Single Valued Neutrosophic Dynamic Vertex Coloring of Some Cartesian Product and Join of SVNG′s

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Abstract

The concept of neutrosophic sets makes it easier to analyse values that are ambiguous or indeterminate. We illustrate the concept of single-valued neutrosophic R-dynamic vertex coloring in this study. In addition, the single valued neutrosophic dynamic vertex coloring of the Cartesian product of path with cycle, cycle with cycle, complete SVNG with complete SVNG and cycle has been determined. Also, we have discussed the single valued neutrosophic dynamic vertex coloring of join of complete SVNG with complete SVNG and cycle, cycle with cycle.
Keywords: Single Valued Neutrosophic Graph; Single Valued Neutrosophic Vertex Coloring; Single Valued Neutrosophic R-dynamic Vertex Coloring; Cartesian product; Join.

1 Introduction

In order to deal with uncertainty, Zadeh [24] developed the phrase 'degree of membership' and defined the idea of fuzzy set in 1965. After ten years, A. Rosenfeld [19] introduced fuzzy graphs, while Munoz et al. [22] pioneered the concept of fuzzy chromatic number in 2004. Atanassov [7] established the notion of intuitionistic fuzzy set by incorporating the degree of non-membership as an independent component in the concept of fuzzy set. In addition, he developed intuitionistic fuzzy graphs in 1999, and Ismail and Rifayathali [14] studied intuitionistic fuzzy graph coloring using (α, β) cuts in 2015. Smarandache [20] established the concept of degree of indeterminacy, it was defined as an independent component, and the idea of neutrosophic set was defined for dealing with partial, uncertain, and conflicting data from a philosophical standpoint information from the real world. In 2010, Wang et al. [23] focused on single valued neutrosophic sets. Dhavaseelan et al. [13] developed and studied the Strong Neutrosophic graph and its features in 2015, and Akram and Shahzadi [1-3] introduced the Single valued neutrosophic idea in 2016. Broumi et al. [8-12] extended their work in Single valued neutrosophic graphs, Isolated single valued graphs, Uniform single valued graphs, Interval valued neutrosophic graphs (IVNG), and Bipolar neutrosophic graphs. In their study published in 2018, Dhavaseelan et al. [13] examined single valued co-neutrosophic graphs and Sinha et al. [21] expanded the scope of the single-valued work for signed digraphs.

Rohini et al. established the notion of single valued neutrosophic vertex coloring, single valued neutrosophic edge coloring, and single valued neutrosophic total coloring of a single valued neutrosophic graph with examples in their research publications [16, 17] published in 2019. In [18], Rohini et al. expanded on their work on single valued neutrosophic vertex coloring and developed the novel concept of single valued neutrosophic irregular vertex coloring. In [15], Bruce Montgomery proposed the concept of r-dynamic coloring. The r-dynamic coloring [4] of a graph is a proper vertex coloring of the graph in which each vertex w receives at least $min\{r, d(w)\}$ different colors from its neighbours. We have presented the concept of single valued neutrosophic R-dynamic vertex coloring in [5] and single valued neutrosophic R-dynamic edge coloring in [6].

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2 Preliminaries

Definition 2.1. [20] Consider Y to be a set of points (objects). The truth membership function $m_N(y)$, an indeterminacy function $i_N(y)$, and a falsity membership (nonmembership) function $f_N(y)$, are then used to characterise the **neutrosophic set** N in Y. The mappings $m_N(y)$, $i_N(y)$ and $f_N(y)$ are real standard or non-standard subsets of $]0^-,1^+[$ which means $m_N(y): Y \rightarrow]0^-,1^+[$, $i_N(y): Y \rightarrow]0^-,1^+[$ and $f_N(y): Y \rightarrow]0^-,1^+[$. Also sum of membership, indeterminacy and falsity function lies in $]0^-,1^+[$.

Definition 2.2. [2] **Single Valued Neutrosophic Graph (SNVG)** $G = (R, S)$ is a pair with the following characteristics: $R : N \rightarrow [0, 1]$ which is a single valued neutrosophic set on N and $S : N \times N \rightarrow [0, 1]$, a single valued neutrosophic relation on N and satisfies
$m_S(wz) \leq min\{m_R(w), m_R(z)\}$
$i_S(wz) \leq min\{i_R(w), i_R(z)\}$
$f_S(wz) \leq max\{f_R(w), f_R(z)\}$
for all $w, z \in N$. The sets R and S are the single valued neutrosophic vertex set and edge set of G respectively. If $m_S(wz) = m_S(zw)$, $i_S(wz) = i_S(zw)$ and $f_S(wz) = f_S(zw)$ for all $w, z \in N$ we say that the single valued neutrosophic relation S is symmetric.

Definition 2.3. [3] If an SVNG $G = (R, S)$ meets the following requirements:
$m_S(wz) = min\{m_R(w), m_R(z)\}$
$i_S(wz) = min\{i_R(w), i_R(z)\}$
$f_S(wz) = max\{f_R(w), f_R(z)\}$
for all $w, z \in R$ it is called a **complete neutrosophic graph (CSVNG)**.

Definition 2.4. [16] If the following criteria are met:
1. $\vee\gamma_h(w) = R, \forall w \in R$
2. $\gamma_h \wedge \gamma_i = 0$
3. For each incident vertices of the edge wz of G, $min\{\gamma_h(m_R(w)), \gamma_h(m_R(z))\} = 0$, $min\{\gamma_h(i_R(w)), \gamma_h(i_R(z))\} = 0$ and $max\{\gamma_h(f_R(w)), \gamma_h(f_R(z))\} = 1, (1 \leq h \leq k)$.
the collection $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_k\}$ of SVN fuzzy sets is referred to as **k-Single Valued Neutrosophic Vertex Coloring (SVNVC)** of an SVNG G = (R, S). This is notated as $\chi^v(G)$ and is referred to as the SVN chromatic number of the SVNG .

Definition 2.5. [5] If the following criteria are met:
1. $\vee\gamma_h(w) = R, \forall w \in R$
2. $\gamma_h \wedge \gamma_i = 0$
3. For each incident vertices of the edge wz of G, $min\{\gamma_h(m_R(w)), \gamma_h(m_R(z))\} = 0$, $min\{\gamma_h(i_R(w)), \gamma_h(i_R(z))\} = 0$ and $max\{\gamma_h(f_R(w)), \gamma_h(f_R(z))\} = 1, (1 \leq h \leq k)$.
4. Every vertex z with n number of incident edges, the corresponding incident vertices of the vertex z receives atleast $min\{R, n\}$ different members(colors) from the set Γ.
the set $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_k\}$ of SVN fuzzy sets is referred to as the **k-Single Valued Neutrosophic R-dynamic Vertex Coloring (SVNRVC)** of a SVNG $G = (R, S)$.
Here, $1 \leq R \leq M$ where M represents the maximum number of incident edges of the vertices of SVNG G.
The SVNRVC of SVNG G with the least value of k is indicated as $\chi^v_R(G)$, is known as Single Valued Neutrosophic R-dynamic chromatic number of the SVNG G.
Furthermore, we define the SVNG G's Single Valued Neutrosophic Dynamic Chromatic Number, $\chi^v_2(G)$ by substituting $R = 2$ in criteria 4, i.e., every vertex z with n incident edges, the incident vertices of the vertex z are assigned at least $min\{2, m\}$ different members (colours) from Γ.

Example: Consider the following SVNG $G = (R, S)$ with SVN vertex set $R = \{w_1, w_2, \cdots, w_8\}$ and SVN edge $S = \{w_jw_k | jk = 12, 23, 34, 41, 15, 26, 37, 48\}$ the membership functions defined as,

$$(m_R(w_j), i_R(w_j), f_R(w_j)) = \begin{cases} (0.2, 0.4, 0.8) & j = 1, 3 \\ (0.1, 0.3, 0.9) & i = 2, 4 \\ (0.7, 0.6, 0.5) & i = 5, 7 \\ (0.4, 0.7, 0.8) & i = 6, 8 \end{cases}$$

$$(m_S(w_jw_k), i_S(w_jv_k), f_S(w_jw_k)) = \begin{cases} (0.1, 0.3, 0.9) & jk = 12, 34 \\ (0.1, 0.1, 0.8) & jk = 23, 41 \\ (0.2, 0.3, 0.7) & jk = 15, 37 \\ (0.1, 0.2, 0.6) & jk = 26, 48 \end{cases}$$

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Here $M = 3$ so $1 \leq R \leq 3$
For $R = 1$ let $\Gamma = \{\gamma_1, \gamma_2\}$ be a family of SVN fuzzy sets defined on R as below:

$$\gamma_1(w_j) = \begin{cases} (0.2, 0.4, 0.8) & for \quad j = 1, 3 \\ (0.4, 0.7, 0.8) & for \quad j = 6, 8 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_2(w_j) = \begin{cases} (0.1, 0.3, 0.9) & for \quad j = 2, 4 \\ (0.7, 0.6, 0.5) & for \quad j = 5, 7 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2\}$ guarantees that the SVNRVC conditions are met. Families with fewer than two points did not meet our criterion when $R = 1$.
For $R = 2$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be a family of SVN fuzzy sets defined on R as below:

$$\gamma_1(w_j) = \begin{cases} (0.2, 0.4, 0.8) & for \quad j = 1, 3 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_2(w_j) = \begin{cases} (0.7, 0.6, 0.5) & for \quad j = 5, 7 \\ (0.4, 0.7, 0.8) & for \quad j = 6, 8 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_3(w_j) = \begin{cases} (0.1, 0.3, 0.9) & for \quad j = 2, 4 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ guarantees that the SVNRVC conditions are met. Families with fewer than three points did not meet our criterion when $R = 2$.
For $R = 3$ let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be a family of SVN fuzzy sets defined on R as below:

$$\gamma_1(w_j) = \begin{cases} (0.2, 0.4, 0.8) & for \quad j = 1, 7 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_2(w_j) = \begin{cases} (0.1, 0.3, 0.9) & for \quad j = 2, 8 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_3(w_j) = \begin{cases} (0.7, 0.6, 0.5) & for \quad j = 3, 5 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_4(w_j) = \begin{cases} (0.4, 0.7, 0.8) & for \quad j = 4, 6 \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ guarantees that the SVNRVC conditions are met. Families with fewer than four points did not meet our criterion when $R = 3$.

Hence $\chi_R^v(G) = \begin{cases} 2 & for \quad R = 1 \\ 3 & for \quad R = 2 \\ 4 & for \quad R = 3 \end{cases}$

Definition 2.6. [9] **Path** P_q in a single valued neutrosophic graph $G = (R, S)$ is an arrangement of different vertices $w_1, w_2, \cdots, w_q$ that satisfy the conditions, for $2 \leq h \leq q \quad m_S(w_{h-1}, w_h), i_S(w_{h-1}, w_h)$ and $f_S(w_{h-1}, w_h)$ are all greater than zero.

Definition 2.7. [9] A **cycle** C_q in a single valued neutrosophic graph $G = (R, S)$ is a sequence of different vertices $w_1, w_2, \cdots, w_q$ that satisfy the conditions for, $2 \leq h \leq q \quad m_S(w_{h-1}, w_h), i_S(w_{h-1}, w_h)$ and $f_S(w_{h-1}, w_h)$ and $m_S(w_1, w_q)$, $i_S(w_1, w_q)$, $f_S(w_1, w_q)$ are all greater than zero.

Definition 2.8. [3] Consider $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ be two single valued neutrosophic graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. Then the cartesian product $G_1 \times G_2$ is defined to be the pair (R, S) such that
1. $m_R(w_1, w_2) = min\{m_{R_1}(w_1), m_{R_2}(w_2)\}$
$i_R(w_1, w_2) = min\{i_{R_1}(w_1), i_{R_2}(w_2)\}$
$f_R(w_1, w_2) = max\{f_{R_1}(w_1), f_{R_2}(w_2)\}$
2. $m_S((w, w_2)(w, z_2)) = min\{m_{R_1}(w), m_{S_2}(w_2 z_2)\}$
$i_S((w, w_2)(w, z_2)) = min\{i_{R_1}(w), i_{S_2}(w_2 z_2)\}$
$f_S((w, w_2)(w, z_2)) = max\{f_{R_1}(w), f_{S_2}(w_2 z_2)\}$ for all $w \in V_1$ and for all $w_2 z_2 \in E_2$.
3. $m_S((w_1, z), (z_1, z)) = min\{m_{S_1}(w_1 z_1), m_{R_2}(z)\}$
$i_S((w_1, z), (z_1, z)) = min\{i_{S_1}(w_1 z_1), i_{R_2}(z)\}$
$f_S((w_1, z), (z_1, z)) = max\{f_{S_1}(w_1 z_1), f_{R_2}(z)\}$ for all $z \in V_2$ and for all $w_1 z_1 \in E_1$.

Definition 2.9. [3] Assume $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ be two single valued neutrosophic graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. Then the join $G_1 + G_2$ is defined to be the pair (R, S) satisfying the criteria

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$$1. m_R(w) = \begin{cases} m_{R_1}(w) & if \quad w \in V_1 \; and \; w \notin V_2 \\ m_{R_2}(w) & if \quad w \in V_2 \; and \; w \notin V_1 \\ max\{m_{R_1}(w), m_{R_2}(w)\} & if \quad w \in V_1 \cap V_2 \end{cases}$$

$$2. \; i_R(w) = \begin{cases} i_{R_1}(w) & if \quad w \in V_1 \; and \; w \notin V_2 \\ i_{R_2}(w) & if \quad w \in V_2 \; and \; w \notin V_1 \\ max\{i_{R_1}(w), i_{R_2}(w)\} & if \quad w \in V_1 \cap V_2 \end{cases}$$

$$3. \; f_R(w) = \begin{cases} f_{R_1}(w) & if \quad w \in V_1 \; and \; w \notin V_2 \\ f_{R_2}(w) & if \quad w \in V_2 \; and \; w \notin V_1 \\ min\{f_{R_1}(w), f_{R_2}(w)\} & if \quad w \in V_1 \cap V_2 \end{cases}$$

$$4. \; m_S(wz) = \begin{cases} m_{S_1}(wz) & if \quad wz \in E_1 \; and \; wz \notin E_2 \\ m_{S_2}(wz) & if \quad wz \in E_2 \; and \; wz \notin E_1 \\ max\{m_{S_1}(wz), m_{S_2}(wz)\} & if \quad wz \in E_1 \cap E_2 \\ min\{m_{R_1}(w), m_{R_2}(z)\} & if \quad wz \in E' \end{cases}$$

$$5. \; i_S(wz) = \begin{cases} i_{S_1}(wz) & if \quad wz \in E_1 \; and \; wz \notin E_2 \\ i_{S_2}(wz) & if \quad wz \in E_2 \; and \; wz \notin E_1 \\ max\{i_{S_1}(wz), i_{S_2}(wz)\} & if \quad wz \in E_1 \cap E_2 \\ min\{i_{R_1}(w), i_{R_2}(z)\} & if \quad wz \in E' \end{cases}$$

$$6. \; f_S(wz) = \begin{cases} f_{S_1}(wz) & if \quad wz \in E_1 \; and \; wz \notin E_2 \\ f_{S_2}(wz) & if \quad wz \in E_2 \; and \; wz \notin E_1 \\ min\{f_{S_1}(wz), f_{S_2}(wz)\} & if \quad wz \in E_1 \cap E_2 \\ max\{f_{R_1}(w), f_{R_2}(z)\} & if \quad wz \in E' \end{cases}$$

3 Single Valued Neutrosophic Dynamic Coloring of Graphs

Here we have discussed the Single Valued Neutrosophic Dynamic Chromatic Number of some graphs like Cartesian product of path with cycle, cycle with cycle, complete SVNG with complete SVNG and cycle; join of complete SVNG with complete SVNG and cycle, cycle with cycle.

Theorem 3.1. *Let $l \geq 2, m \geq 3$, then the single valued neutrosophic dynamic chromatic number of the cartesian product of path P_l with cycle C_m is* $\chi_2^v(P_l \times C_m) = \begin{cases} 3 & when \quad m = 3 \\ 4 & when \quad m \; is \; otherwise \end{cases}$.

Proof. Consider path $P_l = (R_1, S_1)$ with SVN vertex set $R_1 = \{w_1, w_2, \cdots, w_l\}$ and cycle $C_m = (R_2, S_2)$ with SVN vertex set $R_2 = \{z_1, z_2, \cdots, z_m\}$. Then the cartesian product of path P_l with cycle C_m is SVNG $P_l \times C_m = (R, S)$ with SVN vertex set $R = \{(w_j, z_k) : 1 \leq j \leq l, 1 \leq k \leq m\}$ and edge set defined as in the definition 2.8.

Case 1: When $m = 3$.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be the collection of fuzzy sets determined on vertices of $P_l \times C_3$ for $R = 2$ as follows:

$$\gamma_1((w_j, z_k)) = \begin{cases} (m_R((w_i, z_1)), i_R((w_i, z_1)), f_R((w_i, z_1)) & for \quad i \equiv 1(mod\, 3), \\ (m_R((w_j, z_3)), i_R((w_j, z_3)), f_R((w_j, z_3)) & for \quad j \equiv 2(mod\, 3), \\ (m_R((w_k, z_2)), i_R((w_k, z_2)), f_R((w_k, z_2)) & for \quad k \equiv 0(mod\, 3), \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_2((w_j, z_k)) = \begin{cases} (m_R((w_i, z_2)), i_R((w_i, z_2)), f_R((w_i, z_2)) & for \quad i \equiv 1(mod\, 3), \\ (m_R((w_j, z_1)), i_R((w_j, z_1)), f_R((w_j, z_1)) & for \quad j \equiv 2(mod\, 3), \\ (m_R((w_k, z_3)), i_R((w_k, z_3)), f_R((w_k, z_3)) & for \quad k \equiv 0(mod\, 3), \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

$$\gamma_3((w_j, z_k)) = \begin{cases} (m_R((w_i, z_3)), i_R((w_i, z_3)), f_R((w_i, z_3)) & for \quad i \equiv 1(mod\, 3), \\ (m_R((w_j, z_2)), i_R((w_j, z_2)), f_R((w_j, z_2)) & for \quad j \equiv 2(mod\, 3), \\ (m_R((w_k, z_1)), i_R((w_k, z_1)), f_R((w_k, z_1)) & for \quad k \equiv 0(mod\, 3), \\ (0, 0, 1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ guarantees that the SVNRVC conditions are met. Families with fewer than three points did not meet our criterion.

Case 2 : When m is otherwise i. e. , $m \neq 3$.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be the collection of fuzzy sets determined on vertices of $P_l \times C_m$ for $R = 2$ as follows:

Subcase 1 : When m is even.

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$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,2)\ and\ k \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,2)\ and\ k \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 0(mod\,2)\ and\ k \equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 0(mod\,2)\ and\ k \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Subcase 2 : When m is odd.

$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 1(mod\,2)\ but\ k \neq m, \\ & \quad j \equiv 2(mod\,4)\ and\ k = m, \\ & \quad j \equiv 3(mod\,4)\ and\ k \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,4)\ and\ k \equiv 0(mod\,2), \\ & \quad j \equiv 2(mod\,4)\ and\ k = 1, \\ & \quad j \equiv 3(mod\,4)\ and\ k \equiv 1(mod\,2)\ but\ k \neq 1, m, \\ & \quad j \equiv 0(mod\,4)\ and\ k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,4)\ and\ k = m, \\ & \quad j \equiv 2(mod\,4)\ and\ k \equiv 1(mod\,2)\ but\ k \neq 1, m, \\ & \quad j \equiv 3(mod\,4)\ and\ k = 1, \\ & \quad j \equiv 0(mod\,4)\ and\ k \equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 2(mod\,4)\ and\ k \equiv 0(mod\,2) \\ & \quad j \equiv 3(mod\,4)\ and\ k = m, \\ & \quad j \equiv 0(mod\,4)\ and\ k \equiv 1(mod\,2)\ but\ k \neq m \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ guarantees that the SVNRVC conditions are met. Families with fewer than four points did not meet our criterion. □

Theorem 3.2. *Let $l \geq 3, m \geq 3$, then the single valued neutrosophic dynamic chromatic number of the cartesian product of cycle C_l with cycle C_m is* $\chi_2^v(C_l \times C_m) = \begin{cases} 3 & when \quad 3/lm \\ 4 & when \quad otherwise \end{cases}$.

Proof. Consider cycle $C_l = (R_1, S_1)$ with SVN vertex set $R_1 = \{w_1, w_2, \cdots, w_l\}$ and cycle $C_m = (R_2, S_2)$ with SVN vertex set $R_2 = \{z_1, z_2, \cdots, z_m\}$. Then the cartesian product of cycle C_l with cycle C_m is SVNG $C_l \times C_m = (R, S)$ with SVN vertex set $R = \{(w_j, z_k) : 1 \leq j \leq l, 1 \leq k \leq m\}$ and edge set defined as in the definition 2.8.

Case 1: When $3/lm$.

Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ be the collection of fuzzy sets determined on vertices of $C_l \times C_m$ for $R = 2$ as follows:

Subcase 1 : When $l \equiv 0, 2(mod\,3)$ and $m \equiv 0(mod\,3)$.

$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,3)\ and\ k \equiv 1(mod\,3), \\ & \quad j \equiv 2(mod\,3)\ and\ k \equiv 0(mod\,3), \\ & \quad j \equiv 0(mod\,3)\ and\ k \equiv 2(mod\,3) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,3)\ and\ k \equiv 2(mod\,3), \\ & \quad j \equiv 2(mod\,3)\ and\ k \equiv 1(mod\,3), \\ & \quad j \equiv 0(mod\,3)\ and\ k \equiv 0(mod\,3) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,3)\ and\ k \equiv 0(mod\,3), \\ & \quad j \equiv 2(mod\,3)\ and\ k \equiv 2(mod\,3), \\ & \quad j \equiv 0(mod\,3)\ and\ k \equiv 1(mod\,3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Subcase 2 : When $l \equiv 1(mod\,3)$ and $m \equiv 0(mod\,3)$.

$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j \equiv 1(mod\,3)\ and\ k \equiv 1(mod\,3)\ but\ j \neq l, \\ & \quad j \equiv 2(mod\,3)\ and\ k \equiv 0(mod\,3), \\ & \quad j \equiv 0(mod\,3)\ and\ k \equiv 2(mod\,3), \\ & \quad j = l\ and\ k \equiv 0(mod\,3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

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$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 1(mod\,3)\ and\ k\equiv 2(mod\,3)\ but\ j\neq l, \\ & \quad j\equiv 2(mod\,3)\ and\ k\equiv 1(mod\,3), \\ & \quad j\equiv 0(mod\,3)\ and\ k\equiv 0(mod\,3), \\ & \quad j=l\ and\ k\equiv 1(mod\,3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 1(mod\,3)\ and\ k\equiv 0(mod\,3)\ but\ j\neq l, \\ & \quad j\equiv 2(mod\,3)\ and\ k\equiv 2(mod\,3), \\ & \quad j\equiv 0(mod\,3)\ and\ k\equiv 1(mod\,3), \\ & \quad j=l\ and\ k\equiv 2(mod\,3) \\ (0,0,1) & for \quad otherwise \end{cases}$$

The cases when $l\equiv 0(mod\,3)\ and\ m\equiv 0,2(mod\,3)$ are similar to Subcase 1 where we interchange l and m provide the coloring. The cases when $l\equiv 0(mod\,3)\ and\ m\equiv 1(mod\,3)$ are similar to Subcase 2 where we interchange l and m provide the coloring. As a result, the family $\Gamma=\{\gamma_1,\gamma_2,\gamma_3\}$ guarantees that the SVNRVC conditions are met. Families with fewer than three points did not meet our criterion.

Case 2 : When otherwise i. e., $3\nmid lm$.

Let $\Gamma=\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$ be the collection of fuzzy sets determined on vertices of $C_l\times C_m$ for $R=2$ as follows:

Subcase 1 : When l and m are even.

$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 1(mod\,2)\ and\ k\equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 1(mod\,2)\ and\ k\equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 0(mod\,2)\ and\ k\equiv 1(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 0(mod\,2)\ and\ k\equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

Subcase 2 : When l is even and m is odd.

$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 1(mod\,4)\ and\ k\equiv 1(mod\,2)\ but\ k\neq m, \\ & \quad j\equiv 2(mod\,4)\ and\ k=m, \\ & \quad j\equiv 3(mod\,4)\ and\ k\equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 1(mod\,4)\ and\ k\equiv 0(mod\,2), \\ & \quad j\equiv 3(mod\,4)\ and\ k\equiv 1(mod\,2)\ but\ k\neq m, \\ & \quad j\equiv 0(mod\,4)\ and\ k=m \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 2(mod\,4)\ and\ k\equiv 1(mod\,2)\ but\ k\neq m, \\ & \quad j\equiv 1(mod\,4)\ and\ k=m, \\ & \quad j\equiv 0(mod\,4)\ and\ k\equiv 0(mod\,2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & for \quad j\equiv 2(mod\,4)\ and\ k\equiv 0(mod\,2), \\ & \quad j\equiv 2(mod\,4)\ and\ k=m, \\ & \quad j\equiv 0(mod\,4)\ and\ k\equiv 1(mod\,2)\ but\ k\neq m \\ (0,0,1) & for \quad otherwise \end{cases}$$

The cases when l is odd and m is even are similar to Subcase 2 where we interchange l and m provide the coloring. As a result, the family $\Gamma=\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$ guarantees that the SVNRVC conditions are met. Families with fewer than four points did not meet our criterion. □

Theorem 3.3. *Let $m\geq 2$, then the single valued neutrosophic dynamic chromatic number of the cartesian product of complete SVNG K_m with complete SVNG K_m is*

$$\chi_2^v(K_m\times K_m) = \begin{cases} 4 & when \quad m=2 \\ m & when \quad otherwise \end{cases}.$$

Proof. Consider complete SVNG $K_m=(R_1,S_1)$ with SVN vertex set $R_1=\{w_1,w_2,\cdots,w_m\}$ and another complete SVNG $K_m=(R_2,S_2)$ with SVN vertex set $R_2=\{z_1,z_2,\cdots,z_m\}$. Then the cartesian product of complete SVNG K_m with complete SVNG K_m is SVNG $K_m\times K_m=(R,S)$ with SVN vertex set $R=\{(w_j,z_k):1\leq j,k\leq m\}$ and edge set defined as in the definition 2.8.

Case 1 : When $m=2$.

Let $\Gamma=\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$ be the collection of fuzzy sets determined on vertices of $K_2\times K_2$ for $R=2$ as follows:

$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_1,z_1)), i_R((w_1,z_1)), f_R((w_1,z_1)) & \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_1,z_2)), i_R((w_1,z_2)), f_R((w_1,z_2)) & \\ (0,0,1) & for \quad otherwise \end{cases}$$

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$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_2,z_1)), i_R((w_2,z_1)), f_R((w_2,z_1)) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4((w_j,z_k)) = \begin{cases} (m_R((w_2,z_2)), i_R((w_2,z_2)), f_R((w_2,z_2)) \\ (0,0,1) & for \quad otherwise \end{cases}$$
As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ ensures that SVNRVC requirements are met. Families with fewer than four points did not match our defining criteria. Hence $\chi_2^v(K_2 \times K_2) = 4$.
Case 2 : When otherwise.
Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_m\}$ be the collection of fuzzy sets determined on vertices of $K_m \times K_m$ for $R = 2$ as follows:
$$\gamma_1((w_j,z_k)) = \begin{cases} (m_R((w_1,z_1)), i_R((w_1,z_1)), f_R((w_1,z_1)) & , \\ (m_R((w_{m-(j-2)},z_j)), i_R((w_{m-(j-2)},z_j)), f_R((w_{m-(j-2)},z_j)) & when \quad j=2,3,\cdots,m \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & when \quad j+k=3, \\ (m_R((w_{m-(j-3)},z_j)), i_R((w_{m-(j-3)},z_j)), f_R((w_{m-(j-3)},z_j)) & when \quad j=3,4,\cdots,m \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & when \quad j+k=4, \\ (m_R((w_{m-(j-4)},z_j)), i_R((w_{m-(j-4)},z_j)), f_R((w_{m-(j-4)},z_j)) & when \quad j=4,5,\cdots,m \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & when \quad j+k=5, \\ (m_R((w_{m-(j-5)},z_j)), i_R((w_{m-(j-5)},z_j)), f_R((w_{m-(j-5)},z_j)) & when \quad j=5,6,\cdots,m \\ (0,0,1) & for \quad otherwise \end{cases}$$
$\vdots$
$$\gamma_{m-1}((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & when \quad j+k=m, \\ (m_R((w_m,z_m)), i_R((w_m,z_m)), f_R((w_m,z_m)) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_m((w_j,z_k)) = \begin{cases} (m_R((w_j,z_k)), i_R((w_j,z_k)), f_R((w_j,z_k)) & when \quad j+k=m+1, \\ (0,0,1) & for \quad otherwise \end{cases}$$
As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_m\}$ ensures that SVNRVC requirements are met. Families with fewer than m points did not match our defining criteria. Hence $\chi_2^v(K_m \times K_m) = m$ when $m \geq 2$. ☐

Theorem 3.4. *Let $m \geq 3$, then the single valued neutrosophic dynamic chromatic number of the cartesian product of complete SVNG K_m with cycle C_m is $\chi_2^v(K_m \times C_m) = m$.*

Proof. Consider complete SVNG $K_m = (R_1, S_1)$ with SVN vertex set $R_1 = \{w_1, w_2, \cdots, w_m\}$ and cycle $C_m = (R_2, S_2)$ with SVN vertex set $R_2 = \{z_1, z_2, \cdots, z_m\}$. Then the cartesian product of complete SVNG K_m with cycle C_m is SVNG $K_m \times C_m = (R, S)$ with SVN vertex set $R = \{(w_j, z_k) : 1 \leq j, k \leq m\}$ and edge set defined as in the definition 2.8. The single valued neutrosophic dynamic coloring here is same as Case 2 of the previous theorem. ☐

Theorem 3.5. *Let $l, m \geq 2$, then the single valued neutrosophic dynamic chromatic number of the join of complete SVNG K_l with complete SVNG K_m is $\chi_2^v(K_l + K_m) = l + m$.*

Proof. Consider complete SVNG $K_l = (R_1, S_1)$ with SVN vertex set $R_1 = \{w_1, w_2, \cdots, w_l\}$ and complete SVNG $K_m = (R_2, S_2)$ with SVN vertex set $R_2 = \{z_1, z_2, \cdots, z_m\}$. Then the join of complete SVNG K_l with complete graph K_m is SVNG $K_l + K_m = (R, S)$ with SVN vertex set $R = \{w_1, w_2, \cdots, w_l, z_1, z_2, \cdots, z_m\}$. The membership, indeterminacy and falsity functions of the vertices of $K_l + K_m$ is determined using the definition in 2.9 and edge set is also determined in the same way.
Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{l+m}\}$ be the collection of fuzzy sets determined on vertices of $K_l \times K_m$ for $R = 2$ as follows:
For $1 \leq j \leq l$:
$$\gamma_j(w_i) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) \\ (0,0,1) & for \quad otherwise \end{cases}$$
For $1 \leq k \leq m$:
$$\gamma_{l+k}(z_i) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) \\ (0,0,1) & for \quad otherwise \end{cases}$$
As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{l+m}\}$ ensures that SVNRVC requirements are met. Families with fewer than $l + m$ points did not match our defining criteria. Hence $\chi_2^v(K_l + K_m) = l + m$. ☐

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Theorem 3.6. *Let $l \geq 2$ and $m \geq 3$, then the single valued neutrosophic dynamic chromatic number of join of complete SVNG K_l with cycle C_m is* $\chi_2^v(K_l + C_m) = \begin{cases} l+2 & when \quad m \, is \, even \\ l+3 & when \quad m \, is \, odd \end{cases}$.

Proof. Consider complete SVNG $K_l = (R_1, S_1)$ with SVN vertex set $R_1 = \{w_1, w_2, \cdots, w_l\}$ and cycle $C_m = (R_2, S_2)$ with SVN vertex set $R_2 = \{z_1, z_2, \cdots, z_m\}$. Then the join of complete SVNG K_l with cycle C_m is SVNG $K_l + C_m = (R, S)$ with SVN vertex set $R = \{w_1, w_2, \cdots, w_l, z_1, z_2, \cdots, z_m\}$. The membership, indeterminacy and falsity functions of the vertices of $K_l + C_m$ is determined using the definition in 2.9 and edge set is also determined in the same way.
Case 1 : When m is even.
Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{l+2}\}$ be the collection of fuzzy sets determined on vertices of $K_l \times C_m$ when m is even for $R = 2$ as follows:
For $1 \leq j \leq l$:

$$\gamma_j(w_i) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_{l+1}(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 1(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_{l+2}(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{l+2}\}$ ensures that SVNRVC requirements are met. Families with fewer than $l+2$ points did not match our defining criteria. Hence $\chi_2^v(K_l + C_m) = l + 2$ when m is even.
Case 2 : When m is odd.
Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{l+3}\}$ be the collection of fuzzy sets determined on vertices of $K_l \times C_m$ when m is even for $R = 2$ as follows:
For $1 \leq j \leq l$:

$$\gamma_j(w_i) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_{l+1}(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 1(mod\, 2) \; but \; k \neq m \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_{l+2}(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_{l+3}(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_{l+3}\}$ ensures that SVNRVC requirements are met. Families with fewer than $l+3$ points did not match our defining criteria. Hence $\chi_2^v(K_l + C_m) = l + 3$ when m is odd. □

Theorem 3.7. *Let $l, m \geq 3$, then the single valued neutrosophic dynamic chromatic number of join of cycle C_l with cycle C_m is* $\chi_2^v(C_l + C_m) = \begin{cases} 4 & when \quad l \, and \, m \, are \, even \\ 5 & when \quad l \, or \, m \, is \, odd \\ 6 & when \quad l \, and \, m \, are \, odd \end{cases}$.

Proof. Consider cycle $C_l = (R_1, S_1)$ with SVN vertex set $R_1 = \{w_1, w_2, \cdots, w_l\}$ and cycle $C_m = (R_2, S_2)$ with SVN vertex set $R_2 = \{z_1, z_2, \cdots, z_m\}$. Then the join of cycle K_l with cycle C_m is SVNG $C_l + C_m = (R, S)$ with SVN vertex set $R = \{w_1, w_2, \cdots, w_l, z_1, z_2, \cdots, z_m\}$. The membership, indeterminacy and falsity functions of the vertices of $C_l + C_m$ is determined using the definition in 2.9 and edge set is also determined in the same way.
Case 1 : When l and m are even.
Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be the collection of fuzzy sets determined on vertices of $C_l + C_m$ for $R = 2$ as follows:

$$\gamma_1(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j \equiv 1(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_2(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_3(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 1(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

$$\gamma_4(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

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As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ ensures that SVNRVC requirements are met. Families with fewer than four points did not match our defining criteria. Hence $\chi_2^v(C_l + C_m) = l + 3$ when both l and m are even.
Case 2 : When l or m are odd.
First assume the case in which l is odd and m is even. Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ be the collection of fuzzy sets determined on vertices of $C_l \times C_m$ for $R = 2$ as follows:

$$\gamma_1(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j) & for \quad j \equiv 1(mod\, 2)\; but\; j \neq l \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j = l \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 1(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_5(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$

The case in which l is even and m is odd is similar to the former by interchanging l and m to provide the coloring. As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ ensures that SVNRVC requirements are met. Families with fewer than five points did not match our defining criteria. Hence $\chi_2^v(C_l + C_m) = 5$ when l or m is odd.
Case 3 : When l and m are odd.
Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$ be the collection of fuzzy sets determined on vertices of $C_l \times C_m$ for $R = 2$ as follows:

$$\gamma_1(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j \equiv 1(mod\, 2)\; but\; j \neq l \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_2(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_3(w_j) = \begin{cases} (m_R(w_j), i_R(w_j), f_R(w_j)) & for \quad j = l \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_4(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 1(mod\, 2)\; but\; k \neq m \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_5(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k \equiv 0(mod\, 2) \\ (0,0,1) & for \quad otherwise \end{cases}$$
$$\gamma_6(z_k) = \begin{cases} (m_R(z_k), i_R(z_k), f_R(z_k)) & for \quad k = m \\ (0,0,1) & for \quad otherwise \end{cases}$$

As a result, the family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\}$ ensures that SVNRVC requirements are met. Families with fewer than six points did not match our defining criteria. Hence $\chi_2^v(C_l + C_m) = 6$ when both l and m are odd. □

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