



## Multiplicative Interpretation of Neutrosophic Cubic Set on B-Algebra

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### Abstract

Purpose of this paper is to interpret the multiplication of neutrosophic cubic set. Here we define the notation of  $\varkappa$ -multiplication of neutrosophic cubic set and study it with the help of neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal and neutrosophic cubic closed normal ideal. We also study  $\varkappa$ -multiplication under homomorphism and cartesian product through significant characteristics.

**Keywords:** B-algebra, Neutrosophic cubic set,  $\varkappa$ -Multiplication, Cartesian product, Homomorphism.

### 1.Introduction

Theory of existing and non-existing value was first introduced by Zadeh [1,2]. Cubic set was defined by Jun et al. [3] in 2012, which was the modern form of interval-valued fuzzy set. Cubic set with the help of subalgebras, ideals and closed ideals of B-algebra was studied by Senapati et al. [4]. After the defining of BCK-algebra and BCI-algebra by Imai et al. [5] and Iseki [6], cubic set through subalgebras and q-ideals in BCK/BCI-algebra was investigated by Jun et al. [7, 8]. Notion of M-subalgebra on G-algebra is introduced and analyzed by Khalid et al. [9]. Interval-valued fuzzy set on B-algebra was studied by Senapati et al. [10,11]. Intuitionistic fuzzy translation and multiplication of G-algebra were deeply studied by Khalid et al. [19]. Neutrosophic cubic set is the extended form of interval valued intuitionistic fuzzy theory with indeterminacy was introduced by Smarandache [12]. Neutrosophic logics and neutrosophic probability gave the new idea of research were interpret by Smarandache [13]. Neutrosophic cubic was introduced by Jun et al. [14]. Neutrosophic cubic point,  $(\alpha, \beta)$ -fuzzy ideals and neutrosophic cubic  $(\alpha, \beta)$ -ideals were analyzed by Gulistan et al. [15]. A new idea of normal ideal and closed normal ideal under neutrosophic cubic set was given and investigated by Khalid et al. [16]. Neutrosophic cubic set was investigated by Jun et al. [17]. PS fuzzy ideals were studied by Priya et al. [18]. Rosenfeld's fuzzy subgroup was studied by Biswas [20]. B-homomorphism was deeply studied by Neggers et al. [21]. Neutrosophic soft cubic subalgebra was extensively studied by Khalid et al. [22]. A B-algebra is an important logical class of algebra was defined by Neggers et al. [23]. T-Neutrosophic Cubic Set was defined and deeply investigated by Khalid et al. [24].

In this paper, we define  $\varkappa$ -multiplication of neutrosophic cubic set and investigate the neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal (NCNID) and neutrosophic cubic closed normal ideal (NCCNID) under  $\varkappa$ -multiplication with the help of P-intersection, P-union etc. We also study the cartesian product and

homomorphism of  $\alpha$ -multiplication of neutrosophic cubic normal ideal ( $\alpha$ MNCNID) and  $\alpha$ -multiplication of neutrosophic cubic closed normal ideal ( $\alpha$ MNCCNID) with important results.

**2. Preliminaries**

Definition 2.1 [19] A nonempty set  $X$  with a constant  $0$  and  $*$  is said to be  $B$ -algebra if it fulfills these conditions:

- 1:  $t * t = 0$ ,
- 2:  $t * 0 = 0$ , for all  $t \in X$ .
- 3:  $(t * t) * t = t * (t * (0 * t)) \forall t, t, t \in X$ .

Definition 2.2 [21] A nonempty subset  $K$  of  $B$ -algebra  $X$  is called a subalgebra of  $Y$  if  $t * t \in K \forall t, t \in K$ , a mapping  $f : X \rightarrow Y$  of  $B$ -algebra is called  $B$ -homomorphism if  $f(t * t) = f(t) * f(t) \forall t, t \in X$ .

Definition 2.3 [1] Let  $X$  be a collection of elements like  $t$ . Then a FS  $J$  in  $X$  is defined as  $J = \{ \langle t, v_j(t) \rangle \mid t \in X \}$ , where  $v_j(t)$  is called the existenceship value of  $t$  in  $J$  and  $v_j(t) \in [0,1]$ .

For a family  $J_i = \{ \langle t, v_{j_i}(t) \rangle \mid t \in X \}$  of FSs in  $X$ , where  $i \in k$  and  $k$  is index set, Then join ( $\vee$ ) and meet ( $\wedge$ ) are as follows:

$$\bigvee_{i \in k} J_i = (\bigvee_{i \in k} v_{j_i})(t) = \sup\{v_{j_i} \mid i \in k\}$$

and

$$\bigwedge_{i \in k} J_i = (\bigwedge_{i \in k} v_{j_i})(t) = \inf\{v_{j_i} \mid i \in k\},$$

respectively,  $\forall t \in X$ .

Definition 2.4 [2] An IVFS  $B$  is of the form  $B = \{ \langle t, \tilde{v}_B(t) \rangle \mid t \in X \}$ , where  $\tilde{v}_B : X \rightarrow D[0,1]$ , here  $D[0,1]$  is the collection of all subintervals of  $[0,1]$ . The intervals  $\tilde{v}_B(t) = [v_B^-(t), v_B^+(t)] \forall t \in X$  denote the degree of existence of  $t$  to the set  $B$ , also  $\tilde{v}_B^c = [1 - v_B^-(t), 1 - v_B^+(t)]$  shows the complement of  $\tilde{v}_B$ .

For a family  $B_i = \{ \langle t, \tilde{v}_{B_i}(t) \rangle \mid t \in X \}$  of IVFSs in  $X$  where  $k$  is an index set and  $i \in k$ , the union  $G = \bigcup_{i \in k} \tilde{v}_{B_i}(t)$  and the intersection  $F = \bigcap_{i \in k} \tilde{v}_{B_i}(t)$  are defined below:

$$G(t) = \text{rsup}\{\tilde{v}_{B_i}(t) \mid i \in k\}$$

and

$$F(t) = \text{rinf}\{\tilde{v}_{B_i}(t) \mid i \in k\},$$

respectively,  $\forall t \in X$ .

Definition 2.5 [20] Consider two elements  $D_1, D_2 \in D[0,1]$ . If  $D_1 = [t_1^-, t_1^+]$  and  $D_2 = [t_2^-, t_2^+]$ , then  $\text{rmax}(D_1, D_2) = [\max(t_1^-, t_2^-), \max(t_1^+, t_2^+)]$  which is denoted by  $D_1 \vee^r D_2$  and  $\text{rmin}(D_1, D_2) = [\min(t_1^-, t_2^-), \min(t_1^+, t_2^+)]$  which is denoted by  $D_1 \wedge^r D_2$ . Thus, if  $D_i = [t_i^-, t_i^+] \in D[0,1]$  for  $i = 1, 2, 3, \dots$ , then we define  $\text{rsup}_i(D_i) = [\sup_i(t_i^-), \sup_i(t_i^+)]$ , i.e.,  $\vee_i^r D_i = [\vee_i t_i^-, \vee_i t_i^+]$ . Similarly we define  $\text{rinf}_i(D_i) = [\inf_i(t_i^-), \inf_i(t_i^+)]$ , i.e.,  $\wedge_i^r D_i = [\wedge_i t_i^-, \wedge_i t_i^+]$ . Now we call  $D_1 \geq D_2 \iff t_1^- \geq t_2^-$  and  $t_1^+ \geq t_2^+$ . Similarly the relations  $D_1 \leq D_2$  and  $D_1 = D_2$  are defined.

Definition 2.6 [19] A fuzzy set  $B = \{ \langle t, v_B(t) \rangle \mid t \in X \}$  is called a fuzzy subalgebra of  $X$  if  $v_B(t * t) \geq \min\{v_B(t), v_B(t)\} \forall t, t \in X$ .

Definition 2.7 [14] Let  $X$  be a nonempty set. A NCS is  $P_k = (B, \Lambda)$ , where  $B = \{ \langle t; B_T(t), B_I(t), B_F(t) \rangle \mid t \in X \}$  is an interval neutrosophic set in  $X$  and  $\Lambda = \{ \langle t; \lambda_T(t), \lambda_I(t), \lambda_F(t) \rangle \mid t \in X \}$  is a neutrosophic set in  $X$ .

Definition 2.8 [3] Let  $U$  be a universe and cubic set in  $U$ , we mean a structure  $\{ t, \bar{v}_A(t), \lambda_A(t) \mid t \in U \}$  in which  $\bar{v}_A$  is an IVF set in  $U$  and  $\lambda_A$  is a fuzzy set in  $U$ . A cubic set  $A = \{ t, \bar{v}_A(t), \lambda_A(t) \mid t \in U \}$  is simply denoted by  $C(U)$ , which is the set of all cubic sets in  $U$ .

Definition 2.9 [3] Let  $C = \{ \langle t, C(t), \lambda(t) \rangle \}$  be a cubic set, where  $C(t)$  is an IVFS in  $Y$ ,  $\lambda(t)$  is a fuzzy set in  $Y$ . Then  $A$  is cubic subalgebra under  $*$  if it fulfills these axioms:

$$C1: C(t * t) \geq \min\{C(t), C(t)\},$$

$$C2: \lambda(t * t) \leq \max\{\lambda(t), \lambda(t)\} \forall t, t \in X.$$

Definition 2.10 [18] A fuzzy set  $B = \{ \langle t, v_B(t) \rangle \mid t \in X \}$  is called a fuzzy ideal of  $X$  if

$$(i) v_B(0) \geq v_B(t),$$

$$(ii) v_B(t) \geq \min\{v_B(t * t), v_B(t)\} \forall t, t \in X.$$

Definition 2.11 [14] For any  $C_i = (A_i, F_i)$ , where  $A_i = \{ \langle t_1; A_{iT}(t), A_{iI}(t), A_{iF}(t) \rangle \mid t \in Y \}$ ,  $F_i = \{ \langle t_1; F_{iT}(t), F_{iI}(t), F_{iF}(t) \rangle \mid t \in Y \}$  for  $i \in k$ , then

$$P\text{-union: } \bigcup_{i \in k} C_i = (\bigcup_{i \in k} A_i, \bigvee_{i \in k} F_i),$$

$$P\text{-intersection: } \bigcap_{i \in k} C_i = (\bigcap_{i \in k} A_i, \bigwedge_{i \in k} F_i),$$

$$R\text{-union: } \bigcup_R C_i = (\bigcup_{i \in k} A_i, \bigwedge_{i \in k} F_i),$$

$$R\text{-intersection: } \bigcap_R C_i = (\bigcap_{i \in k} A_i, \bigvee_{i \in k} F_i).$$

Definition 2.12 [16] A NCS  $R = (R_{T,I,F}, \lambda_{T,I,F})$  of  $X$  is called a NCNID of  $X$  if it fulfills following axioms:

$$N3. R_{T,I,F}(0) \geq R_{T,I,F}(t * \alpha) \text{ and } \lambda_{T,I,F}(0) \leq \lambda_{T,I,F}(t * \alpha),$$

$$N4. R_{T,I,F}(t * \alpha) \geq \min\{R_{T,I,F}((t * \alpha) * (t * \beta)), R_{T,I,F}(t * \beta)\},$$

$$N5. \lambda_{T,I,F}(t * \alpha) \leq \max\{\lambda_{T,I,F}((t * \alpha) * (t * \beta)), \lambda_{T,I,F}(t * \beta)\}, \forall t, t \in X \text{ and } \alpha, \beta \in [0,1].$$

Let  $R = \{R_{T,I,F}, \lambda_{T,I,F}\}$  be a NCS  $X$  then it is called NCCNID of  $X$  if it fulfills  $N4, N5$  and  $N6: R_{T,I,F}(0 * (t * \alpha)) \geq R_{T,I,F}(t * \alpha)$  and  $\lambda_{T,I,F}(0 * (t * \alpha)) \leq \lambda_{T,I,F}(t * \alpha), \forall t \in X$  and  $\alpha \in [0,1]$ .

Definition 2.13 [16] Let  $R = (R_{T,I,F}, \lambda_{T,I,F})$  and  $B = (B_{T,I,F}, \upsilon_{T,I,F})$  are two NCSs of  $X$  and  $Y$  respectively. The Cartesian product  $R \times B = (X \times Y, R_{T,I,F} \times B_{T,I,F}, \lambda_{T,I,F} \times \upsilon_{T,I,F})$  is defined by  $(R_{T,I,F} \times B_{T,I,F})(t * \alpha, t * \beta) = \min\{R_{T,I,F}(t * \alpha), B_{T,I,F}(t * \beta)\}$  and  $(\lambda_{T,I,F} \times \upsilon_{T,I,F})(t * \alpha, t * \beta) = \max\{\lambda_{T,I,F}(t * \alpha), \upsilon_{T,I,F}(t * \beta)\}$ , where  $R_{T,I,F} \times B_{T,I,F} \mid X \times Y \rightarrow D[0,1]$  and  $\lambda_{T,I,F} \times \upsilon_{T,I,F} \mid X \times Y \rightarrow [0,1] \forall (t, t) \in X \times Y$  and  $\alpha, \beta \in [0,1]$ .

Definition 2.14 [16] A neutrosophic cubic subset  $R \times F = (X \times Y, R_{T,I,F} \times F_{T,I,F}, \lambda_{T,I,F} \times \mu_{T,I,F})$  is called a NCNID if satisfies these conditions:

1.  $(R_{T,I,F} \times F_{T,I,F})(0,0) \geq (R_{T,I,F} \times F_{T,I,F})((t * \alpha), (t * \beta))$  and  $(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})((t * \alpha), (t * \beta)) \forall (t, t) \in X \times Y$  and  $\alpha, \beta \in [0,1]$ .
2.  $(R_{T,I,F} \times F_{T,I,F})(t_1 * \alpha, t_1 * \beta) \geq \text{rmin}\{(R_{T,I,F} \times F_{T,I,F})((t_1 * \alpha, t_1 * \beta) * (t_2 * \alpha, t_2 * \beta)), (R_{T,I,F} \times F_{T,I,F})(t_2 * \alpha, t_2 * \beta)\}$ .
3.  $(\lambda_{T,I,F} \times \mu_{T,I,F})(t_1 * \alpha, t_1 * \beta) \leq \max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((t_1 * \alpha, t_1 * \beta)(t_2 * \alpha, t_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})(t_2 * \alpha, t_2 * \beta)\}$  and  $R \times F$  is closed normal ideal if it satisfies 2, 3, and 4.  $(R_{T,I,F} \times F_{T,I,F})((0,0) * (t_1 * \alpha, t_1 * \beta)) \geq (R_{T,I,F} \times F_{T,I,F})(t * \alpha, t * \beta)$  and  $(\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (t * \alpha, t * \beta)) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})(t * \alpha, t * \beta) \forall (t_1, t_1)$  and  $(t_2, t_2) \in X \times Y$  and  $\alpha, \beta \in [0,1]$ .

Definition 2.15 [9] Let  $\tilde{F}_k = (A_{e_i}, \Lambda_{e_i})$  be a neutrosophic soft cubic set, where  $Y$  is subalgebra. Then  $\tilde{F}_k$  is NSCMSU under binary operation  $*$  where  $t_1, t_2 \in Y$  and  $\alpha, \beta \in [0,1]$  if it fulfills these conditions:

$$A_{e_i}^o((t_1 * \alpha) * (t_2 * \beta)) \geq \text{rmin}\{A_{e_i}^o(t_1 * \alpha), A_{e_i}^o(t_2 * \beta)\} \text{ and } \Lambda_{e_i}^o((t_1 * \alpha) * (t_2 * \beta)) \leq \max\{\Lambda_{e_i}^o(t_1 * \alpha), \Lambda_{e_i}^o(t_2 * \beta)\}.$$

### 3. $\gamma$ -Multiplication of Neutrosophic Cubic Normal Ideal and Closed Normal Ideal

Definition 3.1. Let  $H_b = (H_{T,I,F}, \lambda_{T,I,F})$  be a NCS of  $X$  and  $\gamma \in [0,1]$ . An object of the form  $H_b^M = ({}^M_\gamma H_{T,I,F}^{Hb}, {}^M_\gamma \lambda_{T,I,F}^{Hb})$  is called neutrosophic cubic  $\gamma$  multiplication of  $H_b X$  if it fulfills following axioms:

$${}^M_\gamma H_T^{Hb}(x) = \gamma \cdot H_T^{Hb}(x), \quad {}^M_\gamma \lambda_T^{Hb}(x) = \gamma \cdot \lambda_T^{Hb}(x),$$

$${}^M_\gamma H_I^{Hb}(x) = \gamma \cdot H_I^{Hb}(x), \quad {}^M_\gamma \lambda_I^{Hb}(x) = \gamma \cdot \lambda_I^{Hb}(x),$$

$${}^M_\gamma H_F^{Hb}(x) = \gamma \cdot H_F^{Hb}(x), \quad {}^M_\gamma \lambda_F^{Hb}(x) = \gamma \cdot \lambda_F^{Hb}(x).$$

For convinience we use  ${}^M_\gamma H_{T,I,F}^{Hb} = \gamma \cdot H_{T,I,F}^{Hb}(x)$  and  ${}^M_\gamma \lambda_{T,I,F}^{Hb} = \gamma \cdot \lambda_{T,I,F}^{Hb}(x)$ .

Theorem 3.1 A  $\gamma$ -multiplication of NCCNID of  $B$ -algebra  $X$  is also a  $\gamma$ -multiplication of NCMSU of  $X$ .

Proof. Suppose  $H_b = \{H_{T,I,F}, \lambda_{T,I,F}\}$  be a NCCNID of  $X$ , then for any  $t \in X$ , we have  ${}^M_\gamma H_{T,I,F}(0 * (t * \alpha)) = \gamma \cdot H_{T,I,F}(0 * (t * \alpha)) \geq \gamma \cdot H_{T,I,F}(t * \alpha)$  and  ${}^M_\gamma \lambda_{T,I,F}(0 * (t * \alpha)) = \gamma \cdot \lambda_{T,I,F}(0 * (t * \alpha)) \leq \gamma \cdot \lambda_{T,I,F}(t * \alpha)$ . Now by N4, N6, and through proposition 3.3 of article M subalgebra, we know that  ${}^M_\gamma H_{T,I,F}((t * \alpha) * (t * \beta)) = \gamma \cdot H_{T,I,F}((t * \alpha) * (t * \beta)) \geq \gamma \cdot \text{rmin}\{H_{T,I,F}(((t * \alpha) * (t * \beta)) * (0 * (t * \beta))), H_{T,I,F}(0 * (t * \beta))\} = \gamma \cdot \text{rmin}\{H_{T,I,F}(t * \alpha), H_{T,I,F}(0 * (t * \beta))\} \geq \gamma \cdot \text{rmin}\{H_{T,I,F}(t * \alpha), H_{T,I,F}(t * \beta)\} = \text{rmin}\{\gamma \cdot H_{T,I,F}(t * \alpha), \gamma \cdot H_{T,I,F}(t * \beta)\} = \text{rmin}\{{}^M_\gamma H_{T,I,F}(t * \alpha), {}^M_\gamma H_{T,I,F}(t * \beta)\}$  and  ${}^M_\gamma \lambda_{T,I,F}((t * \alpha) * (t * \beta)) = \gamma \cdot \lambda_{T,I,F}((t * \alpha) * (t * \beta)) \leq \gamma \cdot \max\{\lambda_{T,I,F}(((t * \alpha) * (t * \beta)) * (0 * (t * \beta))), \lambda_{T,I,F}(0 * (t * \beta))\} = \gamma \cdot \max\{\lambda_{T,I,F}(t * \alpha), \lambda_{T,I,F}(0 * (t * \beta))\} \leq \gamma \cdot \max\{\lambda_{T,I,F}(t * \alpha), \lambda_{T,I,F}(t * \beta)\} = \max\{\gamma \cdot \lambda_{T,I,F}(t * \alpha), \gamma \cdot \lambda_{T,I,F}(t * \beta)\} = \max\{{}^M_\gamma \lambda_{T,I,F}(t * \alpha), {}^M_\gamma \lambda_{T,I,F}(t * \beta)\}$ . Hence,  $\gamma$ MNCCNID is  $\gamma$ MNCMSU of  $X$ .

Proposition 3.1 Every  $\gamma$ -multiplication of NCCNID is a  $\gamma$ -multiplication NCNID but the converse is not true in general.

Theorem 3.2 The R-intersection of any set of  $\gamma$ MNCNIDs of X is also a  $\gamma$ MNCNID of X.

Proof. Let  $H_i = \{H_{T,I,F}^i, \lambda_{T,I,F}^i\}$ , where  $i \in k$ , be a  $\gamma$ MNCNID of X and  $t, \mathfrak{t} \in X$ . Then

$$\begin{aligned} (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(0) &= \text{rinf} \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i(0) = \text{rinf} H_{T,I,F}^i(0) . \gamma \\ &\geq \text{rinf} H_{T,I,F}^i(t * \alpha) . \gamma = \text{rinf} \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i(t * \alpha) \\ &= (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(t * \alpha) \\ &\Rightarrow (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(0) \geq (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(t * \alpha) \end{aligned}$$

and

$$\begin{aligned} (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(0) &= \text{sup} \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i(0) = \text{sup} \lambda_{T,I,F}^i(0) . \gamma \\ &\leq \text{sup} \lambda_{T,I,F}^i(t * \alpha) . \gamma = \text{sup} \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i(t * \alpha) \\ &= (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(t * \alpha) \\ &\Rightarrow (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(0) \leq (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(t * \alpha), \end{aligned}$$

now

$$\begin{aligned} (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(t * \alpha) &= \text{rinf} \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i(t * \alpha) = \text{rinf} H_{T,I,F}^i(t * \alpha) . \gamma \\ &\geq \text{rinf} \{ \text{rmin} \{ H_{T,I,F}^i((t * \alpha) * (\mathfrak{t} * \beta)), H_{T,I,F}^i(\mathfrak{t} * \beta) \} \} . \gamma \\ &= \text{rmin} \{ \text{rinf} H_{T,I,F}^i((t * \alpha) * (\mathfrak{t} * \beta)) . \gamma, \text{rinf} H_{T,I,F}^i(\mathfrak{t} * \beta) . \gamma \} \\ &= \text{rmin} \{ \text{rinf} \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i((t * \alpha) * (\mathfrak{t} * \beta)), \text{rinf} \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i(\mathfrak{t} * \beta) \} \\ &= \text{rmin} \{ (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)((t * \alpha) * (\mathfrak{t} * \beta)), (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(\mathfrak{t} * \beta) \} \Rightarrow (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(t * \alpha) \geq \\ &\text{rmin} \{ (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)((t * \alpha) * (\mathfrak{t} * \beta)), (\cap \overset{M}{\underset{\gamma}{H}}_{T,I,F}^i)(\mathfrak{t} * \beta) \} \end{aligned}$$

and

$$\begin{aligned} (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(t * \alpha) &= \text{sup} \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i(t * \alpha) = \text{sup} \lambda_{T,I,F}^i(t * \alpha) . \gamma \\ &\leq \text{sup} \{ \max \{ \lambda_{T,I,F}^i((t * \alpha) * (\mathfrak{t} * \beta)), \lambda_{T,I,F}^i(\mathfrak{t} * \beta) \} \} . \gamma \\ &= \max \{ \text{sup} \lambda_{T,I,F}^i((t * \alpha) * (\mathfrak{t} * \beta)) . \gamma, \text{sup} \lambda_{T,I,F}^i(\mathfrak{t} * \beta) . \gamma \} \\ &= \max \{ \text{sup} \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i((t * \alpha) * (\mathfrak{t} * \beta)), \text{sup} \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i(\mathfrak{t} * \beta) \} \\ &= \max \{ (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)((t * \alpha) * (\mathfrak{t} * \beta)), (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(\mathfrak{t} * \beta) \} \\ &\Rightarrow (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(t * \alpha) \leq \max \{ (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)((t * \alpha) * (\mathfrak{t} * \beta)), (V \overset{M}{\underset{\gamma}{\lambda}}_{T,I,F}^i)(\mathfrak{t} * \beta) \}, \end{aligned}$$



$max\{\lambda_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta), \lambda_{T,I,F}(\mathfrak{t} * \beta)\} . \mathfrak{r} = max\{\lambda_{T,I,F}(0), \lambda_{T,I,F}(\mathfrak{t} * \beta)\} . \mathfrak{r} = \lambda_{T,I,F}(\mathfrak{t} * \beta) . \mathfrak{r} = M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta)$ , so  $M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \alpha) \leq M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta)$ .

Theorem 3.6. Let  $M_{\mathfrak{r}}\mathfrak{H}$  of  $\mathfrak{H} = \{H_{T,I,F}, \lambda_{T,I,F}\}$  is a NCNID of  $X$ .  $\forall \mathfrak{t}, \mathfrak{t} \in X$  and  $\alpha, \beta \in [0,1]$ , then  $\mathfrak{H}$  is a NCMSU of  $X$ .

Proof. Assume that  $M_{\mathfrak{r}}\mathfrak{H}$  is a NCNID of  $X$ ,  $\forall \mathfrak{t}, \mathfrak{t} \in X$  and  $\alpha, \beta \in [0,1]$ . Then  $\mathfrak{r}.H_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta) = M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta) \geq rmin\{M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \beta) * ((\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta)), M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \beta)\} = rmin\{M_{\mathfrak{r}}H_{T,I,F}(0), M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \beta)\} \geq rmin\{M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \alpha), M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \beta)\} = rmin\{H_{T,I,F}(\mathfrak{t} * \alpha) . \mathfrak{r}, H_{T,I,F}(\mathfrak{t} * \beta) . \mathfrak{r}\} = rmin\{H_{T,I,F}(\mathfrak{t} * \alpha), H_{T,I,F}(\mathfrak{t} * \beta)\} . \mathfrak{r} \Rightarrow H_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta) \geq rmin\{H_{T,I,F}(\mathfrak{t} * \alpha), H_{T,I,F}(\mathfrak{t} * \beta)\}$  and  $\mathfrak{r} . \lambda_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta) = M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta) \leq max\{M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta) * ((\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta)), M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta)\} = max\{M_{\mathfrak{r}}\lambda_{T,I,F}(0), M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta)\} \leq max\{M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \alpha), M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta)\} = max\{\lambda_{T,I,F}(\mathfrak{t} * \alpha) . \mathfrak{r}, \lambda_{T,I,F}(\mathfrak{t} * \beta) . \mathfrak{r}\} = max\{\lambda_{T,I,F}(\mathfrak{t} * \alpha), \lambda_{T,I,F}(\mathfrak{t} * \beta)\} . \mathfrak{r} \Rightarrow \lambda_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta) \leq max\{\lambda_{T,I,F}(\mathfrak{t} * \alpha), \lambda_{T,I,F}(\mathfrak{t} * \beta)\} . \mathfrak{r}$ . Hence,  $\mathfrak{H}\{H_{T,I,F}, \lambda_{T,I,F}\}$  is a NCMSU of  $X$ .

**4.  $\mathfrak{r}$ -MULTIPLICATION UNDER HOMOMORPHISM**

Theorem 4.1. Suppose that  $\Gamma|X \rightarrow Y$  is a homomorphic mapping of PS-algebra. If  $M_{\mathfrak{r}}\mathfrak{H}$  of  $\mathfrak{H} = (H_{T,I,F}, \lambda_{T,I,F})$  is a NCNID of  $Y$ , then pre-image  $\Gamma^{-1}(M_{\mathfrak{r}}\mathfrak{H}) = (\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F}), \Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F}))$  of  $M_{\mathfrak{r}}\mathfrak{H}$  under  $\Gamma$  of  $X$  is a NCNID of  $X$ .

Proof. For all  $\mathfrak{t} \in X$  and  $\alpha \in [0,1]$ ,  $\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F})(\mathfrak{t} * \alpha) = M_{\mathfrak{r}}H_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) = H_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) . \mathfrak{r} \leq H_{T,I,F}(\Gamma(0)) . \mathfrak{r} = M_{\mathfrak{r}}H_{T,I,F}(\Gamma(0)) = \Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F})(0)$  and  $\Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F})(\mathfrak{t} * \alpha) = M_{\mathfrak{r}}\lambda_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) = \lambda_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) . \mathfrak{r} \geq \lambda_{T,I,F}(\Gamma(0)) . \mathfrak{r} = M_{\mathfrak{r}}\lambda_{T,I,F}(\Gamma(0)) = \Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F})(0)$ .

Let  $\mathfrak{t}, \mathfrak{t} \in X$ ,  $\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F})(\mathfrak{t} * \alpha) = M_{\mathfrak{r}}H_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) = H_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) . \mathfrak{r} \geq rmin\{H_{T,I,F}(\Gamma(\mathfrak{t} * \alpha) * \Gamma(\mathfrak{t} * \beta)), H_{T,I,F}(\Gamma(\mathfrak{t} * \beta))\} . \mathfrak{r} = rmin\{H_{T,I,F}(\Gamma((\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta))), H_{T,I,F}(\Gamma(\mathfrak{t} * \beta))\} . \mathfrak{r} = rmin\{\Gamma^{-1}(H_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta)) . \mathfrak{r}, \Gamma^{-1}(H_{T,I,F}(\mathfrak{t} * \beta)) . \mathfrak{r}\} = rmin\{\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta)), \Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \beta))\}$  and  $\Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F})(\mathfrak{t} * \alpha) = M_{\mathfrak{r}}\lambda_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) = \lambda_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) . \mathfrak{r} \leq max\{\lambda_{T,I,F}(\Gamma(\mathfrak{t} * \alpha) * \Gamma(\mathfrak{t} * \beta)), \lambda_{T,I,F}(\Gamma(\mathfrak{t} * \beta))\} . \mathfrak{r} = max\{\lambda_{T,I,F}(\Gamma((\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta))), \lambda_{T,I,F}(\Gamma(\mathfrak{t} * \beta))\} . \mathfrak{r} = max\{\Gamma^{-1}(\lambda_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta)) . \mathfrak{r}, \Gamma^{-1}(\lambda_{T,I,F}(\mathfrak{t} * \beta)) . \mathfrak{r}\} = max\{\Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \alpha) * (\mathfrak{t} * \beta)), \Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta))\}$ . Hence,  $\Gamma^{-1}(M_{\mathfrak{r}}\mathfrak{H}) = (\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F}), \Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F}))$  is a NCNID of  $X$ .

Theorem 4.2. Let  $\Gamma|X \rightarrow Y$  be a homomorphic mapping of B-algebra. If  $M_{\mathfrak{r}}\mathfrak{H}_i$  of  $\mathfrak{H}_i = (H_{T,I,F}^i, \lambda_{T,I,F}^i)$  is a NCNID of  $Y$  where  $i \in k$ , then the pre-image  $\Gamma^{-1}(\bigcap_{i \in k} M_{\mathfrak{r}}\mathfrak{H}_i) = (\Gamma^{-1}(\bigcap_{i \in k} M_{\mathfrak{r}}H_{T,I,F}^i), \Gamma^{-1}(\bigcap_{i \in k} M_{\mathfrak{r}}\lambda_{T,I,F}^i))$  is a NCNID of  $X$ .

Proof. We can prove this theorem through Theorem 3.2 and Theorem 4.1.

Theorem 4.3. Let  $\Gamma|X \rightarrow Y$  is an epimorphic mapping of B-algebra. Then  $M_{\mathfrak{r}}\mathfrak{H} = (M_{\mathfrak{r}}H_{T,I,F}, M_{\mathfrak{r}}\lambda_{T,I,F})$  is a NCNID of  $Y$ , if pre-image  $\Gamma^{-1}(M_{\mathfrak{r}}\mathfrak{H}) = (\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F}), \Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F}))$  of  $M_{\mathfrak{r}}\mathfrak{H}$  under  $\Gamma$  of  $X$  is a NCNID of  $X$

Proof. For any  $\mathfrak{t} \in Y$ ,  $\mathfrak{t} \in X$  and  $\alpha, \beta \in [0,1]$  such that  $(\mathfrak{t} * \beta) = \Gamma(\mathfrak{t} * \alpha)$ . Then  $M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t} * \beta) = M_{\mathfrak{r}}H_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) = \Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F})(\mathfrak{t} * \alpha) = \Gamma^{-1}(H_{T,I,F})(\mathfrak{t} * \alpha) . \mathfrak{r} \geq \Gamma^{-1}(H_{T,I,F})(0) . \mathfrak{r} = H_{T,I,F}(\Gamma(0)) . \mathfrak{r} = H_{T,I,F}(0) . \mathfrak{r} = M_{\mathfrak{r}}H_{T,I,F}(0)$  and  $M_{\mathfrak{r}}\lambda_{T,I,F}(\mathfrak{t} * \beta) = M_{\mathfrak{r}}\lambda_{T,I,F}(\Gamma(\mathfrak{t} * \alpha)) = \Gamma^{-1}(M_{\mathfrak{r}}\lambda_{T,I,F})(\mathfrak{t} * \alpha) = \Gamma^{-1}(\lambda_{T,I,F})(\mathfrak{t} * \alpha) . \mathfrak{r} \leq \Gamma^{-1}(\lambda_{T,I,F})(0) . \mathfrak{r} = \lambda_{T,I,F}(\Gamma(0)) . \mathfrak{r} = \lambda_{T,I,F}(0) . \mathfrak{r} = M_{\mathfrak{r}}\lambda_{T,I,F}(0)$ .

Assume  $\mathfrak{t}_1, \mathfrak{t}_2 \in Y$ . Then  $\Gamma(\mathfrak{t}_1 * \alpha) = \mathfrak{t}_1 * \beta$  and  $\Gamma(\mathfrak{t}_2 * \alpha) = \mathfrak{t}_2 * \beta$  for some  $\mathfrak{t}_1, \mathfrak{t}_2 \in X$  and  $\alpha, \beta \in [0,1]$ . Thus  $M_{\mathfrak{r}}H_{T,I,F}(\mathfrak{t}_1 * \beta) = M_{\mathfrak{r}}H_{T,I,F}(\Gamma(\mathfrak{t}_1 * \alpha)) = \Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F})(\mathfrak{t}_1 * \alpha) = \Gamma^{-1}(H_{T,I,F})(\mathfrak{t}_1 * \alpha) . \mathfrak{r} \geq rmin\{\Gamma^{-1}(M_{\mathfrak{r}}H_{T,I,F})(\mathfrak{t}_1 * \alpha)$

$\alpha) * (\mathfrak{t}_2 * \alpha)), \Gamma^{-1}({}^M_{\mathfrak{v}}H_{T,I,F}(\mathfrak{t}_2 * \alpha)). \mathfrak{r} = \text{rmin}\{{}^M_{\mathfrak{v}}H_{T,I,F}(\Gamma((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha))), H_{T,I,F}(\Gamma(\mathfrak{t}_2 * \alpha))\}. \mathfrak{r} =$   
 $\text{rmin}\{H_{T,I,F}(\Gamma(\mathfrak{t}_1 * \alpha) * \Gamma(\mathfrak{t}_2 * \alpha)), H_{T,I,F}(\Gamma(\mathfrak{t}_2 * \alpha))\} = \text{rmin}\{H_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), H_{T,I,F}(\mathfrak{t}_2 * \beta)\}. \mathfrak{r} =$   
 $\text{rmin}\{H_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)). \mathfrak{r}, H_{T,I,F}(\mathfrak{t}_2 * \beta). \mathfrak{r}\} = \text{rmin}\{{}^M_{\mathfrak{v}}H_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), {}^M_{\mathfrak{v}}H_{T,I,F}(\mathfrak{t}_2 * \beta)\} \quad \text{and}$   
 ${}^M_{\mathfrak{v}}\lambda_{T,I,F}(\mathfrak{t}_1 * \beta) = {}^M_{\mathfrak{v}}\lambda_{T,I,F}(\Gamma(\mathfrak{t}_1 * \alpha)) = \Gamma^{-1}({}^M_{\mathfrak{v}}\lambda_{T,I,F})(\mathfrak{t}_1 * \alpha) = \Gamma^{-1}(\lambda_{T,I,F})(\mathfrak{t}_1 * \alpha). \mathfrak{r} \leq \text{max}\{\Gamma^{-1}({}^M_{\mathfrak{v}}\lambda_{T,I,F})(\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha), \Gamma^{-1}({}^M_{\mathfrak{v}}\lambda_{T,I,F})(\mathfrak{t}_2 * \alpha)\}. \mathfrak{r} = \text{max}\{{}^M_{\mathfrak{v}}\lambda_{T,I,F}(\Gamma((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha))), \lambda_{T,I,F}(\Gamma(\mathfrak{t}_2 * \alpha))\}. \mathfrak{r} =$   
 $\text{max}\{\lambda_{T,I,F}(\Gamma(\mathfrak{t}_1 * \alpha) * \Gamma(\mathfrak{t}_2 * \alpha)), \lambda_{T,I,F}(\Gamma(\mathfrak{t}_2 * \alpha))\} = \text{max}\{\lambda_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), \lambda_{T,I,F}(\mathfrak{t}_2 * \beta)\}. \mathfrak{r} =$   
 $\text{max}\{\lambda_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)). \mathfrak{r}, \lambda_{T,I,F}(\mathfrak{t}_2 * \beta). \mathfrak{r}\} = \text{max}\{{}^M_{\mathfrak{v}}\lambda_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), {}^M_{\mathfrak{v}}\lambda_{T,I,F}(\mathfrak{t}_2 * \beta)\}.$   
 Hence,  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} = ({}^M_{\mathfrak{v}}H_{T,I,F}, {}^M_{\mathfrak{v}}\lambda_{T,I,F})$  is a NCNID of  $Y$ .

**5.  $\mathfrak{r}$ -MULTIPLICATION OF CARTESIAN PRODUCT**

Theorem 5.1. Let  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} = ({}^M_{\mathfrak{v}}H_{T,I,F}, {}^M_{\mathfrak{v}}\lambda_{T,I,F})$  and  ${}^M_{\mathfrak{v}}\mathfrak{F} = ({}^M_{\mathfrak{v}}F_{T,I,F}, {}^M_{\mathfrak{v}}\mu_{T,I,F})$  are NCNIDs of  $X$  and  $Y$  respectively. Then  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} \times {}^M_{\mathfrak{v}}\mathfrak{F}$  is a neutrosophic cubic normal ideal of  $X \times Y$ .

Proof. For any  $(\mathfrak{t}, \mathfrak{t}) \in X \times Y$  and  $\alpha, \beta \in [0,1]$ . We have  $({}^M_{\mathfrak{v}}H_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})(0,0) = \mathfrak{r}. (H_{T,I,F} \times F_{T,I,F})(0,0) = \mathfrak{r}. \text{rmin}\{H_{T,I,F}(0), F_{T,I,F}(0)\} \geq \mathfrak{r}. \text{rmin}\{H_{T,I,F}(\mathfrak{t} * \alpha), F_{T,I,F}(\mathfrak{t} * \beta)\} = \text{rmin}\{H_{T,I,F}(\mathfrak{t} * \alpha). \mathfrak{r}, F_{T,I,F}(\mathfrak{t} * \beta). \mathfrak{r}\} = \text{rmin}\{{}^M_{\mathfrak{v}}H_{T,I,F}(\mathfrak{t} * \alpha), {}^M_{\mathfrak{v}}F_{T,I,F}(\mathfrak{t} * \beta)\} = ({}^M_{\mathfrak{v}}H_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})(\mathfrak{t} * \alpha, \mathfrak{t} * \beta)$  and  $({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})(0,0) = \mathfrak{r}. (\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) = \mathfrak{r}. \text{max}\{\lambda_{T,I,F}(0), \mu_{T,I,F}(0)\} \leq \mathfrak{r}. \text{max}\{\lambda_{T,I,F}(\mathfrak{t} * \alpha), \mu_{T,I,F}(\mathfrak{t} * \beta)\} = \text{max}\{\lambda_{T,I,F}(\mathfrak{t} * \alpha). \mathfrak{r}, \mu_{T,I,F}(\mathfrak{t} * \beta). \mathfrak{r}\} = \text{max}\{{}^M_{\mathfrak{v}}\lambda_{T,I,F}(\mathfrak{t} * \alpha), {}^M_{\mathfrak{v}}\mu_{T,I,F}(\mathfrak{t} * \beta)\} = ({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})(\mathfrak{t} * \alpha, \mathfrak{t} * \beta).$

Let  $(\mathfrak{t}_1, \mathfrak{t}_1), (\mathfrak{t}_2, \mathfrak{t}_2) \in X \times Y$  and  $\alpha, \beta \in [0,1]$ . Then  $({}^M_{\mathfrak{v}}H_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})(\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) = \mathfrak{r}. (H_{T,I,F} \times F_{T,I,F})(\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) = \mathfrak{r}. \text{rmin}\{H_{T,I,F}(\mathfrak{t}_1 * \alpha), F_{T,I,F}(\mathfrak{t}_1 * \beta)\} \geq \mathfrak{r}. \text{rmin}\{\text{rmin}\{H_{T,I,F}((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha)), H_{T,I,F}(\mathfrak{t}_2 * \alpha)\}, \text{rmin}\{F_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), F_{T,I,F}(\mathfrak{t}_2 * \beta)\}\} = \mathfrak{r}. \text{rmin}\{\text{rmin}\{H_{T,I,F}((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha)), F_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta))\}, \text{rmin}\{H_{T,I,F}(\mathfrak{t}_2 * \alpha), F_{T,I,F}(\mathfrak{t}_2 * \beta)\}\} = \mathfrak{r}. \text{rmin}\{(H_{T,I,F} \times F_{T,I,F})((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha), (\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), (H_{T,I,F} \times F_{T,I,F})((\mathfrak{t}_2 * \alpha), (\mathfrak{t}_2 * \beta))\} = \text{rmin}\{(H_{T,I,F} \times F_{T,I,F})((\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta)). \mathfrak{r}, (H_{T,I,F} \times F_{T,I,F})(\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta). \mathfrak{r}\} = \text{rmin}\{({}^M_{\mathfrak{v}}R_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})((\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta)), ({}^M_{\mathfrak{v}}R_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})(\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta)\} \quad \text{and} \quad ({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})(\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) = \mathfrak{r}. (\lambda_{T,I,F} \times \mu_{T,I,F})(\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) = \mathfrak{r}. \text{max}\{\lambda_{T,I,F}(\mathfrak{t}_1 * \alpha), \mu_{T,I,F}(\mathfrak{t}_1 * \beta)\} \leq \mathfrak{r}. \text{max}\{\text{max}\{\lambda_{T,I,F}((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha)), \lambda_{T,I,F}(\mathfrak{t}_2 * \alpha)\}, \text{max}\{\mu_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), \mu_{T,I,F}(\mathfrak{t}_2 * \beta)\}\} = \mathfrak{r}. \text{max}\{\text{max}\{\lambda_{T,I,F}((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha)), \mu_{T,I,F}((\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta))\}, \text{max}\{\lambda_{T,I,F}(\mathfrak{t}_2 * \alpha), \mu_{T,I,F}(\mathfrak{t}_2 * \beta)\}\} = \mathfrak{r}. \text{max}\{(\lambda_{T,I,F} \times \mu_{T,I,F})((\mathfrak{t}_1 * \alpha) * (\mathfrak{t}_2 * \alpha), (\mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})(\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta)\} = \text{max}\{({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})((\mathfrak{t}_1 * \alpha, \mathfrak{t}_1 * \beta) * (\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta)), ({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})(\mathfrak{t}_2 * \alpha, \mathfrak{t}_2 * \beta)\}. Hence,  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} \times {}^M_{\mathfrak{v}}\mathfrak{F}$  is a neutrosophic cubic normal ideal of  $X \times Y$ .$

Theorem 5.2. Let  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} = ({}^M_{\mathfrak{v}}H_{T,I,F}, {}^M_{\mathfrak{v}}\lambda_{T,I,F})$  and  ${}^M_{\mathfrak{v}}\mathfrak{F} = ({}^M_{\mathfrak{v}}F_{T,I,F}, {}^M_{\mathfrak{v}}\mu_{T,I,F})$  are two  $\mathfrak{r}$ -multiplications of neutrosophic cubic closed normal ideals of  $X$  and  $Y$  respectively. Then  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} \times {}^M_{\mathfrak{v}}\mathfrak{F}$  is a NCCNID of  $X \times Y$ .

Proof. By Proposition 3.1 and Theorem 5.1,  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} \times {}^M_{\mathfrak{v}}\mathfrak{F}$  is NCNID. Now,  $({}^M_{\mathfrak{v}}H_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})((0,0) * (\mathfrak{t} * \alpha, \mathfrak{t} * \beta)) = (H_{T,I,F} \times F_{T,I,F})((0,0) * (\mathfrak{t} * \alpha, \mathfrak{t} * \beta)). \mathfrak{r} = (H_{T,I,F} \times F_{T,I,F})(0 * (\mathfrak{t} * \alpha), 0 * (\mathfrak{t} * \beta)). \mathfrak{r} = \mathfrak{r}. \text{rmin}\{H_{T,I,F}(0 * (\mathfrak{t} * \alpha)), F_{T,I,F}(0 * (\mathfrak{t} * \beta))\} \geq \mathfrak{r}. \text{rmin}\{H_{T,I,F}(\mathfrak{t} * \alpha), F_{T,I,F}(\mathfrak{t} * \beta)\} = \text{rmin}\{H_{T,I,F}(\mathfrak{t} * \alpha). \mathfrak{r}, F_{T,I,F}(\mathfrak{t} * \beta). \mathfrak{r}\} = \text{rmin}\{{}^M_{\mathfrak{v}}H_{T,I,F}(\mathfrak{t} * \alpha), {}^M_{\mathfrak{v}}F_{T,I,F}(\mathfrak{t} * \beta)\} = ({}^M_{\mathfrak{v}}H_{T,I,F} \times {}^M_{\mathfrak{v}}F_{T,I,F})(\mathfrak{t} * \alpha, \mathfrak{t} * \beta) \quad \text{and} \quad ({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})((0,0) * (\mathfrak{t} * \alpha, \mathfrak{t} * \beta)) = (\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (\mathfrak{t} * \alpha, \mathfrak{t} * \beta)). \mathfrak{r} = (\lambda_{T,I,F} \times \mu_{T,I,F})(0 * (\mathfrak{t} * \alpha), 0 * (\mathfrak{t} * \beta)). \mathfrak{r} = \mathfrak{r}. \text{max}\{\lambda_{T,I,F}(0 * (\mathfrak{t} * \alpha)), \mu_{T,I,F}(0 * (\mathfrak{t} * \beta))\} \leq \mathfrak{r}. \text{max}\{\lambda_{T,I,F}(\mathfrak{t} * \alpha), \mu_{T,I,F}(\mathfrak{t} * \beta)\} = \text{max}\{\lambda_{T,I,F}(\mathfrak{t} * \alpha). \mathfrak{r}, \mu_{T,I,F}(\mathfrak{t} * \beta). \mathfrak{r}\} = \text{max}\{{}^M_{\mathfrak{v}}\lambda_{T,I,F}(\mathfrak{t} * \alpha), {}^M_{\mathfrak{v}}\mu_{T,I,F}(\mathfrak{t} * \beta)\} = ({}^M_{\mathfrak{v}}\lambda_{T,I,F} \times {}^M_{\mathfrak{v}}\mu_{T,I,F})(\mathfrak{t} * \alpha, \mathfrak{t} * \beta). Hence,  ${}^M_{\mathfrak{v}}\mathfrak{H}\mathfrak{b} \times {}^M_{\mathfrak{v}}\mathfrak{F}$  is a neutrosophic cubic closed normal ideal of  $X \times Y$ .$

**6. Conclusion**



In this paper, the notion of  $\tau$ -multiplication of neutrosophic cubic set was introduced and  $\tau$ -multiplication was studied by several useful results. This study will provide the base for further work like t-neutrosophic soft cubic and intuitionistic soft cubic set etc

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