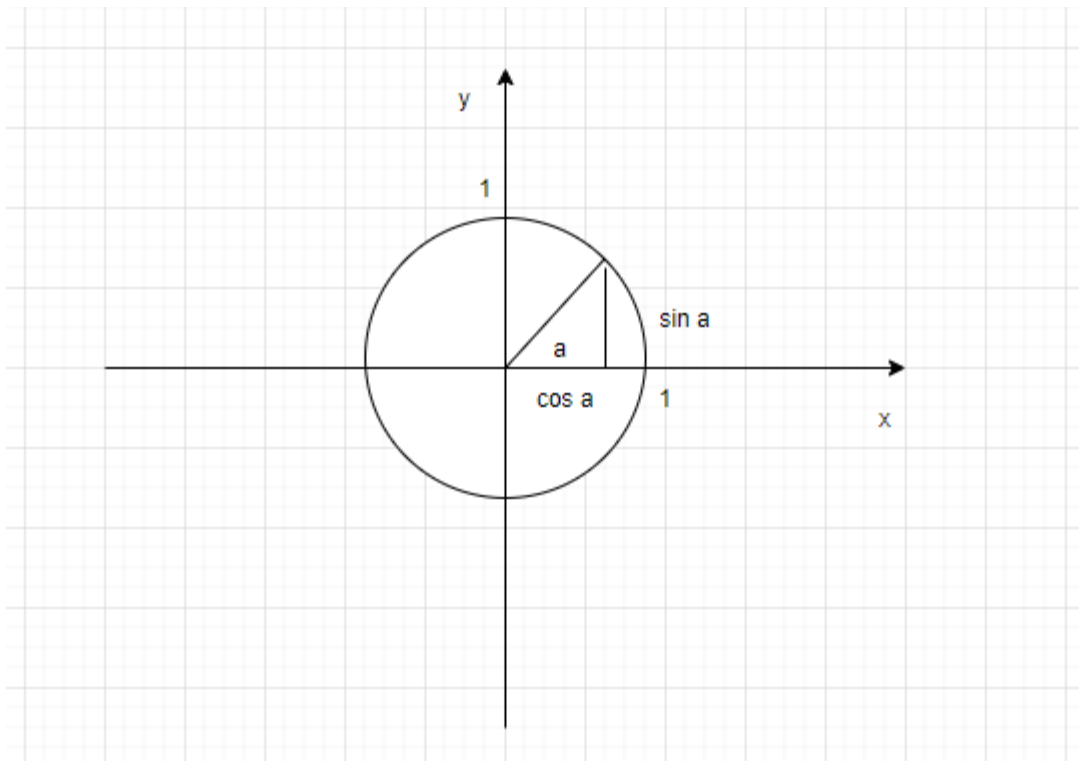


Figure 1: The uncertainty and its fluctuation representation using complex fuzzy set

There are several researchers who paid attention towards dealing the data with complex fuzzy attributes [1-4] and its graphical structure visualization [4-5]. The problem arises while dealing partial ignorance in complex fuzzy attributes beyond the unipolar space [6]. The complex multi-fuzzy context analysis given a way to analyze the multi-attributes and its pattern for knowledge processing tasks. However the problem arises when it contains several opposite, non-opposite or neutral side. In this case, the conflict among two expert may arise while precise representation of complex multi-attribute data sets. To achieve this goal, a method is proposed to represent the complex Plithogenic set with an illustrative example. The motivation is to write uncertainty and fluctuation in Plithogenic set mathematically for decision making process. The objective is to deal to data with complex Plithogenic attributes for knowledge processing tasks and future enhancement.

Other parts of the paper is organized as follows : Section 2 provides basic background about data with Plithogenic set and its algebra. Section 3 contains the proposed method for handling data with Complex Plithogenic attributes with its illustrations in Section 4. Section 5 provides conclusions followed by acknowledgement and references.

## 2. Data with Plithogenic set

In this section, some basic notation related to current paper is given below:

**Definition 1: Neutrosophic Set [20]:** The neutrosophic set consist three independent functions called as truth, indeterminacy and false,  $(T, I, F)$  to represent the uncertainty in attributes. The range of these three independent functions varies between 0 and 1 and mutually exclusive under the conditions  $0 \leq T + I + F \leq 3$ . The Neutrosophic value 0 represents the universal false cases and 3 represents the universal truth cases i.e.  $N = \{ \langle x : T, I, F \rangle : x \in \xi \}$ . It means this set contains triplet having a true,

a false and indeterminacy membership values which can be characterized independently,  $T_N, F_N, I_N$ , independently in  $[0, 1]$  as shown in Figure 2. It can be abbreviated as follows:

$$N = \{ \langle k; T_N(k), I_N(k), F_N(k) \rangle : k \in \xi; T_N(k), I_N(k), F_N(k) \in [0, 1]^+ \} \dots\dots(1)$$

$$\text{Whereas } 0 \leq T_N(k) + I_N(k) + F_N(k) \leq 3^+ \dots\dots\dots(2)$$

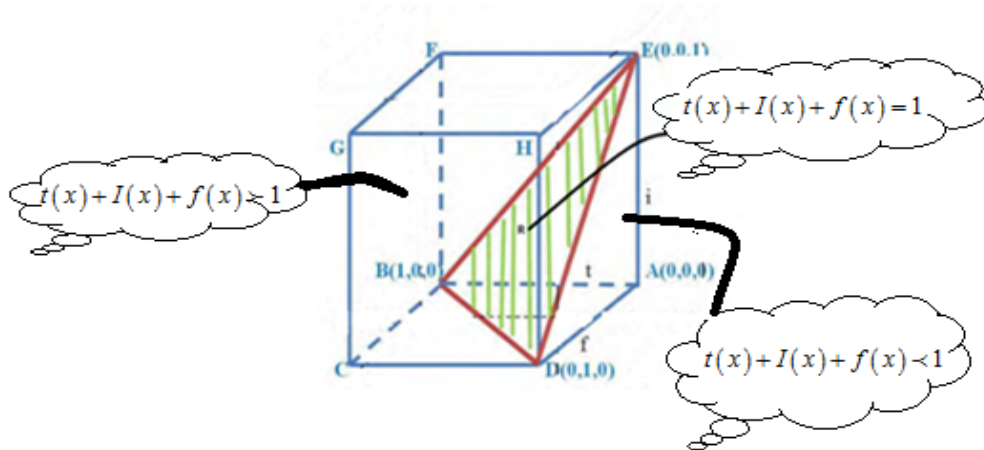


Figure 2. The graphical visualization of neutrosophic environment

**Example 1:** Let us suppose, an expert wants to write the brain drain analysis happened for data science researchers in computer science department. In case 60 percent true data science researchers left the organization, 20 percent data science researchers still working at organization whereas 60 percent partial or uncertain data science researchers due to interdisciplinary research still working in organization. These types of uncertain data can be written precisely using single-valued neutrosophic data sets as  $(0.6, 0.6, 0.2)$ . The problem arises when the experts want precise representation of each expert and its research areas for brain drain analysis. To achieve this goal, the properties of Plithogenic set can be helpful.

**Definition 2. PlithogenicSet [8]:** This set contains five parts to represent the multi-valued attributes of the given data sets. Let us suppose,  $\xi$  be a universe of discourse,  $P$  be a subset of this universe of discourse, “ $a$ ” a multi-valued attribute,  $V$  is the range of the multi-valued attribute, “ $d$ ” be the known (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance with regard to some generic of element  $x$ ’s attribute value to the set  $P$ , and  $c$  is the (fuzzy, intuitionistic fuzzy, neutrosophic) degree of contradiction (dissimilarity) among the attribute values as  $\langle A, \text{Neutral } A, \text{Anti } A \rangle; \langle B, \text{Neutral } B, \text{Anti } B \rangle; \langle C, \text{Neutral } C, \text{Anti } C \rangle$ . It can be represented as a set  $(P, a, V, d, c)$  which named as a Plithogenic Set (**P**). The Plithogenic set is a set **P**  $(P, a, V, d, c)$  in which each element  $x \in P$  is characterized by all attribute’s ( $a$ ) values in  $V = \{v_1, v_2, \dots, v_n\}$ , for  $n \geq 1$  for the degree of appurtenance ( $d$ ). The contradiction degree function ( $c$ ) distinct the Plithogenic set from all of the above set. It represents the between the attribute values in form of fuzzy  $t$ -norm and fuzzy  $t$ -conorm as:

(i)  $c: V \times V \rightarrow [0, 1]$  represents the contradiction degree function among  $v_1$  and  $v_2$ .

It used be noted as  $c(v_1, v_2)$ , and satisfies the following axioms:

(ii)  $c(v_1, v_1) = 0$  i.e. the contradiction among  $v_1$  and  $v_2$  is zero.

(iii)  $c(v_1, v_2) = c(v_2, v_1)$ , the contradiction among  $v_1$  and  $v_2$  or  $v_2$  and  $v_1$  used to be considered as per the commutativity properties. In this paper author focuses on single-valued fuzzy membership to handle the Plithogenic set.

**Example 2:** Let us extend the Example 1, the university wants to analyze the brain drain happened in Computer Science department. To achieve this goal, the university collected feedback of two experts based on resignation happened in the computer science department. The experts consider three major department like Data Science, Machine Learning and Artificial Intelligence left the organization. It is observed that most of the researchers left the organization due to salary increment and promotion criteria. Hence the expert just wanted to write the brain drain based on these criteria. It is well known that management may have contradiction whenever the analysis comes on salary and increment. In this case the Plithogenic set can be useful as shown in Table 1 and Table 2. The entry in Table 1 shows that the expert ( $y_1$ ) agreed that 60 percent brain drain of Data Science researcher, 20 percent brain drain of Machine Learning researcher, 70 percent brain drain of Artificial Intelligence researchers happened due to 80 percent uncertainty in Salary increment and 50 percent uncertainty in Promotion. These data have contradiction with expert ( $y_2$ ) and its analysis as shown in Table 2. The problem arises when the organization wants to analyze the brain drain and its maximum fluctuation in the given phase of time for better understanding. To achieve this goal the neutrosophic cube shown in Figure 2 need to be Squeeze in complex space. To achieve this goal, the extension of Euler formula in third dimension is elaborated in Definition 3.

Table 1: The expert ( $y_1$ ) opinion about brain drain in Computer Science ( $x_1$ ) Departments

Contradiction degree	0	0.33	0.67	0	0.5
Multi-attributes	Data Science	Machine Learning	Artificial Intelligence	Salary Increment	Promotion
Fuzzy degree	0.6	0.2	0.7	0.8	0.5

Table 2: The expert ( $y_2$ ) opinion about brain drain in Computer Science ( $x_1$ ) departments

Contradiction degree	0	0.33	0.67		0	0.5
Multi-attributes	Data Science	Machine Learning	Artificial Intelligence		Salary Increment	Promotion
Fuzzy degree	0.7	0.4	0.6		0.6	0.4

**Definition 3. Three-Dimensional Complex Fuzzy Set** [23-24]: To deal with uncertainty and fluctuation in multi-attribute data sets squeeze of cube shown in Figure 3 is required in complex fuzzy space. Figure 3 shown the squeeze of Figure 2 in unit sphere which can be interpreted using the formula shown in Figure 4.

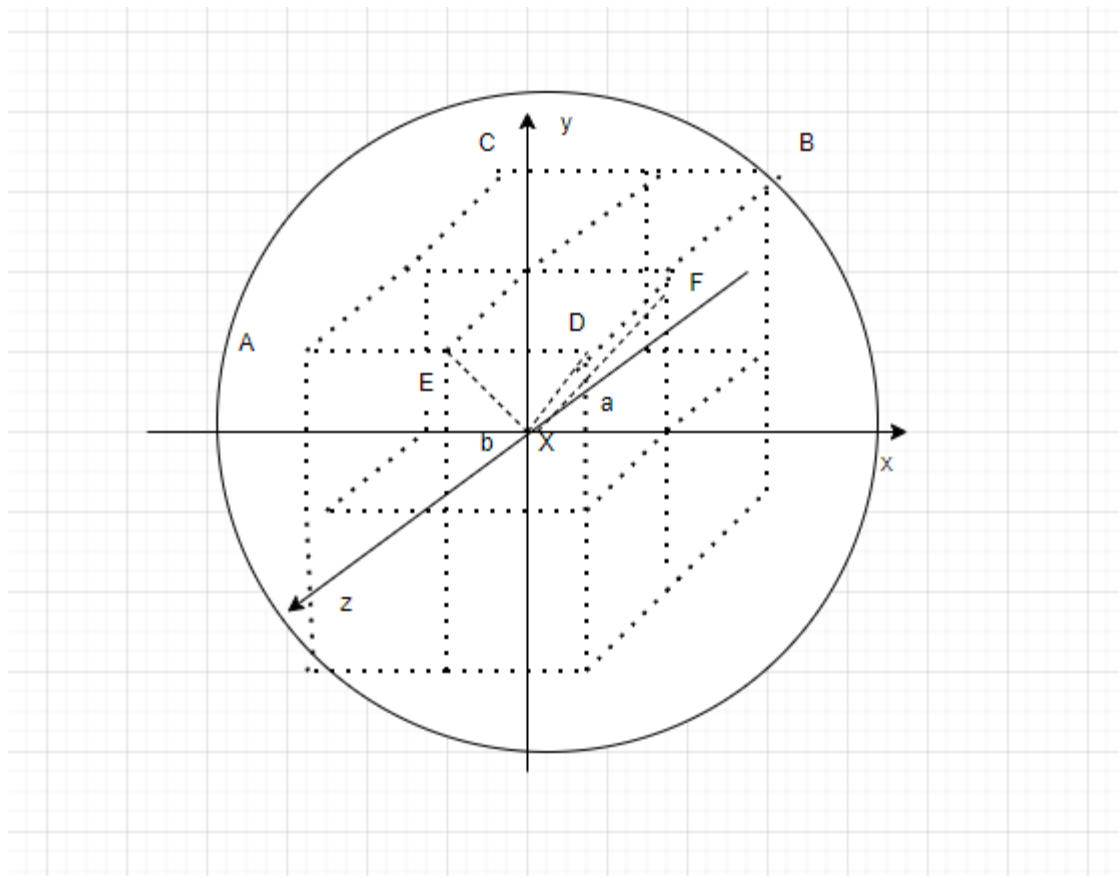


Figure 3: The squeeze of Figure 2 in Unit Sphere

$$\begin{aligned}
 m * \sin a &= y \\
 m * \cos a &= x \\
 n * \sin b &= y \\
 n * \cos b &= z \\
 x^2 + y^2 + z^2 &= 1 \\
 \\
 m &= \frac{\sin b}{\sqrt{(\sin a)^2 + (\cos a)^2 * (\sin b)^2}}
 \end{aligned}$$

Figure 4: The intermediate interpretation of Figure 3

In this way, the data with Plithogenic attributes can be visualized using the complex fuzzy set. In the next section a method is proposed for dealing the Complex Plithogenic set.

**3. A Proposed method for Dealing Data with Complex Plithogenic Set**

This section provides intermediate steps to deal the data with complex Plithogenic attributes as follows:

**Step 1.** Let us consider data with Complex Plithogenic attributes as  $(P, a, V, d, c)$ , where  $P$  is a Plithogenic set,  $a$  is the set of mutli-attribute,  $V$  is the attribute range,  $d$  is the complex fuzzy degree of appurtenance and  $c$  is the degree of contradiction.

**Step 2.** Table 3 shows the complex Plithogenic data set representation in a context.

Table 3: The Representation of Complex Plithogenic Context

Contradiction degree	$c_1$	$c_2$	$c_3$	...	$c_{n-1}$	$c_n$
Multi-attributes	$a_1$	$a_2$	$a_3$	...	$a_{n-1}$	$a_n$
Complex fuzzy degree opinion of expert ( $y_1$ )	$d_1$ ( $a_1 e^{2i w_1}$ )	$d_2$ ( $a_2 e^{2i w_2}$ )	$d_3$ ( $a_3 e^{2i w_3}$ )		$d_{1n}$ ( $a_{n-1} e^{2i w_{n-1}}$ )	$d_n$ ( $a_n e^{2i w_n}$ )
Complex fuzzy degree opinion of expert ( $y_1$ )	$d_1$ ( $a_1 e^{2i w_1}$ )	$d_2$ ( $a_2 e^{2i w_2}$ )	$d_3$ ( $a_3 e^{2i w_3}$ )		$d_{11}$ ( $a_1 e^{2i w_1}$ )	$d_n$ ( $a_n e^{2i w_n}$ )

**Step 3.** The Union among amplitude and phase term for fuzzy degree can be computed, independently as given below:

$$d_{p_1}(a_p, v_p) \vee d_{p_2}(a_p, v_p) = (1 - c_p) \times (d_{p_1}(a_p, v_p) \vee_f d_{p_2}(a_p, v_p)) + c_p(d_{p_1}(a_p, v_p) \wedge_f d_{p_2}(a_p, v_p)) \dots \dots \dots \text{(iii)}$$

**Step 4.** The intersection among amplitude and phase term for fuzzy degree can be computed, independently as given below:

$$d_{p_1}(a_p, v_p) \wedge d_{p_2}(a_p, v_p) = (1 - c_p) \times (d_{p_1}(a_p, v_p) \wedge_f d_{p_2}(a_p, v_p)) + c_p(d_{p_1}(a_p, v_p) \vee_f d_{p_2}(a_p, v_p)) \dots \dots \dots \text{(iv)}$$

**Step 5.** In this way, the union and intersection can be computed for each complex Plithogenic attributes for knowledge processing tasks.

**Time complexity:** Let us suppose, there are  $n$ -number of Complex Plithogenic attributes in the given data set. In this case, the time complexity to compute the Union and Intersection may take  $O(n.m)$  for amplitude and phase term, independently. The degree of contradiction and its computation may take maximum  $O(m.n^2)$  or  $(n.m^2)$  for each data sets for knowledge processing tasks.

#### 4. Illustration

Recently, several researchers paid attention towards dealing the data with Plithogenic sets [8-15] and its graphical visualization [16-19] for knowledge processing tasks. In this process, a problem is addressed while precise measurement of uncertainty and fluctuation in Plithogenic attributes. To achieve this goal, Complex Plithogenic set and its algebra of introduced in Section 3 which is illustrated with an example in this section as follows:

**Example 3:** Let us extend the Example 2 discussed in Section 2 that the university management wants to analyze the brain drain happened in the computer science department within 2 years i.e. 2020-2022. In this case the percentage of researchers left the organization can be represented via amplitude term as  $[0, 1]$ . The time 2 years can be represented via phase term as  $2\pi$ . The expert opinion can be written using the proposed method shown in Section 3 as follows:

**Step 1.** Let us suppose the expert ( $v_1$ ) thinks that 60 percent brain drain in data science researchers happened in eight to nine months of year 2020 without any contradiction. It means the phase term will be  $0.7\pi$ . In this way complex degree of appurtenance can be written as  $0.6e^{i0.7\pi}$  shown in Table 4. In similar way other entries can be interpreted shown in Table 4.

**Step 2.** The Table 5 represents the opinion of expert ( $v_2$ ) for the brain drain analysis in computer science department with the contradiction from expert ( $v_1$ ).



Table 4: The expert ( $y_1$ ) opinion towards the player ( $x_1$ ) to get medal in the given olympic

Contra diction degree	0	0 . 3 3	0.67		0	0 . 5
Multi- attribut es	D a t a  S c i e n c e	M a c h i n e  L e a r n i n g	Artific ial Intelli gence		Sal ary Incr eme nt	P r o m o t i o n
Compl ex Fuzzy degree	0 . 6 $e$ i 0 . 7 $\pi$	0 . 2 $e$ i 0 . 4 $\pi$	0.7 $e^{i0.6\pi}$		0.8 $e^{i0.6$ $\pi$	0 . 5 $e$ i 0 . 4 $\pi$

Table 5: The expert (y<sub>2</sub>) opinion towards the player (x<sub>1</sub>) to get medal in the given olympic

Contradiction degree	0	0 · 3 3	0.67		0	0 · 5
Multi-attributes	D a t a  S c i e n c e	M a c h i n e  L e a r n i n g	Artificial Intelligence		Salary Increment	P r o m o t i o n
Fuzzy degree	0 · 7  $e$ i 0 · 6 π	0 · 4  $e$ i 0 · 2 π	0.6 $e^{i0.7π}$		0.6 $e^{i0.8π}$	0 · 4  $e$ i 0 · 5 π

**Step 3.** The union of experts opinion shown in Table 4 and 5 can be computed using the Step 3 of the proposed method shown in Section 3. Table 6 represents the union among complex Plithogenic set for knowledge processing tasks. It shows that the 88 percent Data Science and 57 percent Artificial Intelligence researchers left the organization due to 92 uncertainty and fluctuation in salary increment in the given year without any contradiction whereas 45 percent happened due to uncertainty in Promotion.

Table 6: The computation of union among complex Plithogenic set shown in Table 4 and 5

Contradiction degree	0	0 . 3 3	0.67	0	0 . 5
Multi-attributes	D a t a  S c i e n c e	M a c h i n e  L e a r n i n g	Artifi cial Intelli gence	Sal ary Inc re me nt	P r o m o t i o n
Complex fuzzy degree of Expert( $\gamma_1$ ) opinion	0 . 6 $e$ i 0 . 7 $\pi$	0 . 2 $e$ i 0 . 4 $\pi$	0.7 $e^{i0.6\pi}$	0.8 $e^{i0.6\pi}$	0 . 5 $e$ i 0 . 4 $\pi$
Complex fuzzy degree of Expert( $\gamma_2$ ) opinion	0 . 7 $e$ i 0 . 6 $\pi$	0 . 4 $e$ i 0 . 2 $\pi$	0.6 $e^{i0.7\pi}$	0.6 $e^{i0.8\pi}$	0 . 4 $e$ i 0 . 5 $\pi$
$\gamma_1 \vee_x \gamma_1$	0 .	0 .	0.57 $e^{i0.57\pi}$	0.9 2	0 .

	8	3			$e^{i0.9}$	4
	8	7			$2\pi$	5
	$e$	$e$				$e$
	$i$	$i$				$i$
	0	0				0
	.	.				.
	8	3				4
	8	7				5
	$\pi$	$\pi$				$\pi$

**Step 4.** The intersection of experts opinion shown in Table 4 and 5 can be computed using the Step 4 of the proposed method shown in Section 3. Table 7 represents the intersection among complex Plithogenic set for knowledge processing tasks. It shows that both expert agreed that approximately 50 percent Data Science and Artificial Intelligence researchers left the organization due to 50 percent uncertainty and fluctuation in Salary increment and promotion at the given incremental time period i.e. july.

Table 7: The intersecion among complex Plithogenic set shown in Table 4 and 5

Contradiction degree	0	0 · 3 3	0.67		0	0 · 5
Multi-attributes	D a t a  S c i e n c e	M a c h i n e  L e a r n i n g	Artifi cial Intelli gence		Sal ary Inc re me nt	P r o m o t i o n
Complex fuzzy degree of Expert( $\gamma_1$ ) opinion	0 · 6 $e$ i 0 · 7 $\pi$	0 · 2 $e$ i 0 · 4 $\pi$	0.7 $e^{i0.6\pi}$		0.8 $e^{i0.6\pi}$	0 · 5 $e$ i 0 · 4 $\pi$
Complex fuzzy degree of Expert( $\gamma_1$ ) opinion	0 · 7 $e$ i 0 · ·	0 · 4 $e$ i 0 · ·	0.6 $e^{i0.7\pi}$		0.6 $e^{i0.8\pi}$	0 · 4 $e$ i 0 · ·

	6 π	2 π			5 π
$y_1 \wedge_x y_1$	0 · 4 2  $e$ $i$ 0 · 4 2 π	0 · 2 3  $e$ $i$ 0 · 2 3 π	$0.73$ $e^{i0.73\pi}$	$0.4$ 8 $e^{i0.4}$ 8π	0 · 4 5  $e$ $i$ 0 · 4 5 π

**Step 5.** In this way the proposed method provides an analysis that the Brain drain in the given organization happened maximum in Data Science researchers and Artificial Intelligence researchers due to uncertainty and fluctuation in Salary increment and Promotion. However it is observed less in Machine Learning researchers. It means they left the organization due to some other issues like work pressure or other jobs which need more analysis. It will help the organization for multi-decision process.

It can be observed that the proposed method provides a mathematical modeling to deal the uncertainty and fluctuation in data with Plithogenic attributes. In future the author work will focus on graphical visualization of complex Plithogenic set and its applications in various fields [23-24].

**5. Conclusions**

This paper introduced Complex Plithogenic set and its algebra for knowledge processing tasks with an illustrative example. It can be observed that that the proposed method is distinct from any of the recent approaches on Plithogenic set. In near future, the author will focus on graphical visualization of complex Plithogenic set and its analysis.

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