Fuzzy Logic Used to Solve ODEs of Second Order Under Neutrosophic Initial Conditions

Sahar M. Jabbar*[1] , Azal Mera [2] , Ameera N. Alkiffai [3]

[1,2] Department of Mathematics, College of Education for Pure Science, University of Babylon, Babil, Iraq.
[3] Department of Mathematics, College of Education for Women, University of Kufa, Al-Najaf, Iraq.
Emails: pure.saher.mohsen@uobabylon.edu.iq; azal.mera@uobabylon.edu.iq;
ameeran.alkiffai@uokufa.edu.iq

Abstract

The Mohand transform method, which has the benefit of unit preservation property over the well-established Laplace transform method, is used in this study to solve the ordinary differential equation of second order with neutrosophic numbers as initial conditions. Moreover, the solution obtained at different (α,β,γ) –cut .

Keywords: Fuzzy number; Neutrosophic number; Neutrosophic triangular number; Strongly –generalized differentiability; Mohand transform; Fuzzy Mohand transform.

1. Introduction

Fuzzy differential equations have seen recent decades use because of their numerous and important applications in a range of domains. In order to keep up with the rapidly expanding and developing field of fuzzy differential equations, this work presents a novel method for solving these kinds of problems.

Florentin Smarandache uses a new notion derived from the world of uncertainty known as "neutrosophic set theory" to address a number of issues. Description, the reason, belongingness of fact values, false values, and values of undetermined [1] are the underlined searchable fields. As a result , neutrosophic is a new logic in the mathematical world which relies on the principle of indeterminacy , and this logic is considered as a generalization of fuzzy logic [2] . In nearly every application area of engineering and research, neutrosophic logic has shown to be one of the most valuable and significant modeling tools. The style may be applied to a variety of real-world occurrences by utilizing differential equations to describe them (see [3], [4], [5], [6], and [7]).

Mohand transform, which was introduced in 2017 by Mohand and Mahgoub [8], is used to solve fuzzy differential equations. The scaling property and the unit-preserving feature are two crucial characteristics of this transform. In this study, we use Mohand transform approach to solve a second-order ordinary differential equation in a neutrosophic setting where the solution is computed at different levels of cut–points .

2. Fuzzy Mohand Transform For Second –Order Differential Equation ($\widehat{\mathcal{M}}$-Transform)

In [9], we defined fuzzy Mohand transform (**$\widehat{\mathcal{M}}$-Transform)** for first order differential equation by :
Let $h(w)$ be a continuous fuzzy – valued function suppose that $\upsilon^2 h(w)e^{-\upsilon w}$ is an improper fuzzy Riemann – integrable on $[0,\infty)$, then
$\upsilon^2 \int_0^\infty h(w)\, e^{-\upsilon w}\, dw$ is called fuzzy Mohand transform and it is denoted by:
$\widehat{\mathcal{M}}[h(w)] = \upsilon^2 \int_0^\infty h(w)\, e^{-\upsilon w} dw\,, (\upsilon > 0 \text{ and integer }).$ Thus ,
$\upsilon^2 \int_0^\infty h(w)\, e^{-\upsilon w}\, dw = (\upsilon^2 \int_0^\infty \underline{h}(w,r)\, e^{-\upsilon w}\, dw\,, \upsilon^2 \int_0^\infty \overline{h}(w,r)\, e^{-\upsilon w}\, dw)$

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

Using the definition of classical Mohand transform:
$\mathcal{M}[\underline{h}(w,r)]=v^2\int_0^\infty \underline{h}(w,r)\,e^{-vw}\ dw$ and $\mathcal{M}[\overline{h}(w,r)]=v^2\int_0^\infty \overline{h}(w,r)\,e^{-vw}\ dw$ then
$\widehat{\mathcal{M}}\,[h(w,r)] = (\mathcal{M}[\underline{h}(w,r)]\,,\mathcal{M}[\overline{h}(w,r)])$.

Theorem 1 [10]

Assume that $h\colon[a,b]\to[0,1]$ ¸ be a function such that
$h(w) = (\underline{h}(w,r),\overline{h}(w,r))$, ∀ r∈ [0,1], then :

1. If h is differentiable of the first form (i), then $\underline{h}(w,r)$ and $\overline{h}(w,r)$ are differentiable functions and $h'(w,r) = \left(\underline{h}'(w,r),\overline{h}'(w,r)\right)$.
2. If h is differentiable of the second form (ii)¸ then $\underline{h}(w,r)$ and $\overline{h}(w,r)$ are differentiable function and $h'(w,r) = (\overline{h}'(w,r),\underline{h}'(w,r))$.

In this paper we will define fuzzy Mohand for second order PDE as following :

Definition 2.1[11]

Let h be a continuous fuzzy –valued function , such that $h(x_\circ) : (a,b)\to\mathbb{R}_f$ and $x_\circ\in(a,b)$.
We say that a mapping h is strongly generalized differentiable at $x_\circ$ if there exists an element $h''(x_\circ)\in\mathbb{R}_f$
such that :

(i) For all $\tau>0$ sufficiently small , $\exists\ h'(x_\circ+\tau)\ominus h'(x_\circ), h(x_\circ)\ominus(h'x_\circ-\tau)$
Where $\lim\limits_{\tau\to0}\frac{h'(x_\circ+\tau)\ominus h'(x_\circ)}{\tau}=\lim\limits_{\tau\to0}\frac{h'(x_\circ)\ominus h'(x_\circ-\tau)}{\tau}=h''(x_\circ)$,
or
(ii) For all $\tau>0$ sufficiently small , $\exists\ h'(x_\circ)\ominus h'(x_\circ+\tau), h'(x_\circ-\tau)\ominus h'(x_\circ)$
Where $\lim\limits_{\tau\to0}\frac{h'(x_\circ)\ominus h'(x_\circ+\tau)}{-\tau}=\lim\limits_{\tau\to0}\frac{h'(x_\circ-\tau)\ominus h'(x_\circ)}{-\tau}=h''(x_\circ)$,
or
(iii) For all $\tau>0$ sufficiently small , $\exists\ h'(x_\circ+\tau)\ominus h'(x_\circ), h'(x_\circ-\tau)\ominus h'(x_\circ)$,where
$\lim\limits_{\tau\to0}\frac{h'(x_\circ+\tau)\ominus h'(x_\circ)}{\tau}=\lim\limits_{\tau\to0}\frac{h'(x_\circ-\tau)\ominus h'(x_\circ)}{-\tau}=h''(x_\circ)$,
(iv) For all $\tau>0$ sufficiently small , $\exists\ h'(x_\circ)\ominus h'(x_\circ+\tau)$, $h'(x_\circ)\ominus h'(x_\circ-\tau)$ where
$\lim\limits_{\tau\to0}\frac{h'(x_\circ)\ominus h'(x_\circ+\tau)}{-\tau}=\lim\limits_{\tau\to0}\frac{h'(x_\circ)\ominus h'(x_\circ-\tau)}{\tau}=h''(x_\circ)$.

Theorem 2 [12]

Let $h(w)$ *and* $h'(w)$ are two differentiable fuzzy – valued functions, such that
$h(w) = (\underline{h}(w,r),\overline{h}(w,r))$, ∀ r∈ [0,1], then:

1. Let $h(w)$ *and* $h'(w)$ are (i)-differentiable, or let $h(w)$ *and* $h'(w)$ are (ii)-differentiable then $\underline{h}(w,r)$ and $\overline{h}(w,r)$ have derivatives in first and second order such that $h''(w)=\left[\underline{h}''(w,r),\overline{h}''(w,r)\right]$.
2. Let $h(w)$ be (i)-differentiable, and $h'(w)$ be (ii)-differentiable or let $h(w)$ be (ii)-differentiable and $h'(w)$ be (i)-differentiable then $\underline{h}(w,r)$ and $\overline{h}(w,r)$ have derivatives in first and second order such that $h''(w)=\left[\overline{h}''(w,r),\underline{h}''(w,r)\right]$.

Theorem 3

Let $h(w)$, $h'(w)$ be continuous neutrosophic valued functions on $[0,\infty]$ and $h''(w)$ be a piecewise continuous neutrosophic valued function on $[0,\infty]$ then ,

a) $\widehat{\mathcal{M}}[h''(w)]=\{v^2\widehat{\mathcal{M}}[h(w)]\ominus v^3h(0)\}\ominus v^2h'(0)$,
where $h(w)$ and $h'(w)$ are (i)- differentiable .
b) $\widehat{\mathcal{M}}[h''(w)]=-v^3h(0)\ominus\{-v^2\widehat{\mathcal{M}}[h(w)]\}-v^2h'(0)$,
where $h(w)$ is (i)- differentiable and $h'(w)$ is (ii) – differentiable
c) $\widehat{\mathcal{M}}[h''(w)]=\left\{-v^3h(0)\ominus\{-v^2\widehat{\mathcal{M}}[h(w)]\}\right\}\ominus v^2h'(0)$,
where $h(w)$ is (ii)- differentiable and $h'(t)$ is (i) – differentiable
d) $\widehat{\mathcal{M}}[h''(w)]=v^2\widehat{\mathcal{M}}[h(w)]\ominus v^3h(0)-v^2h'(0)$
where $h(w)$ and $h'(w)$ are (ii) – differentiable .

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

Proof (a): $h(w)$ and $h'(w)$ are (i) – differentiable

$\{v^2\widehat{\mathcal{M}}[h(w)] \ominus v^3h(0)\} \ominus v^2h'(0) =$

$$(v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h'}(0,r), v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\,\overline{h'}(0,r))$$

Since $\mathcal{M}[\underline{h}''(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h}'(0,r)$,

$\mathcal{M}\left[\overline{h}''(w,r)\right] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\overline{h}'(0,r)$

Since $h(w)$ and $h'(w)$ are (i) – differentiable using Theorem (2) ¸

$\underline{h''}(w,r) = \underline{h}''(w,r)$ ¸ $\overline{h''}=(w,r) = \overline{h}''(w,r)$

Since $h(w)$ is (i) – differentiable using Theorem (1)

$\overline{h'}(0,r) = \overline{h}'(0,r)$ and $\underline{h'}(0,r) = \underline{h}'(0,r)$

$\mathcal{M}[\underline{h''}(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h'}(0,r)$

$\mathcal{M}[\overline{h''}(w,r)] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\overline{h'}(0,r)$

$\{v^2\widehat{\mathcal{M}}[h(w)] \ominus v^3h(0)\} \ominus v^2h'(0) = (\mathcal{M}[\underline{h''}(w,r)], \mathcal{M}[\overline{h''}(w,r)]) = \widehat{\mathcal{M}}[h''(w)]$.

$\widehat{\mathcal{M}}[h''(w)] = \{v^2\widehat{\mathcal{M}}[h(w)] \ominus v^3h(0)\} \ominus v^2h'(0)$

(b) Since $h(w)$ is (i) – differentiable and since $h'(w)$ is (ii) – differentiable

$-v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(w)]\} - v^2h'(0) =$

$(-v^3\overline{h}(0,r) + v^2\mathcal{M}[\overline{h}(w,r)] - v^2\underline{h'}(0,r), -v^3\underline{h}(0,r) + v^2\mathcal{M}[\underline{h}(w,r)] - v^2\underline{h'}(0,r))$

Since $\mathcal{M}\left[\overline{h}''(w,r)\right] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\overline{h}'(0,r)$,

$\mathcal{M}[\underline{h}''(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h}'(0,r)$

Since $h(w)$ is (i) – differentiable and since $h'(w)$ is (ii) – differentiable using theorem (2)

$\overline{h''}(w,r) = \underline{h}''(w,r)$¸ $\underline{h''}(w,r) = \overline{h}''(w,r)$,

Since $h(w)$ is (i)-differentiable using theorem (1)

$\overline{h'}(0,r) = \overline{h}'(0,r)$ and $\underline{h'}(0,r) = \underline{h}'(0,r)$

$\mathcal{M}[\underline{h''}(w,r)] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\overline{h'}(0,r)$

$\mathcal{M}[\overline{h''}(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h'}(0,r)$

$-v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(w)]\} - v^2h'(0) = (\mathcal{M}[\underline{h''}(w,r)], \mathcal{M}[\overline{h''}(w,r)]) = \widehat{\mathcal{M}}[h''(w)]$.

$\widehat{\mathcal{M}}[h''(w)] = -v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(w)]\} - v^2h'(0) =$

(c) Since $h(w)$ is (ii) – differentiable and $h'(w)$ is (i) – differentiable

$\{-v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(w)]\}\} \ominus v^2h'(0) =$

$(-v^3\overline{h}(0,r) + v^2\widehat{\mathcal{M}}[\overline{h}(w,r)] - v^2\underline{h'}(0,r), -v^3\underline{h}(0,r) + v^2\widehat{\mathcal{M}}[\underline{h}(w,r)] - v^2\overline{h'}(0,r))$

Since $\mathcal{M}\left[\overline{h}''(w,r)\right] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\overline{h}'(0,r)$¸

$\mathcal{M}[\underline{h}''(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h}'(0,r)$

Since $h(w)$ is (ii) – differentiable and $h'(w)$ is (i) – differentiable using theorem (2)

$\overline{h''}(w,r) = \underline{h}''(w,r)$¸ $\underline{h''}(w,r) = \overline{h}''(w,r)$

Since $h(w)$ is (ii) – differentiable using theorem (1)

$\overline{h'}(0,r) = \underline{h}'(0,r)$ and $\underline{h'}(0,r) = \overline{h}'(0,r)$

$\mathcal{M}[\underline{h''}(w,r)] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\underline{h'}(0,r)$

$\mathcal{M}[\overline{h''}(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\overline{h'}(0,r)$

$\{-v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(w)]\}\} \ominus v^2h'(0) = \mathcal{M}[\underline{h''}(w,r)], \mathcal{M}[\overline{h''}(w,r)] = \widehat{\mathcal{M}}[h''(w)]$.

$\widehat{\mathcal{M}}[h''(w)] = \{-v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(w)]\}\} \ominus v^2h'(0)$

(d) Since $h(w)$ and $h'(w)$ are (ii) – differentiable

$v^2\widehat{\mathcal{M}}[h(w)] \ominus v^3h(0) - v^2h'(0) =$

$(v^2\,\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\overline{h'}(0,r), v^2\mathcal{M}\,[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\underline{h'}(0,r))$

Since

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

$\mathcal{M}[\underline{h}''(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\underline{h}'(0,r)$
$\mathcal{M}[\overline{h}''(w,r)] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\overline{h}'(0,r)$
Since $h(w)$ and $h'(w)$ are (ii) – differentiable using theorem (2)
$\underline{h''}(w,r) = \underline{h}''(w,r), \overline{h''}(w,r) = \overline{h}''(w,r)$

Since $h(w)$ is (ii) – differentiable using theorem (1)
$\overline{h'}(0,r) = \underline{h}'(0,r), \underline{h'}(0,r) = \overline{h}'(0,r)$
$\mathcal{M}[\underline{h''}(w,r)] = v^2\mathcal{M}[\underline{h}(w,r)] - v^3\underline{h}(0,r) - v^2\overline{h'}(0,r),$
$\mathcal{M}[\overline{h''}(w,r)] = v^2\mathcal{M}[\overline{h}(w,r)] - v^3\overline{h}(0,r) - v^2\underline{h'}(0,r)$
$v^2\widehat{\mathcal{M}}[h(w)] \ominus v^3h(0) - v^2h'(0) = (\mathcal{M}[\underline{h''}(w,r)], \mathcal{M}[\overline{h''}(w,r)]) = \widehat{\mathcal{M}}\,[h''(w)]$.
$\widehat{\mathcal{M}}\,[h''(w)] = v^2\widehat{\mathcal{M}}[h(w)] \ominus v^3h(0) - v^2h'(0)$

Theorem 4

Let $h: R \to G(R)$ be a continuous neutrosophic valued function and denotes by:
$h_T(w) = [\underline{h}_{T\alpha}(w), \overline{h}_{T\alpha}(w)]$, for each $\alpha \in [0,1]$
$h_I(w) = [\underline{h}_{I\beta}(w), \overline{h}_{I\beta}(w)]$, for each $\beta \in [0,1]$
$h_F(w) = [\underline{h}_{F\gamma}(w), \overline{h}_{F\gamma}(w)]$, for each $\gamma \in [0,1]$
Then

1. If h_T is (i) – differentiable, then $\underline{h}_{T\alpha}$ and $\overline{h}_{T\alpha}$ are differentiable function and $h'(w) = [\underline{h}'_{T\alpha}(w), \overline{h}'_{T\alpha}(w)]$.
2. If h_T is (ii) – differentiable, then $\underline{h}_{T\alpha}$ and $\overline{h}_{T\alpha}$ are differentiable function and $h'(w) = [\overline{h}'_{T\alpha}(w), \underline{h}'_{T\alpha}(w)]$.
3. If h_I is (i) – differentiable, then $\underline{h}_{I\beta}(t)$ and $\overline{h}_{I\beta}(t)$ are differentiable function and $h'(w) = [\underline{h}'_{I\beta}(w), \overline{h}'_{I\beta}(w)]$.
4. If h_I is (ii) – differentiable, then $\underline{h}_{I\beta}(t)$ and $\overline{h}_{I\beta}(t)$ are differentiable function and $h'(w) = [\overline{h}'_{I\beta}(w), \underline{h}'_{I\beta}(w)]$.
5. If h_F is (i) – differentiable, then $\underline{h}_{F\gamma}(t)$ and $\overline{h}_{F\gamma}(t)$ are differentiable function and $h'(w) = [\underline{h}'_{F\gamma}(w), \overline{h}'_{F\gamma}(w)]$.
6. If h_F is (ii) – differentiable, then $\underline{h}_{F\gamma}(t)$ and $\overline{h}_{F\gamma}(t)$ are differentiable function and $h'(w) = [\overline{h}'_{F\gamma}(w), \underline{h}'_{F\gamma}(w)]$.

Note : The proof is similar to the proof of Theorem (2) , so we skip it.

3. Neutrosophic Environment for second order ordinary Differential Equation

Let us consider a general ordinary differential equation of second order given as follows:
$y''(t) = f(t, y(t), y'(t))$ ……..(1)
With the initial conditions $y(t_0) = y_0$, $y'(t_0) = z_0$, where $f: [t_0, P] \times R \to R$.
Suppose that the initial values y_0 and z_0 (neutrosophic number) are uncertain and are defined in terms of lower and upper bound of truth , indeterminacy and falsity. Thus from equation (1) , we have the following fuzzy initial value differential equations:
$y''(t) = h(t, y(t), y'(t))$ $\quad 0 \le t \le P$ such that:

$$\left.\begin{aligned} y_T(t_0) &= y_0 = [\underline{y}_{T\alpha}(0), \overline{y}_{T\alpha}(0)] \ , 0 \le \alpha \le 1 \\ y'_T(t_0) &= z_0 = [\underline{z}_{T\alpha}(0), \overline{z}_{T\alpha}(0)] \ , 0 \le \alpha \le 1 \end{aligned}\right\} \quad \ldots\ldots\ldots(2)$$

$$\left.\begin{aligned} y_I(t_0) &= y_0 = [\underline{y}_{I\beta}(0), \overline{y}_{I\beta}(0)] \ , 0 \le \beta \le 1 \\ y'_I(t_0) &= z_0 = [\underline{z}_{I\beta}(0), \overline{z}_{I\beta}(0)] \ , 0 \le \beta \le 1 \end{aligned}\right\} \quad \ldots\ldots\ldots(3)$$

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

$$\left.\begin{array}{l} y_F(t_0) = y_0 = \left[\underline{y}_{F\gamma}(0), \overline{y}_{F\gamma}(0)\right] \ , 0 \le \gamma \le 1 \\ y'_F(t_0) = z_0 = \left[\underline{z}_{F\gamma}(0, \overline{z}_{F\gamma}(0)\right] \ , 0 \le \gamma \le 1 \end{array}\right\} \quad \ldots\ldots\ldots(4)$$

By applying neutrosophic Mohand transform on given second order differential equation ,we have $\mathcal{M}\,[y''(t)] = \mathcal{M}[h(t, y(t), y'(t)\,]$.

Case 1: If $y(t)$ and $y'(t)$ are (i) – differentiable functions or let $y(t)$ and $y'(t)$ are (ii) – differentiable ,then from theorem (4) , we have

$$y''(t) = \left[\underline{y}''(t), \overline{y}''(t)\right]$$

The differential equation is then reduced to the following :

$$\underline{y''}_{T\alpha}(t) = \underline{h}_{T\alpha}(t, y(t), y'(t))\,,\ \underline{y}_{T\alpha}(t_0) = \underline{y}_{T\alpha}(0)\ ,\quad \underline{y'}_{T\alpha}(t_0) = \underline{z}_{T\alpha}(0)$$
$$\overline{y''}_{T\alpha}(t) = \overline{h}_{T\alpha}(t, y(t), y'(t))\,,\ \overline{y}_{T\alpha}(t_0) = \overline{y}_{T\alpha}(0)\ ,\quad \overline{y'}_{T\alpha}(t_0) = \overline{z}_{T\alpha}(0)$$

$$\underline{y''}_{I\beta}(t) = \underline{h}_{I\beta}(t, y(t), y'(t))\,,\ \underline{y}_{I\beta}(t_0) = \underline{y}_{I\beta}(0)\ ,\quad \underline{y'}_{I\beta}(t_0) = \underline{z}_{I\beta}(0)$$
$$\overline{y''}_{I\beta}(t) = \overline{h}_{I\beta}(t, y(t), y'(t))\,,\ \overline{y}_{I\beta}(t_0) = \overline{y}_{I\beta}(0)\ ,\quad \overline{y'}_{I\beta}(t_0) = \overline{z}_{I\beta}(0)$$

$$\underline{y''}_{F\gamma}(t) = \underline{h}_{F\gamma}(t, y(t), y'(t))\,,\ \underline{y}_{F\gamma}(t_0) = \underline{y}_{F\gamma}(0)\ ,\quad \underline{y'}_{F\gamma}(t_0) = \underline{z}_{F\gamma}(0)$$
$$\overline{y''}_{F\gamma}(t) = \overline{h}_{F\gamma}(t, y(t), y'(t))\,,\ \overline{y}_{F\gamma}(t_0) = \overline{y}_{F\gamma}(0)\ ,\quad \overline{y'}_{F\gamma}(t_0) = \overline{z}_{F\gamma}(0)$$

Using neutrosophic Mohand transform for solving , to get :

$$\widehat{\mathcal{M}}[h''(t)\,] = \{v^2\widehat{\mathcal{M}}[h(t)] \ominus v^3 h(t_0)\} \ominus v^2 h'(t_0)$$

Using upper and lower functions , to have

$$\mathcal{M}\left[\underline{h}_{T\alpha}(t, y(t), y'(t))\right] = v^2\mathcal{M}\left[\underline{y}_{T\alpha}(t)\right] - v^3\underline{y}_{T\alpha}(0) - v^2\underline{y'}_{T\alpha}(0)$$
$$\mathcal{M}\left[\overline{h}_{T\alpha}(t, y(t), y'(t))\right] = v^2\mathcal{M}\left[\overline{y}_{T\alpha}(t)\right] - v^3\overline{y}_{T\alpha}(0) - v^2\overline{y'}_{T\alpha}(0)$$
$$\mathcal{M}\left[\underline{h}_{I\beta}(t, y(t), y'(t))\right] = v^2\mathcal{M}\left[\underline{y}_{I\beta}(t)\right] - v^3\underline{y}_{I\beta}(0) - v^2\underline{y'}_{I\beta}(0)$$
$$\mathcal{M}\left[\overline{h}_{I\beta}(t, y(t), y'(t))\right] = v^2\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] - v^3\overline{y}_{I\beta}(0) - v^2\overline{y'}_{I\beta}(0)$$
$$\mathcal{M}\left[\underline{h}_{F\gamma}(t, y(t), y'(t))\right] = v^2\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] - v^3\underline{y}_{F\gamma}(0) - v^2\underline{y'}_{F\gamma}(0)$$
$$\mathcal{M}\left[\overline{h}_{F\gamma}(t, y(t), y'(t))\right] = v^2\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] - v^3\overline{y}_{F\gamma}(0) - v^2\overline{y'}_{F\gamma}(0)$$

To solve this, we will use the inverse neutrosophic Mohand transform to get the following: $\underline{y}_{T\alpha}(t)\,, \overline{y}_{T\alpha}(t), \underline{y}_{I\beta}(t), \overline{y}_{I\beta}(t), \underline{y}_{F\gamma}(t), \overline{y}_{F\gamma}(t)$.

Case 2 : If $y(t)$ be (i)- differentiable and $y'(t)$ be (ii)-differentiable, or let $y(t)$ be (ii)-differentiable and $y'(t)$be (i)-differentiable then from theorem (4), we have :

$$y''(t) = \left[\overline{y}''(t), \underline{y}''(t)\right]$$

The differential equation is then reduced to the following :

$$\underline{y''}_{T\alpha}(t) = \underline{h}_{T\alpha}(t, y(t), y'(t))\,,\ \underline{y}_{T\alpha}(t_0) = \underline{y}_{T\alpha}(0)\ ,\quad \underline{y'}_{T\alpha}(t_0) = \underline{z}_{T\alpha}(0)$$
$$\overline{y''}_{T\alpha}(t) = \overline{h}_{T\alpha}(t, y(t), y'(t))\,,\ \overline{y}_{T\alpha}(t_0) = \overline{y}_{T\alpha}(0)\ ,\quad \overline{y'}_{T\alpha}(t_0) = \overline{z}_{T\alpha}(0)$$

$$\underline{y''}_{I\beta}(t) = \underline{h}_{I\beta}(t, y(t), y'(t))\,,\ \underline{y}_{I\beta}(t_0) = \underline{y}_{I\beta}(0)\ ,\quad \underline{y'}_{I\beta}(t_0) = \underline{z}_{I\beta}(0)$$
$$\overline{y''}_{I\beta}(t) = \overline{h}_{I\beta}(t, y(t), y'(t))\,,\ \overline{y}_{I\beta}(t_0) = \overline{y}_{I\beta}(0)\ ,\quad \overline{y'}_{I\beta}(t_0) = \overline{z}_{I\beta}(0)$$

$$\underline{y''}_{F\gamma}(t) = \underline{h}_{F\gamma}(t, y(t), y'(t))\,,\ \underline{y}_{F\gamma}(t_0) = \underline{y}_{F\gamma}(0)\ ,\quad \underline{y'}_{F\gamma}(t_0) = \underline{z}_{F\gamma}(0)$$
$$\overline{y''}_{F\gamma}(t) = \overline{h}_{F\gamma}(t, y(t), y'(t))\,,\ \overline{y}_{F\gamma}(t_0) = \overline{y}_{F\gamma}(0)\ ,\quad \overline{y'}_{F\gamma}(t_0) = \overline{z}_{F\gamma}(0)$$

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

Using $\widehat{\mathcal{M}}[h''(t)] = \{-v^3 h(t_0) \ominus \{-v^2 \widehat{\mathcal{M}}[h(t)]\}\} \ominus v^2 h'(t_0)$
Using upper and lower functions , to have

$\mathcal{M}[\underline{h}_{T\alpha}(t, y(t), y'(t))] = -v^3 \overline{y}_{T\alpha}(0) + v^2 \mathcal{M}[\overline{y}_{T\alpha}(t)] - v^2 \underline{y'}_{T\alpha}(0)$

$\mathcal{M}[\overline{h}_{T\alpha}(t, y(t), y'(t))] = -v^3 \underline{y}_{T\alpha}(0) + v^2 \mathcal{M}[\underline{y}_{T\alpha}(t)] - v^2 \overline{y'}_{T\alpha}(0)$

$\mathcal{M}[\underline{h}_{I\beta}(t, y(t), y'(t))] = -v^3 \overline{y}_{I\beta}(0) + v^2 \mathcal{M}[\overline{y}_{I\beta}(t)] - v^2 \underline{y'}_{I\beta}(0)$

$\mathcal{M}[\overline{h}_{I\beta}(t, y(t), y'(t))] = -v^3 \underline{y}_{I\beta}(0) + v^2 \mathcal{M}[\underline{y}_{I\beta}(t)] - v^2 \overline{y'}_{I\beta}(0)$

$\mathcal{M}[\underline{h}_{F\gamma}(t, y(t), y'(t))] = -v^3 \overline{y}_{F\gamma}(0) + v^2 \mathcal{M}[\overline{y}_{F\gamma}(t)] - v^2 \underline{y'}_{F\gamma}(0)$

$\mathcal{M}[\overline{h}_{F\gamma}(t, y(t), y'(t))] = -v^3 \underline{y}_{F\gamma}(0) + v^2 \mathcal{M}[\underline{y}_{F\gamma}(t)] - v^2 \overline{y'}_{F\gamma}(0)$

To solve this , we will use the inverse neutrosophic Mohand transform to get the following :
$\underline{y}_{T\alpha}(t)$, $\overline{y}_{T\alpha}(t)$, $\underline{y}_{I\beta}(t)$, $\overline{y}_{I\beta}(t)$, $\underline{y}_{F\gamma}(t)$, $\overline{y}_{F\gamma}(t)$,

4. Illustrative Example

Consider the neutrosophic initial value problem :

$y''(t) - a^2 y(t) = 0$

$y_T(0) = [\alpha - 1, 1 - \alpha]$, $y_T'(0) = [\alpha - 1, 1 - \alpha]$
$y_I(0) = [-\beta, \beta]$, $y_I'(0) = [-\beta, \beta]$
$y_F(0) = [-0.5\gamma, 0.5\gamma]$, $y_F'(0) = [-0.5\gamma, 0.5\gamma]$

Using neutrosophic Mohand transform on given second order differential equation :

$$\widehat{\mathcal{M}}[y''(t)] - a^2 \widehat{\mathcal{M}}[y(t)] = 0$$

Case 1:

$\widehat{\mathcal{M}}[h''(t)] = \{v^2 \widehat{\mathcal{M}}[h(t)] \ominus v^3 h(0)\} \ominus v^2 h'(0)$

For α-cut

$v^2 \mathcal{M}[\underline{y}_T(t, \alpha)] - v^3 \underline{y}_T(0, \alpha) - v^2 \underline{y'}_T(0, \alpha) - a^2 \mathcal{M}[\underline{y}_T(t, \alpha)] = 0$

$v^2 \mathcal{M}[\underline{y}_{T\alpha}(t)] - v^3 (\alpha - 1) - (\alpha - 1) v^2 - a^2 \mathcal{M}[\underline{y}_{T\alpha}(t)] = 0$

$(v^2 - a^2) \mathcal{M}[\underline{y}_{T\alpha}(t)] = (\alpha - 1) v^3 + (\alpha - 1) v^2$

$\mathcal{M}[\underline{y}_{T\alpha}(t)] = \frac{(\alpha - 1) v^3}{(v^2 - a^2)} + \frac{(\alpha - 1) v^2}{(v^2 - a^2)}$

$\underline{y}_{T\alpha}(t) = (\alpha - 1) \cosh at + \frac{(\alpha - 1)}{a} \sinh at$.

$v^2 \mathcal{M}[\overline{y}_{T\alpha}(t)] - v^3 \overline{y}_{T\alpha}(0) - v^2 \overline{y'}_{T\alpha} - a^2 \mathcal{M}[\overline{y}_{T\alpha}(t)] = 0$
$v^2 \mathcal{M}[\overline{y}_{T\alpha}(t)] - (1 - \alpha) v^3 - (1 - \alpha) v^2 - a^2 \mathcal{M}[\overline{y}_{T\alpha}(t)] = 0$
$(v^2 - a^2) \mathcal{M}[\overline{y}_{T\alpha}(t)] = (1 - \alpha)(v^3 + v^2)$
$\mathcal{M}[\overline{y}_{T\alpha}(t)] = \frac{(1 - \alpha) v^3}{(v^2 - a^2)} + \frac{(1 - \alpha) v^2}{(v^2 - a^2)}$
$\overline{y}_{T\alpha}(t) = (1 - \alpha) \cosh at + \frac{(1 - \alpha)}{a} \sinh at$ ■

For β-cut

$v^2 \mathcal{M}[\underline{y}_{I\beta}(t)] - v^3 \underline{y}_{I\beta}(0) - v^2 \underline{y'}_{I\beta}(0) - a^2 \mathcal{M}[\underline{y}_{I\beta}(t)] = 0$

$v^2 \mathcal{M}[\underline{y}_{I\beta}(t)] - v^3 (-\beta) - v^2(-\beta) - a^2 \mathcal{M}[\underline{y}_{I\beta}(t)] = 0$

$(v^2 - a^2) \mathcal{M}[\underline{y}_{I\beta}(t)] = -\beta v^3 - \beta v^2$

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

$\mathcal{M}\left[\underline{y}_{I\beta}(t)\right] = \frac{-\beta v^3}{(v^2-a^2)} - \frac{\beta v^2}{(v^2-a^2)}$

$\underline{y}_{I\beta}(t) = -\beta \cosh at - \frac{\beta}{a}\sinh at$.

$v^2\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] - v^3\overline{y}_{I\beta}(0) - v^2\overline{y'}_{I\beta} - a^2\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] = 0$

$v^2\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] - \beta v^3 - \beta v^2 - a^2\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] = 0$

$(v^2 - a^2)\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] = \beta v^3 + \beta v^2$

$\mathcal{M}\left[\overline{y}_{I\beta}(t)\right] = \frac{\beta v^3}{(v^2-a^2)} + \frac{\beta v^2}{(v^2-a^2)}$

$\overline{y}_{I\beta}(t) = \beta \cosh at + \frac{\beta}{a}\sinh at$ ■

For γ-cut

$v^2\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] - v^3\underline{y}_{F\gamma}(0) - v^2\underline{y'}_{F\gamma}(0) - a^2\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] = 0$

$v^2\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] - (-0.5\gamma)v^3 - (-0.5\gamma)v^2 - a^2\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] = 0$

$(v^2 - a^2)\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] = (-0.5\gamma)v^3 - (0.5)\gamma v^2$

$\mathcal{M}\left[\underline{y}_{F\gamma}(t)\right] = \frac{(-0.5\gamma)v^3}{(v^2-a^2)} - \frac{(0.5\gamma)v^2}{(v^2-a^2)}$

$\underline{y}_{F\gamma}(t) = -0.5\gamma \cosh at - \frac{0.5\gamma}{a}\sinh at$.

$v^2\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] - v^3\overline{y}_{F\gamma}(0) - v^2\overline{y'}_{F\gamma}(0) - a^2\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] = 0$

$v^2\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] - 0.5\gamma v^3 - 0.5\gamma v^2 - a^2\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] = 0$

$(v^2 - a^2)\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] = 0.5\gamma v^3 + 0.5\gamma v^2$

$\mathcal{M}\left[\overline{y}_{F\gamma}(t)\right] = \frac{0.5\gamma v^3}{(v^2-a^2)} + \frac{0.5\gamma v^2}{(v^2-a^2)}$

$\overline{y}_{F\gamma}(t) = 0.5\gamma \cosh at + \frac{0.5\gamma}{a}\sinh at$ ■

Case 2 :

Since $\widehat{\mathcal{M}}[h''(t)] = \left\{-v^3h(0) \ominus \{-v^2\widehat{\mathcal{M}}[h(t)]\}\right\} \ominus v^2h'(0)$

For α-cut

$-v^3\overline{y}_{T\alpha}(0) + v^2\mathcal{M}[\overline{y}_{T\alpha}(t)] - v^2\underline{y'}_{T\alpha}(0) - a^2\mathcal{M}\left[\underline{y}_{T\alpha}(t)\right] = 0$

$-v^3\underline{y}_{T\alpha}(0) + v^2\mathcal{M}\left[\underline{y}_{T\alpha}(t)\right] - v^2\overline{y'}_{T\alpha}(0) - a^2\mathcal{M}[\overline{y}_{T\alpha}(t)] = 0$

$-v^3(1-\alpha) + +v^2\mathcal{M}[\overline{y}_{T\alpha}(t)] - v^2(\alpha - 1) - a^2\mathcal{M}\left[\underline{y}_{T\alpha}(t)\right] = 0$

$-v^3(\alpha - 1) + v^2\mathcal{M}\left[\underline{y}_{T\alpha}(t)\right] - v^2(1-\alpha) - a^2\mathcal{M}[\overline{y}_{T\alpha}(t)] = 0$

$\underline{y}_{T\alpha}(t) = (\alpha - 1)\cos at + \frac{(\alpha-1)}{a}\sin at$, $\overline{y}_{T\alpha}(t) = (1-\alpha)cosat + \frac{(1-\alpha)}{a}sinat$.

For β-cut

$\underline{y}_{I\beta}(t) = -\beta \cos at - \frac{\beta}{a}\sin at$, $\overline{y}_{I\beta}(t) = \beta cosat + \frac{\beta}{a}sinat$.

Also, for γ-cut

$\underline{y}_{F\gamma}(t) = -0.5\gamma \cos at - \frac{0.5\gamma}{a}\sin at$, $\overline{y}_{F\gamma}(t) = 0.5\gamma cosat + \frac{0.5\gamma}{a}sinat$.

6. Conclusion

Due to their diverse and significant applications across a variety of areas, fuzzy differential equations have become increasingly popular in recent years. This paper proposes a novel method to solve such problems, keeping up with the fast expanding and developing field of fuzzy differential equations. The ordinary differential equation of second order with neutrosophic numbers as initial conditions is solved in this work using the

Doi: https://doi.org/10.54216/IJNS.230104
Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023

Mohand transform method, which has the advantage of unit preservation property over the well-known Laplace transform approach. An illustrative example is discussed to show the accuracy of this approach. By solving the ordinary differential equation of third order till n^{th}- order with neutrosophic numbers as initial conditions, we will continue to stay current with the rapidly growing and developing subject of fuzzy differential equations. Additionally, this method was used to solve difficulties in real life.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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Received: June 06, 2023 Revised: August 22, 2023 Accepted: November 08, 2023