On Nil-clean Neutrosophic Rings

P. Prabakaran
Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam - 638401, Tamil Nadu, India.
Emails: prabakaranpvkr@gmail.com

Abstract

A ring is said to be nil-clean if every element of the ring can be written as a sum of an idempotent element and a nilpotent element of the ring. In this paper, we generalize this argument to neutrosophic structure. We introduce the structure of nil-clean neutrosophic ring and some of its elementary properties are presented. Also, we have found the equivalence between classical nil-clean ring R and the corresponding neutrosophic ring $R(I)$, refined neutrosophic ring $R(I_1, I_2)$, and n-refined neutrosophic ring $R_n(I)$.

Keywords: Clean ring; nil-clean ring; neutrosophic ring; refined neutrosophic ring; clean neutrosophic ring; nil-clean neutrosophic ring.

1 Introduction

In 1980, Smarandache first time introduced the neutrosophic theory. The theory has given idea to the construction of the concept of new algebraic structures called the neutrosophic structure. Kandasamy and Smarandache[13] introduced the concept of neutrosophic algebraic structures. In this reference, several neutrosophic algebraic structures are introduced and studied. They introduced some neutrosophic algebraic structures like neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, neutrosophic bigroupoids, and neutrosophic rings. Kandasamy and Smarandache[14] introduced the concept of neutrosophic rings. Agboola et.al.,[4,5] given several aspect related to the neutrosophic algebraic structures and some types of this structure, concerning neutrosophic ring and its elementary properties, neutrosophic polynomial rings, and the unique factorization domain in neutrosophic polynomial rings.

Nicholson[15] introduced the notion of a clean ring. The notion of uniquely clean rings was firstly defined[15] in the commutative case as those rings in which every element is uniquely the sum of a unit and an idempotent. Later on, the authors[16] study such arbitrary rings, again calling them uniquely clean. Diesl[12] modified the definition of a clean ring and obtained an interesting new concept he called nil-clean ring. In his article he proved many fundamental properties as well as developed a general theory of nil-clean rings like uniquely nil-clean rings.

Suryoto and Uidjiani[19] studied the concept of neutrosophic clean ring with many elementary interesting properties. Recently, Abobala,[3] proved that a nuetrosophic ring $R(I)$ is clean if and only if R is clean. Motivated by this works, in this paper we have introduced and studied the notion of nil-clean nuetrosophic rings. Also, we have found the equivalence between classical nil-clean ring R and the corresponding neutrosophic ring $R(I)$, refined neutrosophic ring $R(I_1, I_2)$, and n-refined neutrosophic ring $R_n(I)$.

https://doi.org/10.54216/GJMSA.090105

Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023

2 Preliminary

In this section, we recall notions related to clean rings, nil-clean rings, neutrosophic rings, nil-clean neutrosophic rings. See especially,[3–5,9,16,19] for further details and background.

Definition 2.1. [13] Let $(R, +, \times)$ be a ring, $R(I) = \{a + bI : a, b \in R\}$ is called the neutrosophic ring where I is a neutrosophic element with condition $I^2 = I$.

Example 2.2. The set $Z_2(I) = \{\bar{0}, \bar{1}, I, \bar{1} + I\}$ is called the neutrosophic ring of integers modulo 3 generated by the ring $Z_2 = \{\bar{0}, \bar{1}\}$ and an idempotent element I.

Definition 2.3. [9] Let R be any ring, $x \in R$ be an arbitrary element, then.

(a). x is called a idempotent if $x^2 = x$.

(b). x is called an nilpotent if $x^k = 0$ for some positive integer k.

Definition 2.4. [9] Let R be a ring, then it is called clean if and only if for each $x \in R$, we have $x = e + l$, where e is a unit element and l is an idempotent element of R.

Example 2.5. The ring of integer modulo 3, $Z_3 = \{\bar{0}, \bar{1}, \bar{2}\}$ is a clean ring.

Definition 2.6. [12] Let R be a ring, then it is called nil-clean if and only if for each $x \in R$, we have $x = l + n$, where l is a idempotent element and n is an nil-potent element of R.

Example 2.7. The ring of integer modulo 2, $Z_2 = \{\bar{0}, \bar{1}\}$ is a nil-clean ring.

Definition 2.8. [19] Let $R(I)$ be a neutrosophic ring and $e \in R(I)$. The element e is an idempotent element for multiplivative operation on $R(I)$, if $e^2 = e$. The set of all idempotent elements of $R(I)$ is denoted by $Id(R(I))$.

Example 2.9. In $Z_3(I)$, the neutrosophic ring of integer modulo 3, $Id(Z_3(I)) = \{\bar{0}, \bar{1}, I, \bar{1} + I, \bar{1} + \bar{2}I\}$.

Definition 2.10. [19] Let $R(I)$ be a neutrosophic ring with $1 \neq 0$ and let $u \in R(I)$. The element u is a unit of $R(I)$ if there exists an element $v \in R(I)$ such that $uv = vu = 1$.

Example 2.11. In $Z_7(I)$, the neutrosophic ring of integer modulo 7, $\bar{3} + I$ is a unit in $Z_7(I)$, because we have $(\bar{3} + I)(\bar{5} + \bar{4}I) = (\bar{5} + \bar{4}I)(\bar{3} + I) = 1$.

Definition 2.12. [5] Let $R(I)$ be a neutrosophic ring. An element $x \in R(I)$ is called nilpotent element, if there exists a positive integer k such that $x^k = 0$. The set of all nilpotent elements of $R(I)$ is denoted by $N(R(I))$.

Example 2.13. In $Z_4(I)$, the neutrosophic ring of integer modulo 4, $N(Z_4(I)) = \{\bar{0}, \bar{2}, \bar{2}I, \bar{2} + \bar{2}I\}$.

Definition 2.14. [19] Assume that $R(I)$ is neutrosophic ring and that $x \in R(I)$. The element x is said to be clean if $x = e + u$, with e is an idempotent element and u is a unit of $R(I)$.

Example 2.15. All the elements of the neutrosophic ring $Z_2(I) = \{\bar{0}, \bar{1}, I, \bar{1} + I\}$ are clean.

Definition 2.16. [19] A neutrosophic ring in which all elements are clean, then the ring is called a clean neutrosophic ring. Furthermore, if each element of the neutrosophic ring is uniquely clean, then the ring is called a uniquely clean neutrosophic ring.

Example 2.17. The neutrosophic rings $Z_2(I)$ and $Z_3(I)$ are clean rings. Here, $Z_2(I)$ is uniquely clean ring, but $Z_3(I)$ is not uniquely clean.

Definition 2.18. [19] Let $R(I)$ be a neutrosophic ring and $e \in R(I)$ is an idempotent element. The idempotent e is a called central idempotent if $ex = xe$ for all $x \in R(I)$. The set of all central idempotent of $R(I)$ is denoted by $C(R(I))$.

Example 2.19. In the neutrosophic ring $Z_3(I)$, we have $Id(Z_3(I)) = C(Z_3(I)) = \{\bar{0}, \bar{1}, I, \bar{1} + \bar{2}I\}$.

Theorem 2.20. [1] *Let $R(I)$ be a neutrosophic ring, then $x = a + bI$ is idempotent in $R(I)$ if and only if $a, a + b$ are idempotents in R.*

https://doi.org/10.54216/GJMSA.090105

Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023

Theorem 2.21. [2] *Let $R(I)$ be a neutrosophic ring, then $x = a + bI$ is nilpotent in $R(I)$ if and only if $a, a + b$ are nilpotents in R.*

Definition 2.22. [5] Let $(R, +, \cdot)$ be a ring then $(R(I_1, I_2), +, \cdot)$ is called a refined neutrosophic ring generated by X, I_1, and I_2. Where $I_1^2 = I_1, I_2^2 = I_2, I_1 I_2 = I_2 I_1 = I_1$.

Definition 2.23. [18] Let $(R, +, \cdot)$ be a ring and $I_k, k = 1, 2, , \cdots , n$ be n indeterminacies. We define $R_n(I) = \{a_0 + a_1 I_1 + \cdots + a_n I_n; a_i \in R\}$ to be n-refined neutrosophic ring.
Addition and multiplication on $R_n(I)$ are defined as follows:

$$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i$$

$$\sum_{i=0}^{n} x_i I_i \times \sum_{j=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j$$

where $\times$ is the multiplication defined on the ring R and $xI_0 = x$ for all $x \in R$, $I_j I_i = I_i I_j = I_{min(i,j)}, I_0 I_j = I_j$.

3 Nil-clean Neutrosophic Rings

We begin with the following definition.

Definition 3.1. Let R be any ring, $R(I)$ be its corresponding neutrosophic ring. An element $x \in R(I)$ is said to be nil-clean if $x = l + n$, where l is an idempotent and n is a nilpotent element of $R(I)$. If, in addition, the existing idempotent l is unique, then x is called uniquely nil-clean element.

Example 3.2. Consider the neutrosophic ring

$$Z_3(I) = \{\bar{0}, \bar{1}, \bar{2}, I, 2I, \bar{1} + I, \bar{1} + 2I, \bar{2} + I, \bar{2} + 2I\}.$$

Since $\bar{0}$ is a nil-potent in $Z_3(I)$, so the idempotent elements $\bar{0}, \bar{1}, I, \bar{1} + 2I$ are nil-clean elements of $Z_3(I)$. The only nilpotent elements of $Z_3(I)$ is $\bar{0}$, so $\bar{0}, \bar{1}, I, \bar{1} + 2I$ are uniquely nil-clean elements of $Z_3(I)$.

Definition 3.3. A neutrosophic ring in which all elements are nil-clean, then the ring is called a nil-clean neutrosophic ring. Furthermore, if each element of the neutrosophic ring is uniquely nil-clean, then the ring is called a uniquely nil-clean neutrosophic ring.

Example 3.4. $Z_2(I) = \{\bar{0}, \bar{1}, I, \bar{1} + I\}$ is a nil-clean neutrosophic ring, that is because all the elements in $Z_2(I)$ are idempotents, $\bar{0}$ is a nilpotent element in $Z_2(I)$.

Lemma 3.5. *If x is a nilpotent element of $R(I)$, then $1 + x$ is a unit in $R(I)$.*

Proof. If x is a nilpotent element of $R(I)$ then $x^k = 0$ for some $k > 0$. But then, $(1 + x)(1 - x + x^2 - x^3 + ... + (-1)^{k-1} x^{k-1}) = 1$ and so $1 + x$ is unit in $R(I)$. ☐

Theorem 3.6. *Every nil-clean neutrosophic ring is clean neutrosophic ring.*

Proof. Suppose that $R(I)$ is a nil-clean neutrosophic ring, and let $x \in R(I)$. Then $x - 1$ is an element of $R(I)$ and hence $x - 1 = l + n$, where l is an idempotent element and n is a nilpotent element of $R(I)$.
This implies that, $x = l + (1 + n)$ is a nil-clean element of $R(I)$ because $1 + n$ is a unit element of $R(I)$ by Lemma 3.5. ☐

Example 3.7. Consider, the clean neutrosophic ring

https://doi.org/10.54216/GJMSA.090105
Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023

$$Z_3(I) = \{\bar{0}, \bar{1}, \bar{2}, I, 2I, \bar{1}+I, \bar{1}+2I, \bar{2}+I, \bar{2}+2I\}.$$

The only nilpotent element of $Z_3(I)$ is $\bar{0}$ and $\bar{2}+I$ is not an idempotent element in $Z_3(I)$ so it is not nil-clean.

Theorem 3.8. *Let R be any ring, $R(I)$ be its corresponding neutrosophic ring. $R(I)$ is nil-clean if and only if R is nil-clean.*

Proof. We know that, $R \subset R(I)$. So, if $R(I)$ is nil-clean then R is nil-clean.
Conversely, suppose that R is nil-clean. We have to show that $R(I)$ is nil-clean.
Let $x \in R(I)$. Then, $x = a + bI$, where $a, b, a+b \in R$.
By our assumption, we have $a = l_1 + n_1, b = l_2 + n_2$ and $a + b = l_3 + n_3$, where l_1, l_2, l_3 are idempotent elements of R and n_1, n_2, n_3 are nilpotent elements of R.
Now,

$$\begin{aligned} x &= a + bI \\ &= a + (a + b - a)I \\ &= (l_1 + n_1) + ((l_3 + n_3) - (l_1 + n_1))I \\ &= [l_1 + (l_3 - l_1)I] + [n_1 + (n_3 - n_1)I] \\ &= x_1 + x_2 \end{aligned}$$

where $x_1 = l_1 + (l_3 - l_1)I$ and $x_2 = n_1 + (n_3 - n_1)I$. Since, $l_1 + (l_3 - l_1) = l_3$ is idempotent in R, by Theorem 2.20, $x_1 = l_1 + (l_3 - l_1)I$ is idempotent in $R(I)$. Similarly, $n_1 + n_3 - n_1 = n_3$ is nilpotent in R, by Theorem 2.21, $x_2 = n_1 + (n_3 - n_1)I$ is nilpotent in $R(I)$. Hence, $R(I)$ is nil-clean. ☐

Theorem 3.9. *If $R(I)$ is a neutrosophic ring, then every central idempotent of $R(I)$ is uniquely nil-clean element.*

Proof. We now that, every idempotent element of $R(I)$ are nil-clean. Let x be a central idempotent element of $R(I)$. Then $x = (1 - x) + (2x - 1)$. Suppose that $x = l + n$, where l is an idempotent and n is a nilpotent element of $R(I)$. Since $nx = xn$, we obtain $l + n = (l + n)^2 = l + 2ln + n^2$. So, we have $n = 1 - 2l$ and hence $l = 1 - x$, as reuired. ☐

Lemma 3.10. *Let $R(I)$ be uniquely nil-clean neutrosophic ring. Then all idempotents of $R(I)$ are central.*

Proof. Let $l \in R(I)$ be an idempotent element and x be any element of $R(I)$. Now, the element $l + lx - lxl$ can be written as $l + (lx - lxl)$ or $(l + (lx - lxl)) + 0$ each time as the sum of an idempotent and a nilpotent element of $R(I)$. Since $R(I)$ is uniquely nil-clean, we have $l = l + (lx - lxl)$. This implies that $lx - lxl = 0$ and so $lx = lxl$. In the similar way, we can show that $xl = lxl$. Hence $lx = xl$ as required. ☐

For the boolean neutrosophic ring, we have the following result. A neutrosophic ring $R(I)$, is called boolean neutrosophic ring if $x^2 = x$ for all $x \in R(I)$.

Theorem 3.11. *Every Boolean neutrosophic ring is uniquely nil-clean.*

Proof. If $R(I)$ is a Boolean neutrosophic ring, then $R(I) = Id(R(I))$. Since Boolean rings are abelian, we have $Id(R(I)) = C(R(I))$. This implies that, $R(I) = C(R(I))$. By Theorem 3.9, every element of the ring $R(I)$ are uniquely nil-clean. Hence $R(I)$ is uniquely nil-clean ring. ☐

https://doi.org/10.54216/GJMSA.090105

Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023

4 Refined Nil-clean Neutrosophic rings

In this section, we study the equivalence between classical nil-clean ring R and the corresponding refined neutrosophic ring $R(I_1, I_2)$, and n-refined neutrosophic ring $R_n(I)$. We begin with the following theorem.

Theorem 4.1. [1] *Let $R(I_1, I_2)$ be a refined neutrosophic ring, then*

(i) $x = a + bI_1 + cI_2$ is idempotent in $R(I_1, I_2)$ if and only if $a, a + c, a + b + c$ are idempotents in R.

(ii) $x = a + bI_1 + cI_2$ is nilpotent in $R(I_1, I_2)$ if and only if $a, a + c, a + b + c$ are nilpotents in R.

Remark 4.2. Theorem 4.1 should be extended to the case of n-refined neutrosophic rings.

Theorem 4.3. [17] *Let R be a ring. Then the class of nil-clean rings is closed under homomorphic images.*

Theorem 4.4. *Let R be any ring, $R(I_1, I_2)$ be its corresponding refined neutrosophic ring. $R(I_1, I_2)$ is nil-clean if and only if R is nil-clean.*

Proof. Assume that $R(I_1, I_2)$ is nil-clean. Since R is a homomorphic image of $R(I_1, I_2)$, so R is nil-clean by Theorem 4.3.
Conversely, assume that R is nil-clean, we must prove that $R(I_1, I_2)$ is nil-clean. Let $x = a + bI_1 + cI_2 \in R(I_1, I_2)$ then $a, a + c, a + b + c \in R$. Since R is nil-clean we have $a = l_1 + n_1, a + c = l_2 + n_2, a + b + c = l_3 + n_3$, where l_i are idempotent elements and n_i are nilpotent elements of R. Now,

$$\begin{aligned}
x &= a + bI_1 + cI_2 \\
&= a + [(a + b + c) - (a + c)]I_1 + [(a + c) - a]I_2 \\
&= (l_1 + n_1) + [(l_3 + n_3) - (l_2 + n_2)]I_1 + [(l_2 + n_2) - (l_1 + n_1)]I_2 \\
&= (l_1 + n_1) + [(l_3 - l_2) + (n_3 - n_2)]I_1 + [(l_2 - l_1) + (n_2 - n_1)]I_2 \\
&= (l_1 + (l_3 - l_2)I_1 + (l_2 - l_1)I_2) + (n_1 + (n_3 - n_2)I_1 + (n_2 - n_1)I_2) \\
&= x_1 + x_2.
\end{aligned}$$

where, $x_1 = l_1 + (l_3 - l_2)I_1 + (l_2 - l_1)I_2$ and $x_2 = n_1 + (n_3 - n_2)I_1 + (n_2 - n_1)I_2$. By Theorem 4.1, $l_1, (l_2 - l_1) + l_1 = l_2, l_1 + (l_2 - l_1) + (l_2 - l_1) = l_3$ are idempotents in R. Therefore, x_1 is a idempotent element of R. Also, x_2 is a nilpotent element of R by a similar discussion. Hence $R(I_1, I_2)$ is nil-clean. □

Theorem 4.5. *Let R be any ring, $R_n(I)$ be its corresponding n-refined neutrosophic ring. Then $R_n(I)$ is nil-clean if and only if R is nil-clean.*

Proof. Assume that $R_n(I)$ is nil-clean. Since R is a homomorphic image of $R_n(I)$, so R is nil-clean by Theorem 4.3.
Conversely, assume that R is nil-clean, we must prove that $R_n(I)$ is nil-clean. Let $x = a_0 + a_1I_1 + a_2I_2 + \cdots a_nI_n \in R_n(I)$. Now,

$$\begin{aligned}
x &= a_0 + a_1I_1 + a_2I_2 + \cdots a_nI_n \\
&= a_0 + \left[\sum_{i=0}^{n} a_i - \sum_{i\neq 1}^{n} a_i\right] I_1 + \left[\sum_{i\neq 1}^{n} a_i - \sum_{i\neq 1,2}^{n} a_i\right] I_2 + \cdots \\
&\qquad + [(a_{n-1} + a_n + a_0) - (a_{n-1} + a_0)]\, I_{n-1} + [(a_0 + a_n) - a_0]\, I_n \\
&= (l_0 + m_0) + [(l_1 + m_1) - (l_2 + m_2)]\, I_1 + [(l_2 + m_2) - (l_3 + m_3)]\, I_2 + \cdots \\
&\qquad + [(l_{n-1} + m_{n-1}) - (l_n + m_n)]\, I_{n-1} + [(l_n + m_n) - (l_0 + m_0)]\, I_n \\
&= [l_0 + (l_1 - l_2)I_1 + (l_2 - l_3)I_2 + \cdots (l_{n-1} - l_n)I_{n-1} + (l_n - l_0)I_n] \\
&\qquad + [m_0 + (m_1 - m_2)I_1 + (m_2 - m_3)I_2 + \cdots (m_{n-1} - m_n)I_{n-1} + (m_n - m_0)I_n] \\
&= x_1 + x_2.
\end{aligned}$$

where, l_i are idempotent elements and n_i are nilpotent elements of R. By the Remark 4.2, x_1 is an idempotent element of R, because $l_0, l_0 + (l_1 - l_2) + \cdots + (l_n - l_0) = l_1, l_0 + (l_2 - l_3) + (l_3 - l_4) + \cdots + (l_{n-1} - l_n) + (l_n - l_0) = l_2, \cdots$ are idempotents in R. Also, x_2 is a nilpotent element of R by a similar discussion. Hence $R_n(I)$ is nil-clean. □

https://doi.org/10.54216/GJMSA.090105

Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023

5 Conclusion

In this article, we introduce the the new class of rings called, nil-clean neutrosophic rings is a ring and we have studied various properties of nil-clean neutrosophic rings with proper examples. Also, we have determined necessary and sufficient condition for a neutrosophic ring, refined neutrosophic ring and n-refined neutrosophic ring to be nil-clean.

References

[1] M. Abobala, *On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings*, Journal of New Theory, vol. 33, 2020.

[2] M. Abobala, *A Study of Nil Ideals and Kothey Conjecture in Neutrosophic Rings*, International Journal of Mathematics and Mathematical Sciences, vol. 1, 2021.

[3] M. Abobala, M. Bal and A. Hatip, *A Study Of Some Neutrosophic Clean Rings*, International Journal of Neutrosophic Science (IJNS) Vol. 18, No. 01, pp. 14-19, 2022

[4] A. A. A. Agboola, A. D. Akinola, and O. Y. Oyebola, *Neutrosophic Rings I*, International J.Math combin, Vol 4, pp. 1-14, 2011.

[5] A. A. A. Agboola , E. O. Adeleke and S. A, Akinleye, *Neutrosophic Ring II*, International, J. Math. Combin. 2(1), 2012.

[6] M. Abobala, *On Some Special Substructures of Neutrosophic Rings and Their Properties*, International Journal of Neutrosophic Science", Vol. 4, pp. 72-81, 2020.

[7] E.O. Adeleke, A.A.A. Agboola and F. Smarandache, *Refined Neutrosophic Rings I*, International Journal of Neutrosophic Science, Vol 2 , pp 77-81, 2020.

[8] E.O. Adeleke, A.A.A. Agboola and F. Smarandache, "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol 2, pp 89-94, 2020.

[9] D.D. Anderson, V.P. Camilo, *Commutative rings whose elements are a sum of a unit and idempotent*, Comm. Algebra, 30 (2002), pp. 3327-3336.

[10] S. Breaz, G. Calugareanu, P. Danchev, T. Micu, *Nil-clean matrix rings*, Linear Algebra Appl., 439 (2013), pp. 3115-3119.

[11] T. Chalabathi and K. Kumar, *Neutrosophic Units of Neutrosophic Rings and Fields*, Neutrosophic Sets and Systems, vol 21, pp 5-9, 2018.

[12] A.J. Diesl, *Nil clean rings*, J. Algebra, 383 (2013), pp. 197-211.

[13] W. B. V. Kandasamy and F. Smarandache, *Some Neutrosophic Algebraic Structures and Neutrosophic n-Algebraic Structures* (Arizona: Hexis Phoenix), 2006.

[14] W. B. V. Kandasamy and F. Smarandache, *Neutrosophic Rings* (Arizona: Hexis Phoenix), 2006.

[15] W.K. Nicholson, *Lifting idempotents and exchange rings*, Trans. Amer. Math. Soc., 229 (1977), pp. 269-278.

[16] W.K. Nicholson, Y. Zhou, *Rings in which elements are uniquely the sum of an idempotent and a unit*, Glasg. Math. J., 46 (2004), pp. 227-236.

[17] M. Samiei, *Commutative rings whose proper homomorphic images are nil clean*, Novi Sad J. Math. Vol. 50, No. 1, 2020, 37-44

[18] F. Smarandache and M. Abobala, *n-Refined neutrosophic Rings*, International Journal of Neutrosophic Science, vol. 5, pp. 83-90, 2020.

https://doi.org/10.54216/GJMSA.090105
Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023

[19] H. Suryoto, and T. Uidjiani, *On Clean Neutrosophic Rings*, IOP Conference Series, Journal of Physics, Conf. Series 1217, 2019.

https://doi.org/10.54216/GJMSA.090105
Received: June 12, 2023 Revised: September 11, 2023 Accepted: December 04, 2023